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# MODERN BUSINESS ARITHMETIC

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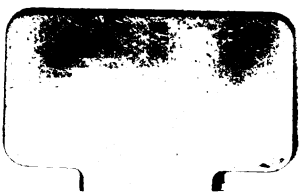

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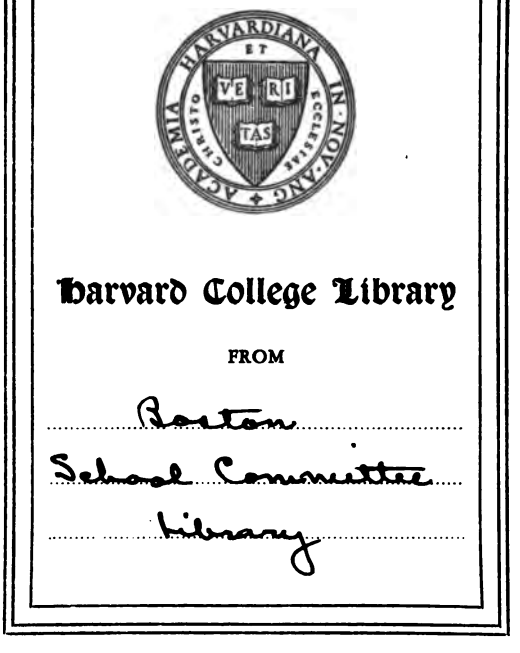


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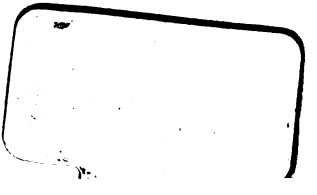

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# MODERN BUSINESS ARITHMETIC

*BRIEF COURSE*

REVISED EDITION

BY

HARRY ANSON FINNEY

PROFESSOR OF ACCOUNTING, NORTHWESTERN UNIVERSITY, CHICAGO
EDITOR, STUDENTS' DEPARTMENT, JOURNAL OF ACCOUNTANCY
FORMERLY LECTURER IN ACCOUNTING, WALTON SCHOOL OF COMMERCE
CHICAGO, ILLINOIS

AND

JOSEPH CLIFTON BROWN

PRESIDENT, STATE TEACHERS COLLEGE, ST. CLOUD, MINNESOTA
FORMERLY OF MATHEMATICS DEPARTMENT
HORACE MANN SCHOOL, NEW YORK CITY

NEW YORK
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# PREFACE

This book presents a short course in the essentials of modern business. Nothing has been included which does not stand the test of business utility. Topics and types of problems which are not commonly used in business practice have been omitted.

The illustrations and model forms were chosen to acquaint the student with actual business conditions. The problems were selected from those which arise in actual business transactions, and the methods of solution are those which are used in business practice. Many of the drill exercises are put in ruled forms such as will be met in the business world; and the student is given practice in the art of presenting business statistics and other material in a neat, concise, and attractive manner. The traditional sequence of topics has been abandoned in some cases in favor of a grouping of topics in which there is a relation of business experience.

The necessity for a high degree of accuracy and facility in computation has been fully recognized. An abundance of adequately motivated material for practice and drill in computation has been provided. The student is required frequently to check results in order to form the habit of checking, so necessary in business, and numerous exercises to be finished within a given time are included to improve speed and accuracy in computation. Since business expediency demands that many problems be solved orally, a large amount of oral work has been included.

Each chapter is adequately motivated. The material is so presented that the student sees the business need for each process and numerous practical problems based on actual business practices are found in every chapter. The problems challenge the interest of the student because they portray actual business situations. The material of the book is not experimental. Its value is vouched for by the enthusiastic testimony of scores of practical business men who have examined it, and by thousands of students who have been prepared in business arithmetic by the use of this text.

From the point of view of both material and method this book is based on actual business conditions and practices.

The authors wish to acknowledge their indebtedness to the numerous teachers, supervisors, and business men who have aided them by offering valuable suggestions and discriminating criticisms, and by furnishing materials.

HARRY ANSON FINNEY
JOSEPH CLIFTON BROWN

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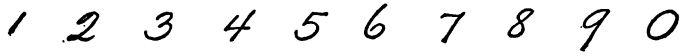
# INTRODUCTION

### To the Student

If you expect to master the arithmetic of the modern business world, you should strive to develop accuracy and speed in computation and neatness in the appearance of your work. A high degree of accuracy is indispensable. Be very careful about all of your work and form the habit of checking your results.

Work as rapidly as you can without reducing your accuracy. The material in this book has been prepared to aid you in the formation and development of proper study habits and to enable you to master the essentials of the arithmetic of modern business in as short a time as possible.

Take pride in the appearance of your work. Most of your papers should be prepared with ink, because that is the way business records are kept. When ruling is to be done, make the lines fine. Make your figures small and similar to those in the following model:



# FUNDAMENTAL PROCESSES

## CHAPTER I

### ADDITION

**1. Drill Tables.** The following table contains the forty-five combinations of two numbers, each of which is less than ten. Practice until you can state these sums in any order, without error, in less than twenty-five seconds. Do not repeat the numbers to be added. State *results only*. Try to excel your own record.

ADDITION DRILL CHART

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 6 | 4 | 6 | 1 | 7 | 4 | 7 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 9 | 8 | 4 | 9 | 2 | 7 | 6 | 9 | 5 | 4 | 5 | 6 | 1 | 8 | 5 |
| 7 | 1 | 8 | 5 | 4 | 8 | 2 | 3 | 4 | 5 | 2 | 1 | 5 | 9 | 4 |
| 8 | 4 | 8 | 9 | 5 | 9 | 3 | 4 | 9 | 5 | 7 | 3 | 6 | 9 | 7 |
| 1 | 6 | 3 | 6 | 1 | 3 | 1 | 5 | 2 | 3 | 5 | 1 | 2 | 4 | 3 |
| 9 | 7 | 3 | 6 | 7 | 8 | 8 | 7 | 2 | 9 | 8 | 6 | 6 | 8 | 7 |

The following group of thirty-six combinations contains the inversions possible, omitting the pairs of equal numbers. Practice until you can state the sums in any order without hesitation.

ADDITION DRILL CHART

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 7 | 9 | 2 | 6 | 3 | 6 | 8 | 7 | 4 | 7 | 8 |
| 3 | 5 | 5 | 1 | 5 | 1 | 1 | 5 | 4 | 1 | 6 | 3 |
| 7 | 4 | 7 | 8 | 9 | 8 | 9 | 9 | 5 | 6 | 9 | 8 |
| 1 | 2 | 2 | 4 | 3 | 6 | 7 | 2 | 2 | 4 | 4 | 7 |
| 9 | 8 | 9 | 3 | 9 | 8 | 6 | 5 | 7 | 5 | 5 | 6 |
| 8 | 1 | 6 | 2 | 1 | 2 | 3 | 4 | 3 | 1 | 3 | 2 |

NOTE. It is suggested that these charts and those which follow be used for frequent drill.

Following are the eighty-one combinations, including inversions, of two numbers, each of which is less than ten. Practice on this exercise until you can state the sums in less than a minute.

ADDITION DRILL CHART

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 1 | 3 | 4 | 4 | 5 | 2 | 4 | 6 | 5 | 1 | 3 | 3 | 5 | 9 | 7 | 5 |
| 1 | 3 | 2 | 4 | 6 | 4 | 7 | 3 | 2 | 9 | 1 | 8 | 3 | 2 | 6 | 8 | 8 |
| 2 | 1 | 1 | 8 | 4 | 8 | 6 | 4 | 8 | 9 | 5 | 3 | 7 | 8 | 4 | 9 | 3 |
| 1 | 8 | 9 | 6 | 5 | 5 | 7 | 7 | 9 | 8 | 6 | 9 | 7 | 8 | 8 | 4 | 7 |
| 7 | 2 | 9 | 1 | 3 | 9 | 8 | 9 | 1 | 6 | 9 | 2 | 4 | 5 | 8 | 6 | 6 |
| 6 | 9 | 1 | 2 | 4 | 5 | 7 | 3 | 7 | 5 | 2 | 6 | 9 | 7 | 2 | 8 | 9 |
| 4 | 2 | 7 | 6 | 2 | 8 | 8 | 7 | 3 | 2 | 1 | 2 | 9 | 1 | 7 | 7 | 5 |
| 1 | 3 | 9 | 6 | 4 | 1 | 3 | 1 | 6 | 2 | 5 | 8 | 7 | 6 | 3 | 4 | 5 |
| 6 | 8 | 4 | 2 | 5 | 6 | 9 | 5 | 1 | 7 | 3 | 7 | 3 | | | | |
| 3 | 4 | 2 | 5 | 1 | 4 | 9 | 3 | 4 | 5 | 5 | 2 | 1 | | | | |

The following exercise contains the nine digits in groups of three. Practice on these combinations until you can state the sums in less than two and a half minutes. Each column should be added both upward and downward.

ADDITION DRILL CHART

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 8 | 7 | 5 | 8 | 4 | 5 | 8 | 9 | 6 | 7 | 5 | 4 | 4 | 6 | 8 | 5 | 4 |
| 6 | 8 | 3 | 5 | 3 | 2 | 5 | 6 | 7 | 5 | 5 | 3 | 3 | 4 | 3 | 2 | 4 | 3 |
| 2 | 2 | 3 | 2 | 2 | 2 | 3 | 3 | 2 | 3 | 3 | 2 | 3 | 3 | 3 | 2 | 2 | 2 |
| 3 | 5 | 3 | 6 | 6 | 4 | 5 | 7 | 6 | 9 | 7 | 8 | 6 | 7 | 9 | 8 | 6 | 6 |
| 2 | 2 | 3 | 3 | 6 | 4 | 4 | 7 | 4 | 8 | 2 | 7 | 2 | 3 | 2 | 7 | 6 | 5 |
| 2 | 2 | 2 | 2 | 2 | 2 | 4 | 2 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 3 | 4 |
| 7 | 8 | 9 | 8 | 9 | 8 | 7 | 7 | 9 | 9 | 6 | 8 | 7 | 9 | 8 | 9 | 5 | 7 |
| 7 | 8 | 6 | 5 | 4 | 8 | 4 | 5 | 4 | 3 | 5 | 3 | 6 | 9 | 4 | 8 | 5 | 4 |
| 3 | 3 | 2 | 3 | 2 | 1 | 2 | 2 | 3 | 2 | 2 | 3 | 2 | 1 | 2 | 3 | 4 | 4 |

NOTE. Certain addition combinations are especially difficult and should therefore receive more emphasis than easy combinations such as 2 and 2, 5 and 5. The most difficult combinations are 8 and 5; 8 and 6; 8 and 7; 8 and 9; 9 and 7; 9 and 8; 7 and 6; and their reverses.

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 8 | 9 | 9 | 9 | 7 | 5 | 9 | 9 | 8 | 9 | 7 | 8 | 8 | 9 | 6 | 7 | 6 |
| 4 | 6 | 9 | 3 | 6 | 4 | 3 | 6 | 8 | 8 | 6 | 7 | 5 | 7 | 5 | 4 | 7 | 6 |
| 4 | 1 | 2 | 3 | 1 | 3 | 3 | 3 | 1 | 4 | 4 | 1 | 4 | 6 | 2 | 2 | 4 | 5 |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 6 | 7 | 8 | 9 | 9 | 9 | 8 | 7 | 8 | 9 | 9 | 5 | 7 | 9 | 9 | 8 | 9 |
| 4 | 6 | 7 | 7 | 7 | 7 | 5 | 5 | 6 | 8 | 7 | 8 | 4 | 6 | 8 | 8 | 5 | 9 |
| 4 | 4 | 6 | 4 | 5 | 7 | 3 | 5 | 5 | 7 | 6 | 5 | 3 | 3 | 6 | 8 | 2 | 8 |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 9 | 8 | 9 | 9 | 9 | 7 | 7 | 8 | 8 | 9 | 8 | 9 | 9 | 9 | 8 | 7 | 8 |
| 5 | 6 | 6 | 9 | 5 | 9 | 6 | 5 | 4 | 6 | 9 | 8 | 5 | 7 | 4 | 7 | 6 | 7 |
| 5 | 5 | 5 | 5 | 5 | 7 | 6 | 4 | 3 | 4 | 4 | 6 | 4 | 1 | 4 | 3 | 4 | 4 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 9 | 9 | 8 | 9 | 6 | 9 | 9 | 7 | 8 | 9 | 9 |
| 7 | 9 | 8 | 6 | 7 | 5 | 6 | 9 | 7 | 8 | 8 | 7 |
| 5 | 6 | 7 | 6 | 4 | 5 | 6 | 3 | 5 | 5 | 4 | 3 |

### Oral Work

Add the following and check results by adding in reverse order. In this, and in all other work in addition, state *the sums only*. For example, in adding 3, 4, 8, 9, say 7, 15, 24.

| | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. | 14. | 15. | 16. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *a.* | 3 | 5 | 7 | 2 | 9 | 1 | 6 | 8 | 5 | 4 | 9 | 3 | 5 | 5 | 7 | 9 |
| *b.* | 7 | 3 | 9 | 1 | 5 | 3 | 7 | 5 | 6 | 9 | 8 | 9 | 3 | 9 | 3 | 8 |
| *c.* | 2 | 8 | 3 | 5 | 7 | 1 | 4 | 9 | 6 | 4 | 6 | 7 | 9 | 4 | 9 | 3 |
| *d.* | 6 | 3 | 8 | 1 | 4 | 2 | 7 | 8 | 9 | 6 | 7 | 8 | 7 | 8 | 2 | 7 |
| *e.* | 6 | 3 | 7 | 5 | 2 | 9 | 5 | 1 | 7 | 4 | 8 | 4 | 1 | 9 | 8 | 5 |
| *f.* | 7 | 8 | 5 | 3 | 9 | 1 | 4 | 7 | 6 | 4 | 3 | 9 | 8 | 7 | 9 | 9 |
| *g.* | 8 | 4 | 9 | 5 | 7 | 3 | 5 | 8 | 5 | 9 | 9 | 6 | 9 | 8 | 9 | 8 |
| *h.* | 9 | 2 | 5 | 3 | 7 | 4 | 8 | 5 | 9 | 6 | 6 | 1 | 6 | 3 | 5 | 6 |

State the sums in each of the following. The first two numbers in each example should be thought of as one number. Thus, in the first example, think 10, 17.

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. | 14. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 5 | 9 | 1 | 2 | 5 | 8 | 2 | 4 | 6 | 3 | 7 | 5 | 9 |
| 2 | 4 | 1 | 6 | 7 | 4 | 1 | 7 | 3 | 2 | 8 | 4 | 3 | 9 |
| 8 | 4 | 6 | 9 | 2 | 9 | 5 | 7 | 3 | 8 | 5 | 3 | 7 | 5 |

NOTE. The sum of the numbers in the rows may be found also.

| 15. | 16. | 17. | 18. | 19. | 20. | 21. | 22. | 23. | 24. | 25. | 26. | 27. | 28. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 5 | 3 | 7 | 2 | 5 | 6 | 9 | 6 | 8 | 5 | 7 | 8 | 4 |
| 6 | 5 | 3 | 8 | 4 | 7 | 4 | 7 | 8 | 5 | 7 | 4 | 8 | 5 |
| 4 | 6 | 2 | 8 | 3 | 5 | 6 | 5 | 7 | 3 | 6 | 9 | 3 | 6 |

| 29. | 30. | 31. | 32. | 33. | 34. | 35. | 36. | 37. | 38. | 39. | 40. | 41. | 42. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 7 | 4 | 9 | 5 | 7 | 4 | 6 | 8 | 3 | 6 | 5 | 7 | 9 |
| 6 | 7 | 3 | 6 | 9 | 5 | 8 | 3 | 6 | 7 | 4 | 5 | 7 | 3 |
| 7 | 4 | 8 | 5 | 3 | 7 | 3 | 8 | 5 | 4 | 2 | 6 | 9 | 4 |

**2. Adding numbers by grouping** may increase your speed. Some rapid computers use group addition. For example,

$$\left.\begin{matrix}8\\3\end{matrix}\right\} 11$$
$$\left.\begin{matrix}4\\2\end{matrix}\right\} 6$$
$$17$$

The acquisition of speed and accuracy in group addition requires much practice. Some computers gain in speed by grouping numbers whose sum is ten.

Use group addition to find the sums in the following exercise.

This work should be done orally. Add both upward and downward.

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. | 14. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 5 | 8 | 4 | 7 | 4 | 8 | 5 | 2 | 6 | 3 | 5 | 4 | 6 |
| 3 | 2 | 2 | 3 | 4 | 3 | 6 | 7 | 3 | 7 | 7 | 8 | 9 | 6 |
| 5 | 7 | 3 | 4 | 8 | 5 | 3 | 7 | 5 | 4 | 9 | 4 | 7 | 3 |
| 4 | 3 | 6 | 2 | 3 | 5 | 7 | 6 | 4 | 8 | 4 | 7 | 3 | 5 |

| 15. | 16. | 17. | 18. | 19. | 20. | 21. | 22. | 23. | 24. | 25. | 26. | 27. | 28. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 7 | 4 | 2 | 5 | 3 | 6 | 2 | 4 | 6 | 8 | 9 | 7 | 5 |
| 3 | 6 | 4 | 8 | 5 | 3 | 7 | 6 | 4 | 8 | 5 | 7 | 3 | 6 |
| 7 | 5 | 3 | 6 | 5 | 8 | 4 | 5 | 6 | 4 | 7 | 3 | 8 | 2 |
| 7 | 6 | 3 | 4 | 1 | 7 | 4 | 7 | 4 | 8 | 9 | 5 | 3 | 7 |

| 29. | 30. | 31. | 32. | 33. | 34. | 35. | 36. | 37. | 38. | 39. | 40. | 41. | 42. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 2 | 6 | 8 | 6 | 4 | 2 | 4 | 6 | 4 | 8 | 5 | 2 | 3 |
| 3 | 6 | 2 | 5 | 8 | 9 | 5 | 3 | 7 | 5 | 3 | 5 | 7 | 9 |
| 9 | 5 | 3 | 6 | 8 | 9 | 5 | 2 | 6 | 2 | 8 | 5 | 3 | 9 |
| 5 | 2 | 7 | 4 | 2 | 7 | 3 | 7 | 9 | 4 | 2 | 7 | 4 | 6 |

| 43. | 44. | 45. | 46. | 47. | 48. | 49. | 50. | 51. | 52. | 53. |
|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 8 | 9 | 5 | 3 | 2 | 6 | 8 | 4 | 3 | 6 |
| 3 | 6 | 8 | 4 | 2 | 6 | 7 | 4 | 3 | 6 | 8 |
| 7 | 4 | 6 | 3 | 7 | 8 | 5 | 3 | 6 | 7 | 9 |
| 4 | 7 | 5 | 2 | 8 | 6 | 4 | 8 | 3 | 7 | 9 |
| 2 | 6 | 4 | 7 | 9 | 6 | 4 | 2 | 6 | 7 | 5 |
| 7 | 4 | 8 | 3 | 9 | 5 | 9 | 3 | 6 | 7 | 5 |

| 54. | 55. | 56. | 57. | 58. | 59. | 60. | 61. | 62. | 63. | 64. |
|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 3 | 2 | 8 | 5 | 3 | 7 | 3 | 7 | 5 | 9 |
| 5 | 3 | 6 | 8 | 7 | 4 | 2 | 6 | 8 | 3 | 7 |
| 4 | 5 | 6 | 4 | 7 | 3 | 8 | 4 | 2 | 8 | 3 |
| 5 | 2 | 3 | 5 | 7 | 8 | 5 | 3 | 6 | 7 | 8 |
| 4 | 7 | 6 | 4 | 8 | 9 | 2 | 6 | 3 | 7 | 5 |
| 2 | 3 | 6 | 7 | 8 | 4 | 2 | 5 | 6 | 3 | 6 |

Find the sums in examples 1–14 by the following method:

In the illustration we say, 72, 112, 119, 139, 143, 183, 186. That is, we add 72 and 40, getting 112, then add 112 and 7, getting 119; then add 119 and 20, getting 139, then 139 and 4, getting 143; then add 143 and 40, getting 183, then 183 and 3, getting 186.

43
24
47
72

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. | 14. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 24 | 46 | 37 | 53 | 58 | 67 | 48 | 39 | 68 | 73 | 66 | 81 | 93 | 65 |
| 33 | 52 | 44 | 63 | 47 | 59 | 73 | 58 | 42 | 86 | 59 | 78 | 38 | 75 |

**3. Recording Addition by Columns.** For convenience in checking, the total of each column may be recorded separately. This will enable you to resume the work where it was discontinued if you are

interrupted after having added one or more columns. Three methods are in use.

| *a.* | *b.* | *c.* |
|---|---|---|
| 369 | 369 | 369 |
| 487 | 487 | 487 |
| 352 | 352 | 352 |
| 896 | 896 | 896 |
| 294 | 294 | 294 |
| 28 | 20 | 28 |
| 37 | 37 | 39 |
| 20 | 28 | 23 |
| 2,398 | 2,398 | |

Illustrations (*a*) and (*b*) show the method of adding each column *without carrying* the tens. The column totals are then added to obtain the final result. Illustration (*c*) shows the method of adding to each column the amount carried from the column at the right.

### Written Work

Make all figures of uniform size and easily legible and place figures of the same order in the same column.

Copy and find the sums. Check each sum by adding in the reverse order.

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| 5,823 | 84,239 | 387,935 | 9,328,577 | 38,421 |
| 4,923 | 42,937 | 512,937 | 4,912,374 | 31,238 |
| 9,382 | 31,629 | 491,327 | 5,823,746 | 312 |
| 5,128 | 49,238 | 395,825 | 4,923,748 | 273,845 |
| 3,258 | 83,153 | 416,239 | 7,239,482 | 3,9547 |
| 7,362 | 42,841 | 482,423 | 1,423,849 | 315,288 |
| 8,127 | 52,384 | 923,857 | 8,237,421 | 4,234 |
| 4,239 | 94,856 | 412,831 | 9,423,748 | 21,582 |

| 6. | 7. | 8. | 9. | 10. | 11. |
|---|---|---|---|---|---|
| 27 | 371 | 419 | 372 | 97 | 996 |
| 83 | 927 | 82 | 496 | 36 | 37 |
| 95 | 482 | 92 | 18 | 547 | 321 |
| 84 | 413 | 412 | 739 | 969 | 493 |
| 82 | 958 | 892 | 847 | 38 | 782 |
| 15 | 285 | 9216 | 96 | 437 | 946 |
| 81 | 381 | 594 | 382 | 692 | 531 |
| 48 | 294 | 5 | 75 | 99 | 28 |
| 79 | 148 | 592 | 989 | 831 | 327 |

Each student should prepare original examples for addition, following the instruction of the teacher. Exchange papers, criticize the form, and find the sums. Check all results. The method explained in article 3 should be used rather frequently in checking.

The ability to give and to take dictation of numbers should be developed. The accountant must acquire the ability to copy and to take from dictation figures for use in computation. Numbers may be dictated by the teacher, or by members of the class.

In preparing certain kinds of reports it is sometimes necessary to add columns horizontally. When additions are made in this way time in recopying of figures is frequently saved. Care must be taken to see that units are added to units, tens to tens, etc.

Add horizontally and vertically. See that the results check.

**1.**

427 + 823 + 579 =
386 + 547 + 387 =
542 + 786 + 395 =
746 + 408 + 548 =
902 + 875 + 386 = ___
__ + __ + __ = __

**2.**

974 + 386 + 277 + 385 =
276 + 843 + 946 + 837 =
478 + 397 + 852 + 478 =
309 + 974 + 706 + 840 =
768 + 897 + 467 + 904 = ___
__ + __ + __ + __ = __

**3.**

9,642 + 8,047 + 9,485 =
8,731 + 9,206 + 3,479 =
4,926 + 7,348 + 7,496 =
8,397 + 9,479 + 8,744 = ___
___ + ___ + ___ = __

**4.**

3,870 + 3,947 + 8,074 =
4,967 + 7,843 + 5,765 =
3,856 + 5,768 + 3,987 =
4,978 + 9,746 + 8,456 = ___
___ + ___ + ___ = __

1. The following are the number of men, women, and children who paid admission to a county fair:

| Paid Admissions | Mon. | Tues. | Wed. | Th rs. | Fri. | Sat. | Totals |
|---|---|---|---|---|---|---|---|
| Men. . . . . | 1,426 | 2,874 | 3,947 | 4,068 | 3,872 | 2,596 | |
| Women . . . | 897 | 1,463 | 1,896 | 2,947 | 2,865 | 1,975 | |
| Children . . . | 483 | 796 | 1,267 | 2,796 | 3,214 | 847 | |
| | | | | | | | |

Rule a suitable blank form and copy the above statistics.

Find the total number of admissions paid each day and the total number of men, of women, and of children for the week.

### Written Work

The following table shows the weekly sales report made by a number of salesmen during a certain week.

| DAY | J.C. OLSEN | D.R. BACON | F.G. BATES | D.O. ESTEY | TOTAL |
|---|---|---|---|---|---|
| MONDAY | 216 45 | 538 74 | 816 98 | 319 47 | |
| TUESDAY | 358 93 | 619 84 | 725 48 | 629 47 | |
| WEDNESDAY | 412 57 | 593 27 | 639 74 | 725 84 | |
| THURSDAY | 385 13 | 634 96 | 744 47 | 594 72 | |
| FRIDAY | 518 92 | 718 96 | 639 37 | 584 38 | |
| SATURDAY | 639 47 | 639 62 | 729 47 | 846 94 | |
| TOTAL | | | | | |

Find the total sales made by each salesman during the week, and enter these totals in the proper places at the bottom of the table.

Find the total sales made each day, and enter these totals in the column at the right.

Find the total sales made by all salesmen during the week.

The grand total should be found in two ways: by adding the totals at the foot of the blank, and those at the right. If these sums agree,

you may assume that the additions are correct; if they do not agree, you should find the error.

**4. How to Rule a Blank Form.** Ruling the forms for your written work will furnish valuable practice as a preparation for bookkeeping in school and in business. Follow the instructions given below:

*a.* Make the columns just wide enough for the figures, — about seven figures to an inch. Remember that the total may have one or two more figures than any of the addends (numbers to be added).

*b.* Make the horizontal lines parallel and about $\frac{1}{4}$ of an inch apart.

*c.* Make the spaces for headings and totals $\frac{3}{8}$ of an inch deep.

*d.* Place the form in the middle of the paper.

**5. How to Enter Statistics.** Much of the clerk's work in many business offices consists of entering statistics on ruled forms.

*a.* Always write figures on the line.

*b.* Point off figures in groups of three.

*c.* Make figures of uniform size and easily legible.

*d.* Keep the figures close to the right side of the column.

*e.* Keep the columns vertical: units above units, tens above tens, etc.

*f.* Write the figures at equal distances from each other.

*g.* Verify the copying by checking with the original.

*h.* Be very careful not to omit a figure or to copy figures in the wrong order.

### Written Work

**1.** The following table shows the delivery records of a store running five wagons. A record of daily deliveries is kept and the week's totals are found. If the summary shows that any one wagon is required to make an unreasonably large number of deliveries, a change in the routes may be necessary. By finding the total number of deliveries made each week, the average cost per delivery may be computed.

Rule a blank form similar to this model; copy the statistics, and find:

*a.* The total number of deliveries made during the week by each wagon.

*b.* The total number of deliveries made each day.

*c.* The total number of deliveries for the week.

| DAY | WAGON NO 1 | WAGON NO 2 | WAGON NO 3 | WAGON NO 4 | WAGON NO 5 | TOTAL |
|---|---|---|---|---|---|---|
| MONDAY | 243 | 319 | 273 | 143 | 241 | |
| TUESDAY | 216 | 326 | 391 | 178 | 216 | |
| WEDNESDAY | 257 | 287 | 294 | 121 | 322 | |
| THURSDAY | 301 | 291 | 341 | 147 | 296 | |
| FRIDAY | 294 | 304 | 303 | 218 | 213 | |
| SATURDAY | 318 | 361 | 420 | 223 | 301 | |
| TOTAL | | | | | | G.T. |

**2.** Rule a blank form similar to the first of the tables on this page, copy the data, and find the totals indicated.

WEEKLY SALES REPORT BY DEPARTMENTS

| DEPT. No. | MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY | TOTAL |
|---|---|---|---|---|---|---|---|
| 1 | $932.87 | $823.75 | $239.75 | $724.16 | $295.16 | $840.19 | |
| 2 | 829.47 | 584.92 | 395.58 | 645.97 | 396.74 | 492.10 | |
| 3 | 723.85 | 486.47 | 523.78 | 419.52 | 296.15 | 492.58 | |
| 4 | 836.47 | 594.38 | 723.91 | 391.16 | 749.15 | 581.07 | |
| 5 | 486.74 | 432.95 | 312.85 | 492.43 | 723.85 | 239.57 | |
| 6 | 728.74 | 483.96 | 472.58 | 439.17 | 396.14 | 211.64 | |
| 7 | 385.75 | 539.59 | 542.87 | 734.67 | 723.96 | 447.92 | |
| 8 | 648.57 | 823.85 | 514.29 | 439.76 | 824.57 | 238.11 | |
| 9 | 923.37 | 239.73 | 629.85 | 329.54 | 385.47 | 294.10 | |
| 10 | 473.48 | 865.94 | 518.59 | 172.39 | 749.80 | 663.19 | |
| Total | | | | | | | Grand Total |

## Problems

Enter the items of the following problems on suitable ruled forms and find the totals as required.

**1.** In a four day contest to raise funds for charitable purposes the four contesting teams reported the following collections: Compute the

daily totals, the totals for each team for the week and the grand total. Team A: Monday, $1,436; Tuesday, $798; Wednesday, $1,748; Thursday, $1,493. Team B; Monday, $2,473; Tuesday, $1,536; Wednesday, $894; Thursday, $1,592. Team C: Monday, $480; Tuesday, $1,735; Wednesday, $1,849; Thursday, $2,178. Team D: Monday, $485; Tuesday, $1,340; Wednesday, $968; Thursday, $1,745.

2. The daily sales for a week in a department store were as follows:

Shoes, $485.50, $792, $535.50, $815.25, $245, and $380.
Furniture, $1,845, $1,455, $2,580, $960, $3,590, and $1,650.
Groceries, $580.40, $965, $740.30, $380, $875.35, and $760.80.
Hardware, $495.25, $815,30, $748.60, $614.85, $786.30, and $514.85.
Clothing, $985, $1,465.50, $1,730.50, $984.50, $658, and $1,149.50.
Notions, $97.40, $132.80, $218.45, $114.15, $112.50, and $242.30.

Compute the total sales for each day for the store, the total sales of each kind for the week, and the grand total for the week.

It is of the utmost importance that business computations be correct. The computer must check his own work. He must know that his results are accurate. The *habit* of checking all computations should be formed.

There are several methods by which addition may be checked. Some of these checks have been noted. One of the common checks is to add each column twice and in reverse order. *Never try to check by adding in the same order.*

## Checking Addition by Casting Out Nines

**6. Excess of Nines.** The remainder after dividing any number by 9 is called the *Excess of Nines.* If the number 47 is divided by 9, the excess of nines is 2. The excess of nines in 30 is 3, because when 30 is divided by 9, there is a remainder of 3. When we " cast out nines " from a number, we divide the number by 9, and indicate the excess.

What is the excess of nines in 17? 14? 39? 45? 117? 23?

**7. Method of Checking.** If two numbers, each exactly divisible by 9, are added, their sum also is divisible by 9. If one number with an excess of 3 is added to a number with an excess of 2, the sum will have an excess of 5. *The excess of nines in a sum is equal to the excess in the sum of the excesses of the numbers added.*

**Examples.** Add 30 and 19; check the result by casting out nines.

SOLUTION: 30 + 19 = 49.
The excess of nines in the sum, 49, is 4.
The excess in 30 is 3;
The excess in 19 is 1.
The sum of the excesses of the numbers added is 4, therefore the addition checks.

For convenience, the work may be arranged as follows:

| | NUMBERS ADDED | EXCESSES |
|---|---|---|
| *a.* | 30 | 3 |
| | 19 | 1 |
| Sum | 49 | 4 |
| Excess of sum | 4 | |

| | NUMBERS ADDED | EXCESSES |
|---|---|---|
| *b.* | 23 | 5 |
| | 13 | 4 |
| Sum | 36 | 9 |
| Excess of sum | 0 | 0 |

| | NUMBERS ADDED | EXCESSES |
|---|---|---|
| *c.* | 49 | 4 |
| | 34 | 7 |
| Sum | 83 | 11 |
| Excess of sum | 2 | 2 |

**8. Short Method of Finding the Excess.** The excess of nines in any number may be found by the following method:

*Add the digits composing the number. If this sum is composed of two or more digits add them. Continue until a result of one digit is secured.*

**Examples.** 1. What is the excess of nines in 25?

SOLUTION: 2 + 5 = 7, the excess.

2. What is the excess of nines in 328?

SOLUTION: 3 + 2 + 8 = 13
1 + 3 = 4, the excess.

The addition of digits to find the excess will be further simplified by observing the following suggestions:

*a.* Ignore 9's, and combinations which add to 9 or multiples of 9.

**Example.** What is the excess of nines in 9,457?

SOLUTION: Ignore 9 and 5 + 4; the excess is 7.

*b.* When the addition results in a number of two digits, add the digits and proceed as before.

**Example.** What is the excess of nines in 7,528?

SOLUTION: $7 + 5 = 12$; $1 + 2 = 3$;
$3 + 2 + 8 = 13$; $1 + 3 = 4$, the excess.

**9. Limitations of the Method.** Casting out nines is not an absolute check for any process. It will not detect an error or omission of 9 or any multiple of 9, neither will it disclose an interchange of digits, such as 763 for 673. If a result is wrong in such a way as to leave the excesses unchanged, this method of checking will not detect the error.

### Written Work

1. Add and check:

| NUMBERS TO BE ADDED | EXCESSES |
|---|---|
| 213 | ? |
| 412 | ? |
| 617 | ? |
| 362 | ? |
| 145 | ? |
| 215 | ? |
| 829 | ? |
| 657 | ? |
| 923 | ? |
| ? | ? |

Add and check by casting out the 9's.

| 2. | 3. | 4. | 5. |
|---|---|---|---|
| 4,796 | 1,436 | 7,029 | 48,987 |
| 8,342 | 9,087 | 8,516 | 72,806 |
| 7,987 | 2,635 | 3,094 | 39,081 |
| 3,246 | 4,097 | 7,290 | 79,420 |
| 2,874 | 386 | 8,096 | 63,945 |

### CHECKING BY CASTING OUT ELEVENS

The method of checking addition by casting out elevens will be clear from the following:

The excess of elevens in any number is computed by finding the difference between the sum of the digits in the odd and in the even

places in the number. Thus, the excess of 11's in 47,853 is 15 − 12 or 3 (15 is the sum of the digits in the odd orders, that is, the 1st, 3d, etc., and 12 is the sum of the digits in the even orders). If the sum of the digits in the even orders exceeds the sum of those in the odd orders, add 11, or a multiple of 11, to the sum of the digits in the odd orders. Thus, the excess of 11's in 9,354 is

$$\begin{aligned} 4 + 3 + 11 &= 18 \\ 5 + 9 &= 14 \\ \text{Excess} & \quad 4 \end{aligned}$$

The method is further illustrated by the following:

| Number | Excess |
|---|---|
| 478 | 5 |
| 397 | 1 |
| 2,436 | 5 |
| 9,854 | 9 |
| 13,165 | 20 |

The excess in the sum (13,165) and in the sum of the excesses (20) is 9.

Add and check by casting out 11's.

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| 7,286 | 2,908 | 8,079 | 19,463 |
| 3,974 | 4,736 | 3,897 | 23,879 |
| 8,706 | 7,895 | 6,543 | 92,785 |
| 4,398 | 6,789 | 7,897 | 43,970 |
| 2,764 | 3,470 | 3,788 | 87,698 |

Add and check by casting out 11's.

| 5. | 6. | 7. | 8. |
|---|---|---|---|
| 1,846 | 9,086 | 4,987 | 894 |
| 973 | 3,908 | 3,896 | 1,798 |
| 8,427 | 7,463 | 7,298 | 3,984 |
| 9,208 | 9,287 | 8,407 | 5,049 |
| 764 | 438 | 328 | 7,648 |
| 3,792 | 9,986 | 1,476 | 3,507 |

**9.** Rule a suitable blank form for the following: At the right of each column of figures rule a narrow column in which to indicate the excesses.

Find the totals as indicated; check additions by casting out nines or elevens.

VALUE OF FARM LANDS, BUILDINGS, AND IMPLEMENTS IN THE UNITED STATES IN A RECENT YEAR

| DIVISION | LAND | EX. | BUILDINGS | EX. | IMPLEMENTS | EX. | TOTAL | EX. |
|---|---|---|---|---|---|---|---|---|
| New England . . . | 382,134,424 | | 336,410,384 | | 50,798,826 | | | |
| Middle Atlantic. . | 1,462,321,005 | | 980,628,098 | | 167,480,384 | | | |
| E. N. Central. . . | 7,231,699,114 | | 1,642,292,480 | | 268,806,550 | | | |
| W. N. Central . . | 10,052,560,913 | | 1,562,104,957 | | 368,635,544 | | | |
| South Atlantic . . | 1,883,349,675 | | 603,086,799 | | 98,230,147 | | | |
| E. S. Central . . . | 1,326,826,864 | | 411,570,975 | | 75,339,333 | | | |
| W. S. Central. . . | 2,716,098,530 | | 412,498,352 | | 119,720,377 | | | |
| Mountain . . . . | 1,174,370,096 | | 145,026,777 | | 49,429,975 | | | |
| Pacific . . . . . | 2,246,313,548 | | 231,832,706 | | 66,408,647 | | | |
| U. S. Totals . . | | | | | | | Grand Total | |

**10.** The following table shows a method used by department stores to determine the cost of the purchases made for various departments. For example, Invoice No. 1, purchased on January 2, included goods for the six departments of the store. Invoice No. 2 included goods for only three departments.

The total amount of the invoice may be found by horizontal addition.

The total purchases for each department may be found by vertical addition.

Rule a blank suitable for this material, enter the data, and find:

*a.* The total of each invoice.

*b.* The total purchases for each department.

*c.* The total purchases for all departments.

| INVOICE NO. | DATE | DEPT. 1 | DEPT. 2 | DEPT. 3 | DEPT. 4 | DEPT. 5 | DEPT. 6 | TOTAL |
|---|---|---|---|---|---|---|---|---|
| 1 | Jan. 2 | 129 30 | 340 60 | 290 39 | 360 89 | 240 30 | 376 80 | |
| 2 | Jan. 3 | 276 90 | | 347 80 | | | 174 44 | |
| 3 | Jan. 3 | | 295 67 | 298 87 | 350 27 | 289 67 | 287 57 | |
| 4 | Jan. 5 | 246 32 | 238 47 | 350 27 | 347 25 | 250 68 | 276 57 | |
| 5 | Jan. 6 | 231 53 | 350 21 | | 378 89 | | 190 23 | |
| 6 | Jan. 8 | 212 23 | 325 67 | 345 67 | 367 78 | 310 57 | 185 67 | |
| 7 | Jan. 8 | 227 85 | | 346 78 | 357 89 | 298 67 | | |
| 8 | Jan. 10 | | | | | | 170 67 | |
| 9 | Jan. 11 | 160 87 | 325 67 | 287 67 | 367 57 | 287 00 | 189 00 | |
| 10 | Jan. 15 | 170 97 | 341 28 | 321 67 | 378 98 | 287 89 | | |
| 11 | Jan. 15 | 180 89 | | 333 33 | | 278 90 | 234 86 | |
| 12 | Jan. 15 | | 345 67 | 345 67 | 328 98 | 300 00 | 350 90 | |
| Total | | | | | | | | |

## General Review in Addition

### Oral Work

1. *a.* $27 + 32 + 14 + 43 + 77$
   *b.* $73 + 24 + 18 + 39 + 56$
   *c.* $64 + 29 + 54 + 46 + 18$
   *d.* $78 + 56 + 47 + 54 + 36$

2. What do you consider the best method for checking addition? Why?

3. Five salesmen for Williams & Company reported daily sales as follows. Compute the total daily sales, the total sales for the week, and the grand total.

| Salesman | Mon. | Tues. | Wed. | Thurs. | Fri. | Sat. | Total |
|---|---|---|---|---|---|---|---|
| M. R. Jones . . . | $248.90 | $368.40 | $790.30 | $240.80 | $179.60 | $280.90 | |
| J. B. Adams . . . | 142.00 | 263.90 | 384.20 | 78.60 | 324.80 | 291.40 | |
| S. W. Evans . . . | 375.60 | 264.80 | 372.95 | 147.80 | 265.30 | 282.90 | |
| A. R. Clark . . . | 246.10 | 312.25 | 417.19 | 541.60 | 324.80 | 197.60 | |
| L. C. Briggs . . . | 472.30 | 198.70 | 216.45 | 325.65 | 287.30 | 214.35 | |

4. The receipts at a retail grocery on Monday were $243.80; Tuesday, $196.50; Wednesday, $342.64; Thursday, $146.90; Friday, $215.80; and Saturday, $280.40. What were the total receipts for the week?

5. The population of the largest ten cities in the United States in 1920 was as indicated below. Compute the total population of these cities.

| City | Population | City | Population |
|---|---|---|---|
| New York . . . . . | 5,621,151 | St. Louis . . . . . | 772,897 |
| Chicago . . . . . | 2,701,705 | Boston . . . . . | 748,060 |
| Philadelphia . . . . . | 1,823,158 | Baltimore . . . . . | 733,826 |
| Detroit. . . . . | 993,739 | Pittsburgh. . . . . | 588,193 |
| Cleveland . . . . . | 796,836 | Los Angeles . . . . . | 576,673 |

## CHAPTER II

### SUBTRACTION

**10. Drill Tables.** Daily practice on the following subtraction combinations will increase your speed and accuracy.

Drill on these combinations until you can state all of the results in any order in less than thirty seconds.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 9 | 6 | 7 | 4 | 7 | 3 | 6 | 8 | 7 | 8 | 4 | 5 | 8 |
| 2 | 1 | 5 | 3 | 1 | 2 | 4 | 4 | 3 | 6 | 4 | 3 | 8 | 2 | 1 | 3 |
| 2 | 7 | 9 | 5 | 9 | 5 | 6 | 5 | 9 | 4 | 4 | 9 | 8 | 2 | 6 | 8 |
| 1 | 7 | 4 | 5 | 5 | 4 | 3 | 2 | 7 | 4 | 3 | 8 | 5 | 2 | 4 | 6 |
| 5 | 8 | 4 | 8 | 6 | 7 | 7 | 9 | 8 | 1 | 3 | 3 | 5 | 7 | 9 | 6 |
| 3 | 8 | 1 | 6 | 2 | 1 | 6 | 1 | 2 | 1 | 2 | 1 | 5 | 7 | 6 | 5 |

Drill on the following until you can state all of the results in any order in less than thirty seconds.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 16 | 14 | 15 | 18 | 13 | 17 | 12 | 11 | 16 | 11 | 12 | 17 | 11 |
| 8 | 6 | 8 | 9 | 4 | 9 | 5 | 9 | 7 | 6 | 6 | 8 | 4 |
| 13 | 12 | 15 | 16 | 11 | 12 | 12 | 12 | 11 | 13 | 15 | 11 | 12 |
| 5 | 3 | 6 | 9 | 3 | 3 | 8 | 7 | 7 | 9 | 7 | 8 | 9 |
| 11 | 12 | 13 | 14 | 14 | 13 | 14 | 15 | 14 | 11 | 13 | 17 | 18 |
| 5 | 4 | 7 | 8 | 7 | 6 | 5 | 9 | 9 | 2 | 8 | 6 | 7 |

In subtraction " borrowing " is frequently necessary.

Place a pad under these examples, and make the subtractions.

| 1. | 2. | 3. | 4. | 5. | 6. |
|---|---|---|---|---|---|
| 479 | 536 | 4,279 | 14,973 | 3,084 | 12,379 |
| 284 | 198 | 3,682 | 8,297 | 1,596 | 3,084 |

| 7. | 8. | 9. | 10. | 11. |
|---|---|---|---|---|
| 42,937 | 946,320 | 37,649 | 947,302 | 794,680 |
| 39,048 | 298,452 | 20,863 | 842,943 | 409,694 |

**11. Checking Subtraction.** Subtraction may be conveniently checked by either of the following methods:

(*a*) Difference + Subtrahend = Minuend.
(*b*) Minuend − Difference = Subtrahend.

**Example.** Subtract 21 from 78.

SOLUTION. 78 Minuend
21 Subtrahend
57 Difference

CHECK. *a*. 57 Difference
21 Subtrahend
78 Minuend

*b*. 78 Minuend
57 Difference
21 Subtrahend

**Written Work**

Perform the subtractions indicated; check the first five by method (*a*), and the last five by method (*b*),

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| 489 | 7,932 | 82,578 | 63,217 | 49,216 |
| 392 | 494 | 58,214 | 41,283 | 21,953 |

| 6. | 7. | 8. | 9. | 10. |
|---|---|---|---|---|
| 58,310 | 40,513 | 53,821 | 79,608 | 184,732 |
| 49,168 | 39,458 | 41,927 | 20,973 | 75,806 |

*Always check results in substraction.*

11. The following data show the sales in the various departments of a large store for the month of August, 1921, and the month of August, 1922.

Rule a blank similar to the following:

COMPARATIVE SALES RECORD

| DEPARTMENT No. | AUGUST, 1921 | AUGUST, 1922 | INCREASE OR DECREASE |
|---|---|---|---|
| 1 | $ 5,483.85 | $ 5,834.75 | |
| 2 | 7,239.74 | 8,238.74 | |
| 3 | 15,493.67 | 14,835.92 | |
| 4 | 9,834.56 | 10,385.47 | |
| 5 | 23,842.12 | 22,147.94 | |
| 6 | 15,858.43 | 16,614.34 | |
| 7 | 6,395.75 | 6,114.32 | |
| 8 | 19,432.86 | 22,324.73 | |
| 9 | 9,445.87 | 10,302.95 | |
| 10 | 43,221.42 | 46,932.56 | |
| 11 | 29,496.56 | 31,416.47 | |
| 12 | 18,853.16 | 17,542.85 | |
| 13 | 22,542.19 | 25,436.83 | |
| 14 | 25,746.91 | 24,427.85 | |
| 15 | 18,422.16 | 21,541.86 | |
| Total | | | |

Enter the sales in the proper column of the blank.

Find the increase or decrease in the amount of the sales in each department for August, 1922, over the sales in the same department for August, 1921.

Enter increases in black ink; decreases in red ink.

Find the net increase for the entire store.

Do the results check?

**12.** The following shows the total population of the United States for the years indicated.

| | | |
|---|---|---|
| 1790 — 3,929,214; | 1840 — 17,069,453; | 1890 — 62,947,714; |
| 1800 — 5,308,483; | 1850 — 23,191,876; | 1900 — 75,994,575; |
| 1810 — 7,239,881; | 1860 — 31,443,321; | 1910 — 91,972,266; |
| 1820 — 9,638,453; | 1870 — 38,558,371; | 1920 — 105,708,771. |
| 1830 — 12,866,020; | 1880 — 50,155,783; | |

Rule a blank form to show the year the census was taken, the population each census year, and the increase in population for each interval of 10 years.

See how simple you can make this blank. Check the results.

**13.** Rule a form, enter the following statistics, find the gross profit of each department, and the gross profit of the entire store.

GROSS PROFITS OF A DEPARTMENT STORE

| DEPT. NO. | SALES | COST OF GOODS SOLD | GROSS PROFIT |
|---|---|---|---|
| 1 | $ 5,629.80 | $4,984.37 | |
| 2 | 7,358.92 | 6,295.16 | |
| 3 | 4,916.09 | 4,192.86 | |
| 4 | 7,329.16 | 6,593.54 | |
| 5 | 10,609.15 | 9,835.17 | |
| 6 | 6,123.18 | 5,902.52 | |
| 7 | 7,212.47 | 6,275.43 | |
| 8 | 9,475.37 | 8,594.48 | |
| 9 | 4,238.16 | 3,725.91 | |
| 10 | 5,824.78 | 5,014.78 | |
| Total | | | |

**14.** Rule a suitable form, enter the following statistics, and find:

*a.* The gross profit by departments.

*b.* The gross profit of the entire store (two ways).

*c.* The net profit or net loss by departments; enter net profit in black, net loss in red.

*d.* Net profit or net loss of the entire store (two ways).

| DEPT. NO. | SALES | COST OF GOODS SOLD | GROSS PROFIT | EXPENSES | NET PROFIT OR LOSS |
|---|---|---|---|---|---|
| 1 | $7,816.40 | $6,715.32 | | $423.85 | |
| 2 | 9,317.42 | 8,492.17 | | 530.08 | |
| 3 | 6,842.56 | 5,914.72 | | 731.96 | |
| 4 | 7,319.62 | 6,593.57 | | 671.42 | |
| 5 | 8,295.17 | 7,942.17 | | 456.72 | |
| 6 | 5,732.88 | 5,101.59 | | 322.75 | |
| 7 | 9,514.82 | 8,899.55 | | 693.15 | |
| 8 | 2,289.74 | 1,856.80 | | 216.77 | |
| 9 | 5,793.66 | 5,135.60 | | 788.51 | |
| 10 | 9,559.38 | 8,625.50 | | 523.80 | |

**12. Subtracting by Adding Complements.** A series of additions and subtractions may be performed by the method of adding complements.

The *complement* of a number is the number which must be added to get the next higher power of ten. Thus, the complement of 6 is 4; the complement of 83 is 17.

If, instead of subtracting a number less than ten from a given number, its complement be added, the result will be 10 too large. Thus, $13 - 6 = 7$ or $13 + 4 = 17$ (a result 10 too large).

If, instead of subtracting *two* numbers less than ten from a given number, their complements are added, the result will be 20 too large; if the complements of three such numbers are added, the result will be 30 too large, etc.

**Examples.** 1. $29 - 6 - 7 = ?$

SOLUTION. $29 + 4 + 3 = 36$. Since two complements were added, the result is 20 too large. Therefore, subtract 20, leaving 16.

2. $38 - 4 - 7 - 8 = ?$

SOLUTION. $38 + 6 + 3 + 2 = 49$ Since three complements were added, subtract 30, leaving 19.

3. $46 - 8 + 4 - 7 = ?$

SOLUTION. $46 + 2 + 4 + 3 = 55$. Since two complements were added, 20 must be subtracted, leaving 35.

The practical value of this method will be shown by the following examples.

1. $36 - 24 + 19 - 12 + 21 - 13 - 6 = ?$

SOLUTION. By combining as indicated in the units' column, beginning at the top (complements are marked with an *) we have

$$6 + 6^* + 9 + 8^* + 1 + 7^* + 4^* = 41$$

Since four complements were added, the result is 40 too large. Therefore write 1 and drop the 4.

By combining as indicated in the tens' column we have

$$3 + 8^* + 1 + 9^* + 2 + 9^* = 32$$

Since three complements were added, the result is 30 too large. Write 2 and drop the 3.

Result, 21.

$$\begin{array}{r} 36 \\ -\ 24 \\ \hline 19 \\ -\ 12 \\ \hline 21 \\ -\ 13 \\ -\ \ 6 \\ \hline 21 \end{array}$$

2. $985 + 234 - 126 - 34 - 125 - 174 + 386 = ?$

985
234
− 126
− 34
− 125
− 174
386
1,146

SOLUTION. Combining in the units' column, beginning at the top we have

$$5 + 4 + 4^* + 6^* + 5^* + 6^* + 6 = 36$$

Write the 6. Since four complements were added, the result is 40 too large. Therefore we must drop the 3 in 36, and also subtract 1 from the 8 at the top of the tens' column.

Combining in the tens' column we have

$$7 + 3 + 8^* + 7^* + 8^* + 3^* + 8 = 44$$

Since four complements were added, we must deduct 40, leaving 4 which is written in the tens' column of the result.

Combining in the hundreds' column we have

$$9 + 2 + 9^* + 9^* + 9^* + 3 = 41$$

Three complements were added, the result is therefore 30 too large. Subtract 30 and write 11.

Result, 1146.

The complement method is valuable when an occasional subtraction is to be made in work which consists largely of addition. Subtractions are made on the adding machine by adding the complement of the number to be subtracted, and deducting one from the next higher order.

Find the results by the addition of complements:

1. $46 - 4 - 7 =$
2. $72 - 8 - 6 - 3 =$
3. $39 - 7 + 3 - 9 =$
4. $48 - 17 - 6 =$
5. $81 - 23 - 42 =$
6. $49 - 22 + 15 - 11 =$
7. $85 - 26 - 35 + 62 - 16 =$
8. $643 - 289 + 364 =$
9. $781 - 247 + 64 =$
10. $1046 - 987 + 649 =$

| 11. | 12. | 13. |
|---|---|---|
| 2,346 | 1,932 | 923,578 |
| + 1,267 | + 9,845 | − 5,284 |
| − 321 | − 932 | + 28,956 |
| + 6,964 | + 2,122 | − 123,749 |
| − 1,235 | − 2,375 | + 83,219 |
| − 2,960 | + 692 | + 125,982 |
| − 985 | − 1,243 | − 429,764 |

### Written Work

**1.** The following model shows the ruling of a ledger in which a bank keeps its accounts with depositors. Deposits are added to the balance of the previous day, and checks are subtracted, to find the new balance to the credit of the depositor's account.

Find the daily balances:

WILLIAM HATFIELD

| DATE | DEPOSITS | CHECKS | BALANCE |
|---|---|---|---|
| June 1 | | | $328.57 |
| | $ 85.68 | $ 14.25 | ? |
| 2 | 125.80 | 65.90 | |
| | | 12.73 | ? |
| 3 | 245.85 | 127.89 | |
| | | 114.56 | |
| | | 35.87 | ? |
| 4 | 319.45 | 38.51 | |
| | 26.82 | 102.50 | ? |
| 5 | | 95.90 | |
| | | 141.66 | |
| | | 95.68 | ? |
| 6 | 450.00 | 132.88 | |
| | | 6.20 | ? |
| 8 | 139.75 | 216.45 | |
| | | 39.46 | |
| | | 19.99 | |
| | | 213.55 | |
| | | 66.82 | ? |

**2.** The following blank shows a convenient method of keeping a record of cash received and paid by a small business.

Rule a blank similar to the model and enter the data.

| DATE | CASH RECEIVED | | CASH PAID | | | DAILY CASH |
|---|---|---|---|---|---|---|
| | Cash Sales | On Account | Purchases | On Account | Expenses | Balance |
| Feb. 1 | | | | | | $216.27 |
| 1 | $215.78 | $124.35 | $ 80.75 | $ 68.55 | $ 8.75 | ? |
| 2 | 294.80 | 95.88 | 157.47 | 113.25 | 3.73 | ? |
| 3 | 188.47 | 148.23 | 95.48 | 316.57 | 27.49 | ? |
| 4 | 218.43 | 49.47 | 17.49 | 42.39 | 9.12 | ? |
| 5 | 388.92 | 112.67 | 221.81 | 76.29 | 75.84 | ? |
| 6 | 478.07 | 290.04 | 88.12 | 448.14 | 6.90 | ? |
| | ? | ? | ? | ? | ? | ? |

*a.* Find the daily cash balances.

To the balance of the preceding day add the receipts from cash sales and receipts on account, and subtract the various amounts listed under "Cash Paid."

Thus, \$216.27 + 215.78 + 124.35 − 80.75 − 68.55 − 8.75 = ?

*b.* Find the totals for each column.

**3** Add upward; subtract across. Do the results check?

| *a.* | | *b.* | |
|---|---|---|---|
| 7,463 − 2,847 = | | 174,638 − 94,273 = | |
| 5,928 − 3,804 = | | 38,270 − 27,409 = | |
| 9,604 − 2,870 = | | 70,563 − 2,879 = | |
| 3,962 − 1,436 = | | 924,360 − 14,287 = | |
| 9,287 − 5,426 = | | 7,503 − 2,769 = | |
| − = | | − = | |

In weighing articles contained in bags, barrels, or any container whose weight is not to be included, the total weight is called the *gross weight* and the amount to be deducted is called *tare.* The difference between gross weight and tare is called *net weight.*

Thus, if the gross weight of a wagon loaded with wheat is 3,840 pounds and the weight of the wagon is 1,360 pounds, the net weight is 2,480 pounds.

**4.** From a certain farm the following loads of wheat were hauled to an elevator. The gross weight includes the weight of the wagon. Compute the number of bushels of wheat hauled. Gross weight 3,840, tare 1,380; gross weight 3,960, tare 1,420; gross weight 4,285, tare 1,490; gross weight 4,180, tare 1,410; gross weight 4,585, tare 1,450.

Rule blank forms, enter the data, and make the required computations in the first two problems below.

**1.** On September 22, Adams and Johnson opened business with \$242.18 cash. The receipts and payments during the day were as follows: Receipts, \$42.50, \$18.30, \$112.90, \$5.80, \$11.48, \$79.43, \$106.35, \$25.40, \$10.80. Payments, \$38.50, \$109.45, \$6.40, \$2.50, \$71.95, \$14.60, \$165.80. How much cash did they have at the end of the day?

**2.** On October 3d George Scott had a bank balance of \$432.85. The following checks were drawn and deposits made. Checks, October 4, \$82.40, \$41.50; October 5, \$114.35; October 11, \$145.90; October 12, \$5.60, \$8.45, \$7.50; Deposits, October 5, \$94.80; October 6

$124.10, October 8, $34.67, $118.11; October 10, $14.92, $103.87. What was his balance on October 12th?

3. A contractor is to remove 17,840 cubic yards of earth. During 9 weeks he removes the following number of cubic yards: 482; 697; 365; 495; 672; 543; 748; 836; 765. How many cubic yards are still to be removed?

4. A wheat elevator contains 242,800 bushels of wheat. The following number of bushels are taken from the elevator and loaded into cars: 1,520, 1,430, 1,465, 1,560, 1,610, 1,490, 1,572, 1,463, 1,524, 1,585, 1,605, 1,425, 1,495, 1,580, 1,535. How many bushels of wheat are left in the elevator?

5. The following table gives the population of certain states for 1910 and 1920. Compute the increase in each by horizontal subtraction and add the increases. Devise a method for checking the totals.

| | 1910 | 1920 | Increase |
|---|---|---|---|
| California | 2,377,549 | 3,426,861 | |
| Georgia | 2,609,121 | 2,895,832 | |
| Kansas | 1,690,949 | 1,769,257 | |
| Missouri | 3,293,335 | 3,404,055 | |
| New York | 9,113,614 | 10,384,829 | |
| Ohio | 4,767,121 | 5,759,394 | |
| Pennsylvania | 7,665,111 | 8,720,017 | |
| Total | | | |

## General Review of Addition and Subtraction

Subtract and check results.

| | | | |
|---|---|---|---|
| 1. 47,806<br>30,597 | 2. 70,409<br>38,763 | 3. 18,947<br>3,879 | 4. 90,260<br>83,976 |
| 5. $879.40<br>438.90 | 6. $1,878.30<br>1,499.80 | 7. $2,840.36<br>1,092.09 | 8. $4,086.50<br>2,989.70 |

9. The following table gives the population of the twelve largest cities of the United States in 1920 and the population of these cities in 1910. Compute the increase for each city, the total increase, and the total population of the twelve cities for 1920 and 1910.

| City | 1910 | 1920 | Increase |
|---|---|---|---|
| New York | 4,766,883 | 5,621,151 | |
| Chicago | 2,185,283 | 2,701,705 | |
| Philadelphia | 1,549,008 | 1,823,158 | |
| Detroit | 465,766 | 993,739 | |
| Cleveland | 560,663 | 796,836 | |
| St. Louis | 687,029 | 772,897 | |
| Boston | 670,585 | 748,060 | |
| Baltimore | 558,485 | 733,826 | |
| Pittsburgh | 533,905 | 588,193 | |
| Los Angeles | 319,198 | 576,673 | |
| San Francisco | 416,912 | 508,410 | |
| Buffalo | 423,715 | 506,775 | |
| Total | | | |

**10.** State the difference between each pair of numbers, as 14 and 87.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *a* | 14 | 87 | 46 | 92 | 86 | 92 | 39 | 82 | 80 | 47 | 28 | 96 |
| *b* | 45 | 64 | 97 | 83 | 27 | 69 | 63 | 78 | 92 | 86 | 47 | 39 |
| *c* | 47 | 82 | 26 | 31 | 43 | 72 | 74 | 91 | 37 | 49 | 87 | 46 |
| *d* | 39 | 91 | 84 | 37 | 81 | 46 | 87 | 98 | 87 | 78 | 39 | 14 |

**11.** Add each pair of numbers, in the above list, as 14 and 87.

**12.** What change should be given for a five-dollar bill if the following purchases were made?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *a.* | $1.54 | *e.* | $ .87 | *i.* | $ .49 | *m.* | $1.40 |
| *b.* | 1.30 | *f.* | .45 | *j.* | 1.16 | *n.* | .82 |
| *c.* | 1.18 | *g.* | 1.72 | *k.* | 1.58 | *o.* | .15 |
| *d.* | .95 | *h.* | 1.18 | *l.* | .83 | *p.* | 1.28 |

**13.** Find the results by the complement method:

*a.* 142 + 187 + 274 − 85 − 28
*b.* 948 + 738 − 460 + 895
*c.* 348 + 749 − 685 + 480
*d.* 1,546 + 1,197 − 1,430 + 840

**14.** A real estate agent bought a tract of land containing 100,000 square feet. He sold lots containing the following number of square

feet: 9,000, 9,750, 10,500, 7,500, 15,000, 12,000, 12,300. How many square feet of the tract were not sold?

15. Add orally:

*a.* 34 + 62 + 18 + 37 + 43 + 29
*b.* 83 + 26 + 42 + 39 + 72 + 43
*c.* 25 + 34 + 62 + 37 + 46 + 82
*d.* 63 + 38 + 41 + 64 + 55 + 27

16 Make a problem involving addition of at least five numbers of four digits each and check the result by casting out nines.

17. Make another problem as required above and check the result by casting out elevens.

18. State several methods for checking addition. Which method do you *habitually* use for checking column addition? Why?

# CHAPTER III

## MULTIPLICATION

ONE engaged in business uses multiplication frequently, and the efficient clerk and accountant must have acquired speed and accuracy in the process.

The student has learned the multiplication tables to $10 \times 10$ or to $12 \times 12$. The following is the multiplication tables to $15 \times 15$. This should be thoroughly mastered.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 | 90 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | 91 | 98 | 105 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 | 104 | 112 | 120 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 | 117 | 126 | 135 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 | 143 | 154 | 165 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 156 | 168 | 180 |
| 13 | 26 | 39 | 52 | 65 | 78 | 91 | 104 | 117 | 130 | 143 | 156 | 169 | 182 | 195 |
| 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 | 140 | 154 | 168 | 182 | 196 | 210 |
| 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | 225 |

NOTE. Drill on the more difficult combinations until they are thoroughly mastered.

The result obtained by multiplication is called the *product*. The multiplicand and the multiplier are called the *factors* of the product. The product divided by either factor gives the other factor.

### Written Multiplication

Multiply 387 by 426:

$$\begin{array}{r} 387 \\ 426 \\ \hline 2322 \\ 774 \\ 1548 \\ \hline 164862 \end{array}$$

Multiply 426 by 387:

$$\begin{array}{r} 426 \\ 387 \\ \hline 2982 \\ 3408 \\ 1278 \\ \hline 164862 \end{array}$$

It is apparent from the illustration above that the product is not altered if the order of the factors is reversed. This method is sometimes used in checking multiplication.

Make the following multiplications and add the products. Add the multiplicands and multiply this total by 473.

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| 3,842 | 3,097 | 9,270 | 13,768 |
| 473 | 473 | 473 | 473 |

Make the following multiplications and add the products. Add the multiplicands and multiply this total by 709.

| 5. | 6. | 7. | 8. |
|---|---|---|---|
| 3,846 | 1,908 | 4,362 | 2,940 |
| 709 | 709 | 709 | 709 |

NOTE. The teacher is urged to assign several examples similar to those suggested above, as drill in multiplication.

### 13. Drill Tables.

Multiply across; add upward:

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| $74 \times 436 =$ | $83 \times 423 =$ | $25 \times 624 =$ | $35 \times 624 =$ |
| $74 \times 523 =$ | $83 \times 157 =$ | $25 \times 726 =$ | $35 \times 706 =$ |
| $74 \times 287 =$ | $83 \times 284 =$ | $25 \times 37 =$ | $35 \times 753 =$ |
| $74 \times 492 =$ | $83 \times 307 =$ | $25 \times 869 =$ | $35 \times 496 =$ |
| $74 \times ? =$ | $83 \times 596 =$ | $25 \times 493 =$ | $35 \times 548 =$ |
| | $83 \times ? =$ | $25 \times 468 =$ | $35 \times 784 =$ |
| | | $25 \times ? =$ | $35 \times ? =$ |

**Oral Work**

Use 2, 3, 4, 5, 6, 7, 8, and 9 as multipliers. State the results for each column in less than 20 seconds.

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| 3 | 18 | 5 | 4 | 9 |
| 5 | 10 | 8 | 9 | 11 |
| 7 | 4 | 12 | 15 | 16 |
| 14 | 7 | 15 | 20 | 30 |
| 11 | 9 | 7 | 17 | 25 |
| 6 | 15 | 30 | 8 | 14 |
| 8 | 16 | 22 | 13 | 17 |
| 9 | 20 | 16 | 18 | 11 |
| 15 | 12 | 25 | 22 | 16 |

### Written Work

1. A factory made an investigation of the number of articles of a certain kind manufactured by each of its employees. It found that:

9 men produced 46 articles each.
9 men produced 48 articles each.
11 men produced 53 articles each.
15 men produced 55 articles each.
18 men produced 59 articles each.
23 men produced 61 articles each.
25 men produced 62 articles each.
25 men produced 63 articles each.
21 men produced 64 articles each.
17 men produced 65 articles each.
17 men produced 66 articles each.
14 men produced 68 articles each.
12 men produced 69 articles each.
8 men produced 70 articles each.

Rule a form with a heading similar to the following:

PRODUCTION RECORD

| NUMBER OF MEN | NUMBER OF ARTICLES MADE BY EACH | TOTAL |
|---|---|---|
| | | |

Enter the statistics, find the number of articles produced by each group of employees, and the total number of articles produced by all of the employees.

2. Nine workmen were employed in the manufacture of different articles, and were paid a certain number of cents for each piece completed. Complete the following table, finding the wages earned by each workman.

DAILY PIECEWORK LABOR COST

| WORKMAN NO. | NUMBER OF ARTICLES MADE | WAGE RATE PER PIECE | WAGES EARNED |
|---|---|---|---|
| 1 | 17 | $0.27 | |
| 2 | 19 | .22 | |
| 3 | 38 | .16 | |
| 4 | 79 | .06 | |
| 5 | 32 | .15 | |
| 6 | 28 | .18 | |
| 7 | 64 | .07 | |
| 8 | 81 | .05 | |
| 9 | 29 | .21 | |

**14. Checking Multiplication.** Multiplication may be checked by several methods. The following methods are commonly used.

*a.* Repeating the multiplication and assuming that if the same product is obtained the work is correct. This is not a reliable check because an error is likely to be repeated.

*b.* Interchanging multiplicand and multiplier.

*c.* Dividing the product by the multiplier to obtain the multiplicand, or by the multiplicand to obtain the multiplier.

*d.* Casting out nines.

*e.* Casting out elevens.

**15. Casting Out Nines.** The method is as follows: *Find the excess of nines in the multiplicand and in the multiplier. Find the product of these excesses. Find the excess of nines in this product. It should equal the excess of nines in the result.*

**Example.** Multiply 23 by 16; check by casting out nines.

SOLUTION. 23 5 = the excess of nines in 23.
16 7 = the excess of nines in 16.
368 35 = the product of these excesses.
CHECK. 8 = the excess in this product.
The excess of nines in 368 is also 8. The multiplication, therefore, checks.

**16. Casting Out Elevens.** The check is illustrated below.

1874 . 4
976 8
11244 32 10
13118
16866
1829024 10

### Written Work

Without performing the multiplications determine the probable correctness of the following products, by means of casting out nines:

| 1. | 2. | 3. | 4. | 5. | 6. | 7. |
|---|---|---|---|---|---|---|
| 25 | 82 | 36 | 286 | 4,172 | 344 | 733 |
| 38 | 35 | 87 | 37 | 39 | 281 | 492 |
| 950 | 2,870 | 3,132 | 10,382 | 162,698 | 96,664 | 362,636 |

| 8. | 9. | 10. |
|---|---|---|
| 398 | 43,962 | 34,276 |
| 241 | 47,835 | 21,578 |
| 95,918 | 2,102,922,270 | 738,627,528 |

Multiply and check by casting out elevens:

| 11. | 12. | 13. | 14. | 15. | 16. |
|---|---|---|---|---|---|
| 347 | 279 | 627 | 132,879 | 63,154 | 893,567 |
| 861 | 439 | 123 | 2,378 | 9,837 | 17,839 |

**17.** One square mile contains 640 acres. How many acres in each of the following states?

| State | Area in Square Miles |
|---|---|
| Rhode Island | 1,248 |
| New York | 49,204 |
| Pennsylvania | 45,126 |
| Maryland | 12,327 |
| Georgia | 59,265 |
| Ohio | 41,040 |
| Illinois | 56,665 |
| Minnesota | 84,682 |
| Colorado | 103,948 |
| California | 158,297 |

If your state is not included in the above list, compute the number of acres in it.

**18.** A book of 294 pages contains an average of 348 words to a page. How many words are in the book?

**19.** Each page of the book contains an average of 34 lines. How many lines are in the book?

**20.** A certain field yielded 52 bushels of corn to the acre. How many bushels did the field of 19 acres yield?

**21.** Determine the present retail prices of each of the following and compute the total cost.

| | | |
|---|---|---|
| 7 lb. coffee | 17 lb. chocolate | 25 lb. flour |
| 100 lb. sugar | 11 lb. cocoa | 12 lb. navy beans |
| 2 lb. raisins | 21 lb. rice | 14 bu. potatoes |
| 4 lb. tea | 13 lb. starch | |

**22.** Determine the present retail price of each of the following and compute the total cost.

| | |
|---|---|
| 17 yd. percale | 18 yd. flannel |
| 13 yd. denim | 17 yd. muslin (unbleached) |
| 12 yd. madras | 18 yd. cheese cloth |

**23.** Multiply each of the following numbers by 347, then add the multiplicands and multiply this sum by 347. Does the result check?

847 1,926 427 893

**24.** How many of these examples in multiplication can you copy and perform in six minutes?

| | | | |
|---|---|---|---|
| *a.* 7,480<br>392 | *b.* 1,706<br>927 | *c.* 4,957<br>307 | *d.* 2,759<br>629 |
| *e.* 7,986<br>640 | *f.* 9,043<br>792 | *g.* 8,739<br>708 | *h.* 7,382<br>547 |
| *i.* 9,086<br>429 | *j.* 6,980<br>609 | *k.* 3,708<br>473 | *l.* 9,287<br>654 |

**25.** Check each product in example 24.

**26.** What method do you *habitually* use in checking multiplication? Why?

### Short Methods of Multiplication

These short methods will be found to be very practical. Master two or three of them thoroughly before taking up others. Use those that you have mastered whenever you have opportunity to do so. It is not necessary that all of these short methods be mastered before the succeeding chapters are studied.

**17. To multiply by 10, 100, 1,000, 10,000, etc.**

*a.* When the multiplicand is an integer. *Annex to the multiplicand as many zeros as there are zeros in the multiplier.*

Thus, to multiply an integer by 10, annex one zero; to multiply by 100, annex two zeros.

**Examples.** 1. $37 \times 10 = 370$. 2. $29 \times 100 = 2{,}900$.

*b.* When the multiplicand is a decimal fraction. *Move the decimal point as many places to the right as there are zeros in the multiplier.*

It may be necessary to annex zeros in order to move the decimal point the desired number of places.

**Examples.** 1. Multiply .1357 by 100.

SOLUTION. Move the decimal point two places to the right, 13.57.

2. Multiply 32.46 by 1000.

SOLUTION. In order to move the decimal point three places to the right it is necessary to annex one zero. The result is 32,460.

**Oral Work**

Multiply as indicated:

1. $37{,}946 \times 100$.
2. $5{,}293 \times 10{,}000$.
3. $639 \times 100{,}000$.
4. $120 \times 1{,}000$.
5. $.376 \times 1{,}000$.
6. $1.349 \times 1{,}000$.
7. $27.9637 \times 100$.
8. $.00932 \times 10$.
9. $.0873 \times 1{,}000$.
10. $.7032 \times 1{,}000$.
11. $3.69 \times 1{,}000$.
12. $.0027 \times 100{,}000$.
13. $1{,}427.834 \times 10{,}000$.
14. $625.086 \times 1{,}000$.

**18. To multiply numbers ending with zeros.**

*a.* When both numbers are integers. *Multiply the numbers represented by the significant figures. To the product thus obtained, annex as many zeros as there are final zeros in both the multiplicand and multiplier*

**Example.** Multiply 3,400 by 1,200.

SOLUTION. $34 \times 12 = 408$. Annexing four zeros, we obtain the product 4080000.

Perform the following multiplications. Whenever possible, do the work orally.

1. $169 \times 300$.
2. $210 \times 300$.
3. $4{,}567 \times 700$.
4. $1{,}390 \times 1{,}200$.
5. $2{,}300 \times 1{,}500$.
6. $19{,}000 \times 16$.
7. $3{,}194 \times 23{,}000$.
8. $420 \times 3{,}400$.

*b.* When one of the numbers is a decimal fraction. *Multiply the numbers represented by the significant figures. Move the decimal point as many places to the right in the product as there are final zeros in the integer.* (This may necessitate annexing zeros.)

**Examples.**

1. Multiply .486 by 300.

SOLUTION. $3 \times .486 = 1.458$.
Move the decimal point two places to the right, the result is 145.8.

2. Multiply 3.2 by 400.

SOLUTION. $4 \times 3.2 = 12.8$.
Move the decimal point two places to the right, the result is 1280.

**Written Work**

Perform the following multiplications:

1. $.47 \times 200$.
2. $3.786 \times 4{,}000$.
3. $17.682 \times 500$.
4. $.0746 \times 3{,}000$.
5. $.072 \times 6{,}000$.
6. $.382 \times 1{,}200$.
7. $.0837 \times 14{,}000$.
8. $.042 \times 170$.
9. $.0036 \times 1{,}500$.
10. $4.26 \times 7{,}000$.

**19. To multiply by 9, 99, 999, etc.**

*a.* To multiply by 9. *Annex one zero to the number to be multiplied, thus multiplying it by* 10; *from this result subtract the number to be multiplied.*

**Example.** Multiply 346 by 9.

SOLUTION.

$$\begin{array}{r} 3460 \\ 346 \\ \hline 3114 \end{array}$$

*b.* To multiply by 99. *Annex two zeros to the number to be multiplied, thus multiplying it by* 100; *from this result subtract the number to be multiplied.*

**Example.** Multiply 293 by 99.

SOLUTION.

$$\begin{array}{r} 29300 \\ 293 \\ \hline 29007 \end{array}$$

**Written Work**

Multiply each of the following numbers by 9, 99, and 999:

**1.** 632. **2.** 748. **3.** 135. **4.** 737. **5.** 427. **6.** 166.

**20. To multiply by numbers slightly smaller than 10, 100, 1,000, 10,000, etc.**

A modification of the short method explained in the preceding section may be used to multiply by numbers slightly smaller than 10, 100, 1,000, 10,000, etc.

To multiply by 98. *Annex two zeros to the number to be multiplied and from this result subtract twice the number to be multiplied.*

How can the short method be used if you are to multiply by 97, 96, 95, or 998?

**Written Work**

Apply short methods to the following examples:

| | | |
|---|---|---|
| **1.** $675 \times 96$. | **2.** $350 \times 95$. | **3.** $535 \times 91$. |
| **4.** $687 \times 97$. | **5.** $94 \times 3.4$. | **6.** $995 \times 82$. |
| **7.** $634 \times 994$. | **8.** $.48 \times 997$. | **9.** $23 \times 99$. |
| **10.** $48 \times 997$. | **11.** $12 \times 988$. | **12.** $47 \times 998$. |

**21. To multiply by 11.**

*a.* When the multiplicand contains two digits.

*Place between these two digits, their sum.*

**Example.** $34 \times 11 = 374$.

When the sum of the two digits is 10 or more, 1 must be carried to the digit at the left.

**Example.** $68 \times 11 = 748$.

### Written Work

Multiply each of the following numbers by 11:

| | | | | | |
|---|---|---|---|---|---|
| **1.** 27. | **2.** 63. | **3.** 93. | **4.** 74. | **5.** 26. | **6.** 35. |
| **7.** 22. | **8.** 87. | **9.** 28. | **10.** 46. | **11.** 75. | **12.** 96. |
| **13.** 37. | **14.** 36. | **15.** 57. | **16.** 85. | **17.** 98. | **18.** 72. |

*b.* When the multiplicand contains three or more digits:

*The units' digit of the multiplicand is the units' digit of the product; the sum of the units' and tens' digits is the tens' digit of the product; the sum of the tens' and hundreds' digits is the hundreds' digit of the product, etc.*

Whenever the sum of two digits is ten or more, 1 must be carried.

**Example.** 1. Multiply 793 by 11.

SOLUTION.

3 (the units' digit of the multiplicand) becomes the units' digit of the product.

$9 + 3 = 12$ (carry the 1)

$1 + 7 + 9 = 17$ (carry the 1)

$1 + 7 = 8$

8723

2. Multiply 52,635 by 11.

SOLUTION. $52635 \times 11 = 578985$.

### Written Work

Multiply each of the following by 11:

| | |
|---|---|
| **1.** 363. | **2.** 271. |
| **3.** 823. | **4.** 456. |
| **5.** 3,742. | **6.** 876,394. |
| **7.** 3,578,962. | **8.** 34,579. |
| **9.** 263,789. | **10.** 123,496,287. |

**22. To multiply by 111.**

$362{,}941 \times 111 = ?$

When the multiplication is performed in the customary manner, the multiplicand is repeated as follows:

362,941
362,941
362,941

When the short method is applied, *the units' digit of the multiplicand is the units' digit of the product; the sum of the units' and tens' digits of the multiplicand is the tens' digit of the product; the sum of the units', tens', and hundreds' digits of the multiplicand is the hundreds' digit of the product; the sum of the tens', hundreds', and thousands' digits of the multiplicand is the thousands' digit of the product, etc.*

The excess above 10 is always to be carried to the next sum.

Without recopying, write the products obtained by multiplying each of the following by 111:

| | | |
|---|---|---|
| **1.** 729,361. | **2.** 124,396. | **3.** 1,793,862. |
| **4.** 5,374. | **5.** 235,692. | **6.** 8,354,927. |

**23. To multiply two numbers ending in 5.**

*a.* When the sum of the digits at the left of the 5's is an even number.

*Multiply the digits at the left of the 5's; to this product add one half the sum of these digits; to this result annex 25.*

**Examples.** 1. Multiply 65 by 25.

SOLUTION. $2 \times 6 = 12$, the product of the digits at the left of the 5's.
$\frac{1}{2}$ of $(2 + 6) = 4$
16
1625, result obtained by annexing 25.

2. Multiply 625 by 445.

SOLUTION. $62 \times 44 = 2728$
$\frac{1}{2}$ of $(62 + 44) = 53$
2781
278125, result obtained by annexing 25.

**Written Work**

Multiply as indicated:

| | |
|---|---|
| **1.** $35 \times 75$. | **2.** $95 \times 75$. |
| **3.** $35 \times 55$. | **4.** $25 \times 45$. |
| **5.** $325 \times 45$. | **6.** $725 \times 65$. |
| **7.** $835 \times 175$. | **8.** $145 \times 165$. |
| **9.** $195 \times 115$. | **10.** $225 \times 185$. |

*b.* When the sum of the digits at the left of the 5's is an odd number.

*Multiply the digits at the left of the 5's; to this product add one half the sum of these digits, dropping the ½; to this result annex 75.*

**Examples.** 1. Multiply 75 by 45.

SOLUTION. $4 \times 7 = 28$, the product of the digits at the left of the 5's.

$\frac{1}{2}$ of $(4 + 7) = 5\frac{1}{2}$

$33\frac{1}{2}$

3375, the result obtained by dropping the fraction ½, and annexing 75.

2. Multiply 325 by 475.

SOLUTION. $32 \times 47 = 1504$

$\frac{1}{2}$ of $(32 + 47) = 39\frac{1}{2}$

1543 adding and dropping the ½.

154375, result secured by annexing 75.

**Written Work**

Multiply as indicated:

1. $75 \times 65$.
2. $125 \times 135$.
3. $95 \times 85$.
4. $145 \times 175$.
5. $225 \times 75$.
6. $165 \times 135$.
7. $145 \times 215$.
8. $435 \times 125$.

**24. To multiply two numbers, when certain digits of the multiplier are contained an integral number of times in other digits of the multiplier.**

Multiply 224 by 279.

Since 9 is contained 3 times in 27, first multiply 224 by 9, then multiply this product by 3.

Thus,

$$\begin{array}{r} 224 \\ 279 \\ \hline 2016 = 9 \times 224 \\ 6048 = 3 \times 2016 \\ \hline 62496 \end{array}$$

When using this method, be careful to place the product of the second multiplication in the proper position.

**Example.** 1. Multiply 341 by 618.

| | |
|---|---|
| 341 | |
| 618 | |
| 2046 | $= 6 \times 341$ |
| 6138 | $= 3 \times 2046$ |
| 210738 | |

SOLUTION. Since 18 is a multiple of 6, multiply first by 6. Place the right-hand figure of the product, 2046, directly under the 6 of the multiplier. The product of $18 \times 341$ can now be obtained by multiplying 2046 by 3. The right-hand figure of this product, 6138, is placed directly under the 8 of the multiplier.

### Written Work

Multiply the following, stating by what numbers you multiplied in order to take advantage of the short method:

| | | |
|---|---|---|
| 1. $468 \times 243$. | 2. $1{,}235 \times 981$. | 3. $719 \times 427$. |
| 4. $687 \times 654$. | 5. $739 \times 848$. | 6. $7{,}362 \times 1{,}248$. |
| 7. $1{,}247 \times 1{,}864$. | 8. $146{,}387 \times 315$. | 9. $1{,}235 \times 819$. |

**25.** The **supplement** of a number is the difference between the number and the next lower power of 10. The supplement of 15 is 5; the supplement of 134 is 34; the supplement of 1,042 is 42.

**26. To multiply two numbers each of which is a little larger than 100.**

*To either of the numbers add the supplement of the other; to this sum annex the product of the supplements.*

**Example.** Multiply 131 by 103.

SOLUTION. $131 + 3 = 134$
(or $103 + 31 = 134$).

Annex 93 ($31 \times 3$).
The result is 13493.

The same rule may be applied to numbers a little larger than 1,000.

**Example.** Multiply 1,062 by 1,006.

SOLUTION $1062 + 6 = 1068$
$62 \times 6 = 372$
1068372

NOTE. When the supplements are based on 100 and the product of the supplements is a number of only one digit, a zero must be put in the tens' place. For example: $102 \times 103 = 10{,}506$. Similarly, when the supplements are based on 1,000 and the product of the supplements is a number of less than three digits, zeros must be put in the proper places.

### Written Work

| | | |
|---|---|---|
| 1. $104 \times 120$. | 2. $127 \times 102$. | 3. $113 \times 106$. |
| 4. $114 \times 105$. | 5. $114 \times 106$. | 6. $109 \times 106$. |
| 7. $126 \times 107$. | 8. $109 \times 111$. | 9. $1{,}007 \times 1{,}003$. |
| 10. $1{,}009 \times 1{,}012$. | 11. $1{,}214 \times 1{,}006$. | 12. $1{,}206 \times 1{,}012$. |
| 13. $1{,}112 \times 1{,}006$. | 14. $1{,}416 \times 1{,}009$. | 15. $1{,}374 \times 1{,}005$. |
| 16. $1{,}674 \times 1{,}012$. | | |

**27.** The **complement** of a number is the difference between the next higher power of 10 and the number. The complement of 92 is 8; the complement of 89 is 11; the complement of 996 is 4.

**28. To multiply two numbers both slightly less than 100.**

*From either number subtract the complement of the other number. To this result annex the product of the complements.*

**Example.** Multiply 96 by 93.

SOLUTION. $96 - 7 = 89$ (or $93 - 4 = 89$).
$4 \times 7 = 28$ (product of the complements).
Annex 28 to 89 and the result is 8,928.

NOTE. When the complements are based on 100 and the product of the complements is less than 10, a zero must be put in tens' place. For illustration: $98 \times 97 = 9{,}506$. Similarly, the tens' and hundreds' places must be filled when the complements are based on 1,000.

### Written Work

Multiply as indicated. Perform orally as much of this work as possible.

| | | |
|---|---|---|
| 1. $98 \times 95$. | 2. $97 \times 84$. | 3. $87 \times 91$. |
| 4. $95 \times 80$. | 5. $93 \times 85$. | 6. $87 \times 97$. |
| 7. $88 \times 87$. | 8. $91 \times 92$. | 9. $86 \times 94$. |

The same method may be applied to numbers slightly less than 1,000.

**Example.** Multiply 996 by 987.

SOLUTION. $996 - 13$ (the complement of 987) $= 983$.
$4 \times 13 = 52$ (the product of the complements).
Since the complements are based on 1000 and the product of the complements is a number of only two digits, a zero must be put in hundreds' place. The result is 983052.

### Written Work

Multiply:

| | | | |
|---|---|---|---|
| 1. 994<br>992 | 2. 996<br>984 | 3. 983<br>991 | 4. 987<br>992 |
| 5. 991<br>981 | 6. 983<br>982 | 7. 978<br>983 | 8. 981<br>988 |

**29. To multiply any two numbers in the teens.**

*To either of the numbers add the units' digit of the other number and annex a zero.*

*To this result add the product of the units' digits.*

**Example.** Multiply 15 by 17.

SOLUTION. $15 + 7 = 22$; annex a zero, 220
$5 \times 7 =$ 35
255

### Oral Work

Multiply:

| | | | | | |
|---|---|---|---|---|---|
| 1. 17<br>12 | 2. 13<br>12 | 3. 16<br>18 | 4. 17<br>13 | 5. 14<br>13 | 6. 19<br>15 |
| 7. 13<br>16 | 8. 17<br>19 | 9. 18<br>12 | 10. 13<br>19 | 11. 17<br>16 | 12. 15<br>17 |

**30. To multiply by 25, 50, or 75.**

Since 25 is $\frac{1}{4}$ of 100, the product obtained by multiplying any number by 25 is $\frac{1}{4}$ of the product obtained by multiplying the number by 100.

The student should be asked to discover a short method to be used when 50 or 75 are multipliers.

### Written Work

Multiply each number in the first row by 25, each in the second row by 50, and in the third by 75.

| | | | | | |
|---|---|---|---|---|---|
| *a.* | 482 | 398 | 1,436 | 297 | 849 |
| *b.* | 154 | 697 | 2,843 | 792 | 786 |
| *c.* | 4,092 | 378 | 2,864 | 923 | 871 |

# CHAPTER IV

## DIVISION

### Oral Work

**31. Short Division.**

1. Divide by 2: 16, 28, 76, 248, 368, 926, 1,054.
2. Divide by 3: 27, 36, 57, 75, 417, 732, 873.
3. Divide by 4: 32, 48, 64, 72, 96, 196, 384, 748.
4. Divide by 5: 75, 95, 145, 545, 725, 965, 1,025, 1,370.
5. Divide by 6: 72, 96, 126, 366, 528, 732, 1,044.
6. Divide by 7: 56, 91, 147, 203, 476, 924, 1,575.
7. Divide by 8: 72, 96, 144, 360, 424, 792, 1,240.
8. Divide by 9: 54, 98, 171, 243, 378, 567, 981.

State the remainder when each of the following numbers is divided by 3, 4, 5, 7, 8, and 9.

| 9. | 10. | 11. | 12. | 13. |
|---|---|---|---|---|
| 473 | 9,846 | 2,638 | 4,020 | 19,206 |
| 692 | 3,723 | 7,284 | 7,302 | 47,308 |
| 876 | 9,264 | 3,047 | 7,960 | 23,074 |
| 479 | 3,749 | 2,636 | 8,403 | 95,306 |

14. How many pounds of meat can be bought for \$1.26 at 18¢ per pound?

15. How many weeks in 91 days? 175 days? 266 days?

16. The dividend is 176 and the quotient is 8. What is the divisor?

17. The divisor is 9, the quotient is 7, and the remainder is 3. What is the dividend?

18. Six dozen oranges were bought for \$2.40. What was the price per dozen?

19. How many yards in 186 feet? 924 feet? 5,280 feet?

20. A grocer buys 9 dozen eggs for \$2.88. What is the cost per dozen?

21. If you are to discharge a debt of \$8.40 in 6 equal weekly payments, how much must you pay each week?

**32. Long Division.**

**Examples.** 1. Divide 4,625 by 37. 2. Divide 47,285 by 327.

SOLUTION.

$$\begin{array}{r|l} & 125 \\ 37 & 4625 \\ & 37 \\ \hline & 92 \\ & 74 \\ \hline & 185 \\ & 185 \\ \hline \end{array}$$

SOLUTION.

$$\begin{array}{r|l} & 144\frac{197}{327} \\ 327 & 47285 \\ & 327 \\ \hline & 1458 \\ & 1308 \\ \hline & 1505 \\ & 1308 \\ \hline & 197 \end{array}$$

(The quotient should be written above the dividend in long division.)

**33. Checks.** Division may be checked by several methods. The method stated below is easily understood and generally used.

*Multiply the quotient by the divisor and add the remainder, if any, to this product to obtain the dividend.*

Division may also be checked by the following methods:

1. Divide the dividend by the quotient to get the divisor.
2. Check by casting out the 9's or the 11's.
3. Divide the dividend in turn by the various factors of the divisor.

**Written Work**

Perform the following divisions and check each result:

1. 1,758 ÷ 144.
2. 15,762 ÷ 37.
3. 73,627 ÷ 425.
4. 78,264 ÷ 738.
5. 508,573 ÷ 97.
6. 66,816 ÷ 928.
7. 41,228 ÷ 44.
8. 2,235,812 ÷ 284.
9. Divide the sum of 478, 392, 648, 971, and 1,483 by 27.
10. Divide the sum of 3,024, 4,763, 8,297, 9,468, and 293 by 38.
11. Complete:

$$\begin{array}{rl} 37 \times 29 = & \\ 42 \times 29 = & \\ 36 \times 29 = & \\ 94 \times 29 = & \\ 402 \times 29 = & \\ \text{Total} = & \underline{\qquad\qquad} \end{array}$$

**12.** The total circulation of a certain daily newspaper for 23 consecutive days was 524,423 copies. What was the average daily circulation?

**13.** At $2.18 a yard, how many yards of cloth can be bought for $37.06?

**14.** The dividend is 14,286, the quotient is 18, and the remainder is 12. What is the divisor?

**15.** The product of two numbers is 135,468. One of the numbers is 426. What is the other?

**16.** In a factory 437 men produce 5,681 articles in a week. What is the average weekly production for each employee?

**17.** At the rate of 43 miles an hour, how long will it take a train to run 473 miles?

**18.** Into how many states as large as Rhode Island (1,248 sq. mi.) could Texas (265,896 sq. mi.) be divided?

**19.** The expenses for 5 months for a family of 5 amounted to $258 for food, $170 for rent, $216 for clothes, $147 for amusements and other expenses. What was the average monthly expense for the family and the average for each person?

**34. Pointing Off the Quotient.** When it is necessary to continue a division to tenths or hundredths, place a decimal point to the right of the units' figure of the dividend and the quotient, annex zeros to the dividend, and continue the process.

**Example.** Divide 2,916 by 26.

SOLUTION.

$$26 \overline{)2916.0} \quad 112.1$$

**Written Work**

The following table gives the area of each of the states in square miles, and also the population of each state.

Review the instructions given on page 10 for ruling a blank form and for entering statistics.

Prepare a form similar to the following:

| State and Division | Land Area in Square Miles | Population 1920 | Population per Square Mile |
|---|---|---|---|
| *New England* | | | |
| Maine | 29,895 | 768,014 | |
| New Hampshire | 9,031 | 443,083 | |
| Vermont | 9,124 | 352,428 | |
| Massachusetts | 8,039 | 3,852.356 | |
| Rhode Island | 1,067 | 604,397 | |
| Connecticut | 4,820 | 1,380,631 | |
| *Middle Atlantic* | | | |
| New York | 47,654 | 10,384,829 | |
| New Jersey | 7,514 | 3,155,900 | |
| Pennsylvania | 44,832 | 8,720,017 | |
| *East North Central* | | | |
| Ohio | 40,740 | 5,759,394 | |
| Indiana | 36,045 | 2,930,390 | |
| Illinois | 56,043 | 6,485,280 | |
| Michigan | 57,480 | 3,668,412 | |
| Wisconsin | 55,256 | 2,632,067 | |
| *West North Central* | | | |
| Minnesota | 80,858 | 2,387,125 | |
| Iowa | 55,586 | 2,404,021 | |
| Missouri | 68,727 | 3,404,055 | |
| North Dakota | 70,183 | 645,680 | |
| South Dakota | 76,868 | 636,547 | |
| Nebraska | 76,808 | 1,296,372 | |
| Kansas | 81,774 | 1,769,257 | |
| *South Atlantic* | | | |
| Delaware | 1,965 | 223,003 | |
| Maryland | 9,941 | 1,449,661 | |
| District of Columbia | 60 | 437,571 | |
| Virginia | 40,262 | 2,309,187 | |
| West Virginia | 24,022 | 1,463,701 | |
| North Carolina | 48,740 | 2,559,123 | |
| South Carolina | 30,495 | 1,683,724 | |
| Georgia | 58,725 | 2,895,832 | |
| Florida | 54,861 | 968,470 | |
| *East South Central* | | | |
| Kentucky | 40,181 | 2,416,630 | |
| Tennessee | 41,687 | 2,337,885 | |
| Alabama | 51,279 | 2,348,174 | |
| Mississippi | 46,362 | 1,790,618 | |

| State and Division | Land Area in Square Miles | Population 1920 | Population per Square Mile |
|---|---|---|---|
| *West South Central* | | | |
| Arkansas | 52,525 | 1,752,204 | |
| Louisiana | 45,409 | 1,798,509 | |
| Oklahoma | 69,414 | 2,028,283 | |
| Texas | 262,398 | 4,663,228 | |
| *Mountain* | | | |
| Montana | 146,201 | 548,889 | |
| Idaho | 83,354 | 431,866 | |
| Wyoming | 97,594 | 194,402 | |
| Colorado | 103,658 | 939,629 | |
| New Mexico | 122,503 | 360,350 | |
| Arizona | 113,810 | 333,903 | |
| Utah | 82,184 | 449,396 | |
| Nevada | 109,821 | 77,407 | |
| *Pacific* | | | |
| Washington | 66,836 | 1,356,621 | |
| Oregon | 95,607 | 783,389 | |
| California | 155,652 | 3,426,861 | |

Note. The class should be divided into groups and each group held responsible for a geographical division.

Complete as much of the table as the teacher assigns, showing the population per square mile of each state, of each section, and of the United States.

Carry results to one decimal place.

### General Exercises for Drill

#### I. Estimated Time 7 Minutes

1. Add:

| *a.* | *b.* | *c.* | *d.* | *e.* | *f.* | *g.* | *h.* |
|---|---|---|---|---|---|---|---|
| 27 | 14 | 29 | 73 | 29 | 83 | 72 | 94 |
| 36 | 33 | 43 | 99 | 48 | 92 | 86 | 55 |
| 92 | 27 | 38 | 26 | 73 | 17 | 93 | 76 |
| 23 | 19 | 64 | 10 | 27 | 64 | 14 | 87 |
| 49 | 86 | 27 | 18 | 65 | 20 | 27 | 31 |
| 85 | 35 | 85 | 35 | 42 | 78 | 38 | 49 |

2. Multiply:

| *a.* | *b.* | *c.* |
|---|---|---|
| 4,736 | 9,463 | 7,427 |
| 3 | 5 | 3 |

3. Subtract:

| *a.* | *b.* | *c.* | *d.* |
|---|---|---|---|
| 72,049 | 4,706 | 32,964 | 92,807 |
| 32,608 | 2,985 | 7,193 | 1,968 |

4. Add across and down:

| *a.* | *b.* |
|---|---|
| 7 + 3 + 9 + 2 = | 24 + 7 + 6 = |
| 5 + 4 + 8 + 3 = | 13 + 3 + 3 = |
| 4 + 5 + 1 + 9 = | 17 + 4 + 7 = |
| 7 + 6 + 5 + 1 = | 9 + 15 + 3 = |
| 1 + 9 + 3 + 4 = | 18 + 9 + 5 = |
| 8 + 1 + 2 + 5 = | 13 + 2 + 1 = |
| 3 + 2 + 7 + 8 = | 7 + 19 + 8 = |
| 9 + 7 + 4 + 6 = ___ | 15 + 3 + 9 = ___ |
| + + + = | + + = |

## II. Estimated Time 9 Minutes

1. Add across and down:

741 + 437 + 563 + 234
526 + 318 + 725 + 238
382 + 746 + 128 + 407
387 + 209 + 504 + 916
328 + 460 + 937 + 659
473 + 295 + 243 + 175
___ + + + +

2. Multiply:

| *a.* | *b.* | *c.* | *d.* |
|---|---|---|---|
| 7,460 | 9,047 | 739 | 7,036 |
| 38 | 29 | 86 | 43 |

3. $143 \times 6 =$
$427 \times 5 =$
$793 \times 8 =$
$429 \times 7 =$
$384 \times 3 =$
$827 \times 6 =$
$538 \times 9 =$

4. Divide:

*a.* $326 \overline{)6{,}846}$ *b.* $22 \overline{)6{,}996}$

## III. Estimated Time 8 Minutes

Add across and down:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7 | 9 | 7 | 3 | 5 | 8 | 1 | 8 |
| 4 | 2 | 5 | 9 | 2 | 4 | 7 | 3 |
| 3 | 8 | 3 | 6 | 4 | 5 | 3 | 6 |
| 6 | 4 | 7 | 7 | 8 | 9 | 8 | 9 |
| 2 | 3 | 9 | 4 | 7 | 7 | 6 | 7 |
| 9 | 5 | 8 | 8 | 9 | 6 | 4 | 2 |
| 8 | 7 | 6 | 5 | 3 | 3 | 5 | 4 |
| 4 | 6 | 4 | 1 | 1 | 4 | 9 | 8 |
| 7 | 1 | 9 | 2 | 2 | 1 | 2 | 5 |
| 6 | 9 | 2 | 8 | 6 | 2 | 7 | 1 |

Subtract:

| *a.* | *b.* | *c.* |
|---|---|---|
| 74,036 | 92,407 | 769,082 |
| 21,809 | 38,795 | 432,767 |

Multiply:

| *a.* | *b.* | *c.* |
|---|---|---|
| 3,742 | 9,427 | 3,897 |
| 37 | 45 | 28 |

Divide:

*a.* $6 \overline{)748{,}632}$ *b.* $8 \overline{)938{,}424}$

## IV. Estimated Time 8 Minutes

**1.** Add:

| *a.* | *b.* | *c.* | *d.* | *e.* | *f.* | *g.* | *h.* |
|---|---|---|---|---|---|---|---|
| 37 | 24 | 33 | 51 | 26 | 72 | 43 | 18 |
| 24 | 13 | 15 | 32 | 39 | 46 | 75 | 37 |
| 36 | 43 | 31 | 44 | 72 | 92 | 76 | 11 |
| 15 | 25 | 17 | 65 | 43 | 54 | 28 | 26 |
| 13 | 17 | 65 | 23 | 86 | 28 | 35 | 34 |
| 11 | 38 | 22 | 25 | 54 | 63 | 54 | 83 |

**2.** Subtract:

| *a.* | *b.* | *c.* | *d.* |
|---|---|---|---|
| 4,372 | 7,246 | 9,264 | 3,164 |
| 2,439 | 3,817 | 5,728 | 1,476 |

**3.** Add from left to right:

| *a.* | *b.* |
|---|---|
| 7 + 3 + 2 + 4 = | 6 + 9 + 4 + 8 = |
| 3 + 7 + 5 + 6 = | 13 + 17 + 19 + 12 = |
| 4 + 8 + 9 + 1 = | 15 + 13 + 12 + 16 = |
| 3 + 7 + 2 + 4 = | 5 + 7 + 8 + 6 = |
| 8 + 3 + 7 + 5 = | 13 + 27 + 32 + 17 = |
| 9 + 3 + 2 + 8 = | 48 + 27 + 3 + 2 = |
| 5 + 7 + 3 + 8 = | 7 + 9 + 2 + 3 = |

**4.** Multiply:

| *a.* | *b.* | *c.* | *d.* |
|---|---|---|---|
| 3,724 | 4,923 | 6,728 | 3,107 |
| 2 | 3 | 2 | 3 |

## V. Estimated Time 12 Minutes

**1.** Divide:

| *a.* | *b.* | *c.* |
|---|---|---|
| 5 ⟌ 742,385 | 8 ⟌ 943,824 | 9 ⟌ 736,848 |

**2.** Multiply:

| *a.* | *b.* | *c.* |
|---|---|---|
| 1,437 | 1,943 | 1,048 |
| 246 | 632 | 453 |

**3.** Subtract:

| *a.* | *b.* | *c.* | *d.* |
|---|---|---|---|
| 472,387 | 940,360 | 472,863 | 796,048 |
| 29,389 | 397,685 | 30,940 | 397,059 |

**4.** Add across and down:

$$\begin{array}{l} 943 + 790 + 408 + 941 = \\ 276 + 463 + 312 + 806 = \\ 841 + 287 + 637 + 420 = \\ 732 + 934 + 924 + 387 = \\ 286 + 742 + 836 + 209 = \underline{\quad} \\ \quad + \quad + \quad + \quad = \end{array}$$

**5.** Divide:

*a.* $423 \overline{)13{,}536}$ *b.* $91 \overline{)46{,}228}$

### General Exercises in the Fundamental Operations

**1.**
$439 \times 57 =$
$826 \times 57 =$
$409 \times 57 =$
$392 \times 57 =$
$870 \times 57 =$
$439 \times 57 =$
$? \times 57 = ?$

**2.**
$409 \times 36 =$
$872 \times 36 =$
$791 \times 36 =$
$879 \times 36 =$
$472 \times 36 =$
$748 \times 36 =$
$? \times 36 = ?$

**3.**
$807 \times 83 =$
$904 \times 83 =$
$296 \times 83 =$
$475 \times 83 =$
$917 \times 83 =$
$796 \times 83 =$
$? \times 83 = ?$

**4.** Add without rewriting:

*a.* $47 + 36 + 82 + 94$
*b.* $83 + 21 + 72 + 76$
*c.* $42 + 63 + 17 + 84$
*d.* $56 + 43 + 27 + 18$
*e.* $26 + 35 + 42 + 23$
*f.* $81 + 42 + 63 + 35$

**5.** The number of automobiles in use in certain states in 1919 and 1920 is indicated in the following table. Compute the increase in each

state for 1920, and compute the total number in use in the states listed.

Do the results check?

| | 1919 | 1920 | Gain |
|---|---|---|---|
| Connecticut | 84,902 | 100,550 | |
| Delaware | 12,066 | 16,600 | |
| Idaho | 31,925 | 46,360 | |
| Illinois | 389,135 | 497,318 | |
| Iowa | 327,500 | 405,182 | |
| Kansas | 186,109 | 253,896 | |
| Texas | 250,083 | 298,234 | |
| Wisconsin | 196,844 | 280,452 | |

**6.** The highest and the lowest points of each continent are stated in the table below. Compute the greatest elevation above the lowest point for each continent.

| | Highest Point (Ft.) | Lowest Point (Ft.) (Below sea level) |
|---|---|---|
| North America | 20,300 | 276 |
| South America | 23,080 | Sea level |
| Europe | 15,782 | 86 |
| Asia | 29,002 | 1,290 |
| Africa | 19,320 | 150 |

**7.** How many of the following examples in division can you finish in six minutes?

*a.* 89,175 ÷ 39
*b.* 2,137,512 ÷ 372
*c.* 187,931 ÷ 68
*d.* 784,267 ÷ 269
*e.* 199,988 ÷ 578
*f.* 1,728 ÷ 144
*g.* 148,785 ÷ 455
*h.* 3,157 ÷ 287
*i.* 275,258 ÷ 458
*j.* 198,001 ÷ 67
*k.* 210,897 ÷ 321
*l.* 64,170 ÷ 345

**8.** If you are told the divisor, quotient, and remainder how can you find the dividend?

**9.** If you know your balance in the bank at the end of October, the amount withdrawn during the month and the amount deposited

during the month, how can you compute the balance which you had in the bank at the beginning of the month?

10. If you are told the remainder, the quotient, and the dividend, how can you find the divisor?

11. What do you consider the best check for addition? subtraction? multiplication? division?

12. A plumber has the following stock record of pipe at the time of taking inventory. Complete the table by filling the blanks.

| Kind of Pipe | Feet On Hand Jan. 1 | Feet Bought during Year | Used during Year | Inventory Dec. 31 |
|---|---|---|---|---|
| ½ in. | 745 | 2,230 | | 540 |
| ¾ in. | 1,472 | 1,750 | | 1,200 |
| 1 in. | 2,485 | 1,480 | | 1,260 |
| 1¼ in. | 890 | 2,690 | | 480 |
| 1½ in. | 2,150 | 1,890 | | 1,050 |
| 2 in. | 1,340 | 2,400 | | 1,280 |
| 3 in. | 520 | 1,200 | | 215 |
| 4 in. | 260 | 200 | | 120 |

13. A cashier in a lunch room takes in checks as indicated below. Compute the total amount taken in.

| Check | Number | Check | Number | Check | Number |
|---|---|---|---|---|---|
| 15¢ | 18 | 25¢ | 47 | 60¢ | 9 |
| 18¢ | 7 | 34¢ | 18 | 72¢ | 3 |
| 20¢ | 14 | 45¢ | 17 | 75¢ | 5 |
| 23¢ | 34 | 55¢ | 13 | 82¢ | 4 |

14. A steamer has a storage capacity of 192,000 cubic feet for coal. How many long tons of coal can be carried in the steamer, if a long ton occupies 43 cubic feet?

# CHAPTER V

## AVERAGE

**35. Simple Average.** A boy walked 9 miles on Monday, 5 miles on Tuesday, and 10 miles on Wednesday. If he had divided the trip into three equal distances, how many miles would he have walked each day? In other words, what was the *average* distance walked each day?

A man owned a five-acre field. One acre yielded 71 bushels of potatoes, another 77 bushels, the third 85 bushels, the fourth 93 bushels, and the fifth 64 bushels. What was the total crop? What was the *average* crop per acre?

What two processes are usually involved in computing an *average?*

The average of several quantities is determined by dividing the sum of the quantities by the number of quantities added.

### Written Work

**1.** The noon temperatures in a certain city for a week were as follows: Monday, 62°; Tuesday, 70°; Wednesday, 74°; Thursday, 68°; Friday, 62°; Saturday, 70°; Sunday, 76°. What was the average noon temperature for the week?

**2.** A newsdealer sold papers as indicated in the table. Find the average number of each sold.

| | Mon. | Tues. | Wed. | Thurs. | Fri. | Sat. | Avg. |
|---|---|---|---|---|---|---|---|
| Sentinel . . . | 39 | 47 | 63 | 82 | 74 | 67 | |
| Argus . . . . | 13 | 19 | 27 | 31 | 29 | 26 | |
| Tribune . . . | 39 | 42 | 48 | 51 | 39 | 41 | |
| Recorder. . . | 28 | 43 | 56 | 29 | 37 | 29 | |

**3.** The receipts of a certain department of a store for one week were as follows: Monday, $396.40; Tuesday, $283.70; Wednesday, $468.14; Thursday, $184.68; Friday, $369.58; Saturday, $296.40. Compute the average daily receipts.

4. A certain city reported the rainfall by months, beginning with January, as follows: 1.2, 1.4, 1.4, 1.8, 1.0, .4, .5, .7, 1.0, 1.2, 1.3, 1.5 inches. Compute the average monthly rainfall.

5. A polar expedition recorded the noon temperatures for six consecutive days as follows: 45°, 52°, 48°, 47°, 51°, and 55°. All temperatures were below zero. Compute the average daily temperature.

6. A subscription was taken to secure funds to purchase a gift.

| | | |
|---|---|---|
| 3 men gave | $1.00 | each |
| 1 man gave | .75 | |
| 2 men gave | 1.50 | each |
| 1 man gave | .60 | |
| 4 men gave | .90 | each |

Find the average amount given.

7. The following table shows the number of men employed and the total weekly wages in each of the four departments of a factory. Find the average wage in each department and the average wage for the four departments.

| Department | Number of Men Employed | Total Wage | Average Wage |
|---|---|---|---|
| A | 37 | $ 960 | |
| B | 12 | 290 | |
| C | 42 | 1,054 | |
| D | 16 | 318 | |

### Oral Review

1. What is the difference in the meaning of the following statements?

*a.* Each of the 300 employees in our factory earns $2.50 per day.

*b.* The average daily wages of employees in our factory is $2.50 per day.

2. What would you have to know and how would you proceed to find:

*a.* The average weight of twenty boxes?

*b.* The average value of a herd of cattle?

*c.* The average number of miles a train traveled per hour, going from Chicago to St. Louis?

*d.* The average age of the students in your class?

*e.* The average daily sales of a clerk?

**3.** What could you find if you were told:

*a.* The average value of farm land per acre in your state, and the number of acres of farm land?

*b.* The average daily sales of a clerk during the week days of June?

*c.* The average monthly grocery bill of your family?

**4.** If you were told:

*a.* The daily circulation of the *Evening Herald* for the twenty-six week days of July, and the average daily circulation of the *Evening Transcript* for the same month, how could you compare their circulation?

*b.* The average daily wages of A, who worked 265 days last year and the average daily wages of B, who worked 303 days, how could you find the yearly wages of each, and which one earned the larger amount?

*c.* The difference between the average daily outputs of two shoe factories, how could you find the difference in their production for a year of 300 working days?

**5.** What would you have to know and what would you do to find the average daily speed of an ocean liner on a given voyage?

**6.** Several boys worked for a farmer picking strawberries. One earned $5.00 more than the average of the other boys' earnings. What else must you know and what would you do to find the total earnings of all the boys? What else would you have to know and how would you find the average wage cost of picking a quart of strawberries?

**7.** What facts would you need to know and how would you proceed to find the average height and the average weight of the boys and of the girls in your class?

**Written Work**

**1.** A merchant kept a record of the deliveries of goods made to his customers for a week. The record follows:

| Day | Deliveries |
|---|---|
| Monday | 213 |
| Tuesday | 187 |
| Wednesday | 208 |
| Thursday | 221 |
| Friday | 168 |
| Saturday | 251 |

*a.* What was the average number of deliveries per day?

*b.* The expense of running the delivery cars for a week, including care of cars, repairs and wages, and interest on the money invested in cars, was $84.75. What was the average cost per delivery?

2. Five clerks in a store sold the following amounts of goods during a month:

| | |
|---|---|
| A. | $1,246.50 |
| B. | 1,076.85 |
| C. | 944.90 |
| D. | 1,388.20 |
| E. | 1,109.75 |

*a.* What was the average amount of sales per clerk?

*b.* Which clerks sold more than the average? Which clerks sold less than the average? Did any clerk sell just the average?

## CHAPTER VI

### FACTORS AND MULTIPLES

**36. Terms.** An **integer** is a number of whole units, such as 1, 7, 12.

The **factors** of a number are the integers which, multiplied together, produce the given number. Thus, 3 and 5 are factors of 15; 3 and 6 are factors of 18.

A **factor** of a number is a divisor of that number.

A number which is not exactly divisible by any other number (except 1) is called a **prime number.** Thus, 1, 3, 5, 7, 11, and 13 are prime numbers.

Numbers are said to be **prime to each other** when they have no common factor except 1. Thus, 10 and 27 are **prime to each other,** although neither is a prime number.

**37. Test of Divisibility of Numbers.** A number is divisible by:

*a.* Two, if it ends with 0, 2, 4, 6, or 8.

*b.* Three, if the sum of its digits is divisible by 3.

*c.* Four, if the number expressed by its last two digits is divisible by 4.

*d.* Five, if it ends in 0 or 5.

*e.* Six, if it is divisible by both 2 and 3.

*f.* Eight, if the number expressed by its last three digits is divisible by 8.

*g.* Nine, if the sum of its digits is divisible by 9.

*h.* Ten, if its right-hand digit is zero.

*i.* Eleven, if the difference between the sums of the numbers represented by the odd and even orders of digits is divisible by 11. Thus, 16,280 is divisible by 11, since $(8 + 6) - (0 + 2 + 1)$ is divisible by 11.

There is no simple method for testing divisibility by 7.

**38. Factoring** is the process of separating a number into its factors.

#### Oral Work

**1.** Learn the prime numbers from 1 to 100 so that you can recognize them at sight.

**2.** Apply the tests of divisibility in finding the prime factors of the following:

| | | | | | |
|---|---|---|---|---|---|
| 28 | 160 | 728 | 478 | 76 | 720 |
| 42 | 320 | 640 | 96 | 84 | 37 |
| 72 | 48 | 386 | 84 | 90 | 145 |
| 36 | 360 | 31 | 92 | 360 | 390 |
| 98 | 280 | 100 | 81 | 760 | 625 |

**3.** What numbers between 161 and 200 are divisible by 9?

**4.** What numbers between 746 and 800 are divisible by 6?

**5.** Name the factors of 36 which are not prime to each other.

**39. Cancellation** is the process of shortening certain computations involving division by removing or canceling equal factors from both dividend and divisor.

**Example.** Divide the product of 4, 9, 8, 36, 24, and 7 by the product of 18, 2, 8, 3, 14, and 4.

SOLUTION.
$$\frac{\overset{2}{\cancel{4}} \times \cancel{9} \times \cancel{8} \times \overset{18}{\cancel{36}} \times \overset{12}{\cancel{24}} \times \cancel{7}}{\underset{2}{\cancel{18}} \times \cancel{2} \times \cancel{8} \times \cancel{3} \times \underset{2}{\cancel{14}} \times \cancel{4}} = 36.$$

In all computations involving only multiplication and division, cancellation should be used when possible. Indicate the multiplication and division as in the illustration above, then cancel the common factors.

**Written Work**

Use cancellation when possible

**1.** $\dfrac{24 \times 36 \times 15}{16 \times 5 \times 9}$.

**2.** $\dfrac{4 \times 37 \times 16 \times 5 \times 60}{48 \times 32 \times 74}$.

**3.** $\dfrac{48 \times 32 \times 100 \times 360}{16 \times 50 \times 72}$.

**4.** $\dfrac{27 \times 64 \times 96 \times 38}{19 \times 16 \times 9 \times 2}$.

**5.** $\dfrac{130 \times 14 \times 18 \times 121 \times 15}{7 \times 27 \times 13 \times 11}$.

**6.** $\dfrac{144 \times 32 \times 63 \times 7}{16 \times 9 \times 28}$.

**7.** $\dfrac{1{,}728 \times 360 \times 100 \times 32 \times 3}{18 \times 144 \times 64 \times 75}$.

**8.** $\dfrac{21 \times 72 \times 160 \times 340 \times 27}{180 \times 36 \times 35}$.

**40. Greatest Common Divisor.** An integer that is a factor of two or more numbers is called a **common divisor,** or a **common factor** of those numbers.

The **greatest common divisor** of two or more numbers is the greatest factor common to the numbers. "Greatest common divisor" is usually expressed as g. c. d.

**Example.** Find the greatest common divisor of 12, 20, and 36.

SOLUTION.
$$12 = 2 \times 2 \times 3$$
$$20 = 2 \times 2 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3.$$

The factor 2 occurs twice in all the numbers and none of the other factors occurs in all the numbers, hence, 4 is the g. c. d. of 12, 20, and 36.

To find the g. c. d. of two or more numbers, *separate the numbers into prime factors and find the product of the prime factors common to the numbers.*

**Written Work**

Find the g. c. d. of:

**1.** 12, 18, 24.
**2.** 24, 60, 72.
**3.** 15, 20, 30.
**4.** 60, 90, 100.
**5.** 84, 32, 60.
**6.** 60, 96, 120.
**7.** 18, 32, 48.
**8.** 27, 36, 45.
**9.** 360, 120, 40.
**10.** 121, 88, 242.

**11.** Find the g. c. d. of 8 ft. and 12 ft.

**12.** Find the g. c. d. of $48 and $60.

**13.** What is the length of the longest logs, all of the same length, that can be cut from pieces of timber whose lengths are 36, 54, and 72 feet respectively?

**41. Least Common Multiple.** A **multiple** of a number is an integral number of times that number. Thus, 28 is a multiple of 7. 60 is a multiple of 12.

A **common multiple** of two or more numbers is a number that is a multiple of each of them. It is therefore divisible by each of them.

The **least common multiple** of two or more numbers is the least number that is a multiple of each of them.

Thus, 60 is the least common multiple of 12, 15, and 30.

**Example.** Find the least common multiple (l. c. m.) of 18, 20, and 24.

SOLUTION.

$$18 = 2 \times 3 \times 3$$
$$20 = 2 \times 2 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3.$$
$$\text{l.c.m.} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360.$$

To find the least common multiple of two or more numbers, *separate each number into its prime factors. Find the product of these factors, using each factor the greatest number of times it occurs in any one of the given numbers.*

**Written Work**

Find the l. c. m. of the following:

1. 12, 15.
2. 8, 12.
3. 6, 15.
4. 7, 8, 12.
5. 8, 9, 12, 15.
6. 24, 36, 60.
7. 8, 12, 16.
8. 36, 24, 75.
9. 360, 345.
10. 75, 130, 190.
11. 425, 345, 336.
12. 360, 240, 420.

Find the g. c. d. and the l. c. m. of the following:

13. 60, 80, 95.
14. 36, 75, 48.
15. 480, 360, 120.
16. 24, 30, 36.

# CHAPTER VII

## COMMON FRACTIONS

**42. Terms.** A **fraction** is one or more of the equal parts of a unit.

A **common fraction** is usually expressed by writing one figure above and one below a short line; thus, $\frac{2}{3}$ or $^2/_3$.

The **numerator** of a fraction is the number which shows how many of the equal parts of the unit are taken. It is written above the line.

The **denominator** of a fraction is the number which shows into how many equal parts the unit is divided. It is written below the line.

The numerator and the denominator are called the **terms** of the fraction. Thus, 3 and 4 are the terms of the fraction $\frac{3}{4}$.

A common fraction may be either *proper* or *improper*.

A **proper fraction** is one whose numerator is less than its denominator, as $\frac{3}{4}$.

An **improper fraction** is one whose numerator is equal to or greater than its denominator as $\frac{4}{4}$ or $\frac{5}{4}$.

A **mixed number** consists of a whole number and a fraction, as $5\frac{2}{3}$.

### Oral Work

Which is greater, $\frac{1}{4}$ or $\frac{1}{7}$? $\frac{1}{9}$ or $\frac{1}{10}$? $\frac{1}{20}$ or $\frac{1}{12}$?

How is the value of a fraction affected by increasing the numerator, the denominator remaining the same?

### Reduction of Fractions

**43. Reducing Fractions to Lower Terms.** When the numerator and the denominator contain one or more common factors, the fraction may be reduced to a fraction of equivalent value expressed in lower terms.

Thus, $\frac{10}{12}$ may be reduced to the equivalent fraction $\frac{5}{6}$, by dividing both terms by 2. Similarly, $\frac{18}{24} = \frac{3}{4}$; $\frac{7}{21} = \frac{1}{3}$.

State a rule for reducing fractions to lower terms.

**44. Reducing Fractions to Higher Terms.** Fractions may be raised to equivalent fractions in higher terms, *by multiplying both numerator and denominator by the same number.*

**Example.** Express $\frac{3}{5}$ as an equivalent fraction whose denominator is 60.

SOLUTION. The given denominator, 5, must be multiplied by 12 to obtain the desired denominator, 60. Therefore, multiply both terms of the fraction by 12, and obtain the equivalent fraction $\frac{36}{60}$.

*Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.*

**45. Reducing Improper Fractions to Mixed Numbers.** An improper fraction may be reduced to a mixed number. The following example shows the method:

**Example.** Reduce $\frac{19}{4}$ to a mixed number.

SOLUTION. $19 \div 4 = 4\frac{3}{4}$.

State a rule for changing an improper fraction to a mixed number.

**46. Reducing Mixed Numbers to Improper Fractions.** A mixed number may be reduced to an improper fraction. The following example shows the method.

**Example.** Reduce $13\frac{3}{7}$ to an improper fraction.

SOLUTION. $7 \times 13 = 91$. $91 + 3 = 94$, the numerator of the improper fraction. The denominator of the fraction in the mixed number is retained as the denominator of the improper fraction.

$$13\tfrac{3}{7} = \tfrac{94}{7}.$$

### Oral Work

1. Reduce to lowest terms: $\frac{4}{12}$, $\frac{3}{18}$, $\frac{5}{10}$, $\frac{6}{24}$, $\frac{18}{30}$, $\frac{24}{28}$, $\frac{15}{30}$, $\frac{72}{84}$.
2. Express each of the following fractions with the denominator 16: $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{4}{8}$, $\frac{9}{4}$, $\frac{7}{8}$, $\frac{5}{2}$, $\frac{5}{1}$, 3.
3. Express with the denominator 36: $\frac{4}{9}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{4}{12}$, $\frac{1}{3}$, $\frac{8}{9}$, $\frac{7}{3}$, $\frac{8}{4}$, $\frac{1}{9}$, $\frac{7}{2}$.
4. Reduce to twenty-fourths: $\frac{5}{8}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{2}$, $\frac{7}{6}$, $\frac{1}{12}$, 2.
5. Reduce to lowest terms: $\frac{8}{12}$, $\frac{9}{15}$, $\frac{4}{16}$, $\frac{8}{10}$, $\frac{9}{12}$, $\frac{14}{16}$.
6. Reduce to fiftieths: $\frac{2}{5}$, $\frac{9}{10}$, $\frac{7}{25}$, $\frac{19}{10}$, $\frac{5}{2}$, $\frac{7}{10}$.
7. Reduce to improper fractions: $3\frac{1}{5}$, $5\frac{1}{3}$, $7\frac{1}{4}$, $8\frac{2}{3}$, $4\frac{3}{5}$, $7\frac{2}{7}$, $9\frac{4}{5}$, $8\frac{1}{4}$.
8. Reduce the following fractions to equivalent fractions in lowest terms: $\frac{9}{24}$, $\frac{16}{88}$, $\frac{20}{85}$, $\frac{18}{144}$, $\frac{28}{210}$, $\frac{27}{63}$, $\frac{30}{150}$, $\frac{21}{15}$, $\frac{72}{180}$, $\frac{24}{36}$, $\frac{8}{12}$, $\frac{18}{21}$.
9. Change the following fractions as indicated:

$\frac{3}{4}$ to an equivalent fraction whose denominator is 20.
$\frac{7}{8}$ to an equivalent fraction whose denominator is 64.
$\frac{4}{9}$ to an equivalent fraction whose denominator is 63.
$\frac{2}{15}$ to an equivalent fraction whose denominator is 72.
$\frac{6}{11}$ to an equivalent fraction whose denominator is 121.
$\frac{5}{12}$ to an equivalent fraction whose denominator is 84.

**10.** Change the following improper fractions to mixed numbers: $\frac{8}{3}$, $\frac{17}{6}$, $\frac{39}{7}$, $\frac{25}{9}$, $\frac{31}{2}$, $\frac{84}{11}$, $\frac{488}{15}$, $\frac{300}{29}$, $\frac{341}{17}$, $\frac{83}{13}$, $\frac{216}{13}$, $\frac{235}{12}$.

**11.** Change the following mixed numbers to equivalent improper fractions: $3\frac{2}{3}$, $6\frac{4}{7}$, $8\frac{1}{8}$, $4\frac{3}{5}$, $8\frac{16}{25}$, $10\frac{17}{35}$, $14\frac{8}{13}$, $12\frac{7}{12}$, $18\frac{3}{20}$, $9\frac{5}{24}$.

**12.** When we reduce $\frac{8}{12}$ to $\frac{2}{3}$, do we increase or decrease the size of the fractional unit? Do we increase or decrease the number of fractional units?

**13.** Which fraction is the greatest, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{13}{15}$?

## Addition and Subtraction of Fractions

**47. Finding the Least Common Denominator.** Only like quantities can be added or subtracted. It is not possible to add or subtract 4 lb. and 2 oz. or $\frac{1}{2}$ and $\frac{1}{4}$ until they are reduced to the same denomination.

The common denominator should be the smallest number which will exactly contain all of the denominators. It may usually be found by inspection.

Any number which contains all of the factors of another number will also contain that number. Thus, 24 contains all of the factors of 12, and it therefore contains 12.

The common denominator must, therefore, contain the prime factors of the denominators.

Hence, to find the least common denominator of two or more fractions, *find the prime factors of the denominators; multiply the different prime factors, using each factor the greatest number of times it is contained in any denominator.*

**Problem.** What is the least common denominator of fractions with denominators 8, 9, and 12?

Solution.

$8 = 2 \times 2 \times 2.$
$9 = 3 \times 3.$
$12 = 2 \times 2 \times 3.$
l.c.m. of denominators $= 2 \times 2 \times 2 \times 3 \times 3.$

**48. Reducing Fractions to a Common Denominator.** After the least common denominator has been found, the fractions may be reduced to this denominator.

**Example.** Express $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{5}{6}$ with a common denominator.

SOLUTION. The least common denominator is 30.

$$\frac{3}{5} = \frac{18}{30}.$$
$$\frac{2}{3} = \frac{20}{30}.$$
$$\frac{5}{6} = \frac{25}{30}.$$

State the process of reducing fractions to a common denominator.

**Written Work**

Express the fractions in each example as fractions having the least common denominator:

1. $\frac{1}{2}$ and $\frac{1}{3}$.
2. $\frac{1}{3}$ and $\frac{1}{4}$.
3. $\frac{1}{4}$ and $\frac{1}{8}$.
4. $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{12}$.
5. $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{20}$.
6. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{12}$.
7. $\frac{3}{7}$, $\frac{2}{11}$, $\frac{5}{42}$.
8. $\frac{7}{12}$, $\frac{9}{32}$, $\frac{13}{56}$, $\frac{4}{9}$, $\frac{5}{14}$.
9. $\frac{3}{20}$, $\frac{17}{36}$, $\frac{8}{49}$, $\frac{12}{35}$.
10. $\frac{11}{18}$, $\frac{9}{28}$, $\frac{1}{42}$, $\frac{11}{15}$.
11. $\frac{17}{96}$, $\frac{23}{42}$, $\frac{25}{63}$, $\frac{19}{27}$.

**49. Rule for Addition of Fractions.**

*Reduce the fractions to a common denominator. Add the numerators to form the numerator of the sum. The common denominator is the denominator of the sum. Reduce the result to the simplest form.*

Proper fractions should be reduced to their lowest terms.
Improper fractions should be changed to mixed numbers.

**Examples.** 1. Add $\frac{2}{3}$ and $\frac{3}{4}$.

SOLUTION. The least common multiple of the denominators is 12.

$$\frac{2}{3} = \frac{8}{12}.$$
$$\frac{3}{4} = \frac{9}{12}.$$
$$\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}.$$

2. Add $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{10}$.

SOLUTION. The least common multiple of the denominators is 60.

$$\frac{3}{4} = \frac{45}{60}.$$
$$\frac{5}{6} = \frac{50}{60}.$$
$$\frac{7}{10} = \frac{42}{60}.$$
$$\frac{45}{60} + \frac{50}{60} + \frac{42}{60} = \frac{137}{60} = 2\frac{17}{60}.$$

**50. Rule for Subtraction of Fractions.**

*The process is the same as for Addition of Fractions except that the numerators are subtracted instead of added after the fractions have been reduced to a common denominator.*

**Oral Work**

State the sum, then the difference of the following:

| | | | | |
|---|---|---|---|---|
| **1.** $\frac{1}{2}$, $\frac{1}{3}$. | **2.** $\frac{1}{3}$, $\frac{1}{4}$. | **3.** $\frac{1}{5}$, $\frac{1}{7}$. | **4.** $\frac{1}{4}$, $\frac{1}{6}$. | **5.** $\frac{1}{3}$, $\frac{1}{5}$. |
| **6.** $\frac{1}{2}$, $\frac{1}{8}$. | **7.** $\frac{1}{7}$, $\frac{1}{9}$. | **8.** $\frac{1}{9}$, $\frac{1}{10}$. | **9.** $\frac{2}{3}$, $\frac{2}{7}$. | **10.** $\frac{5}{6}$, $\frac{3}{4}$. |
| **11.** $\frac{7}{9}$, $\frac{2}{3}$. | **12.** $\frac{5}{6}$, $\frac{4}{5}$. | **13.** $\frac{4}{7}$, $\frac{1}{3}$. | **14.** $\frac{3}{4}$, $\frac{3}{5}$. | **15.** $\frac{3}{5}$, $\frac{3}{7}$. |

NOTE. The Short Methods for the Addition and Subtraction of Fractions, explained on pages 72–75, may be studied at this time.

**51. Rule for Addition of Mixed Numbers.**

*Add the fractions.*
*If the sum is an improper fraction, reduce it to a mixed number.*
*Add the sum of the fractions to the sum of the integers.*

**Example.** Add $3\frac{3}{4}$ and $4\frac{7}{8}$.

SOLUTION.

$$\frac{3}{4} + \frac{7}{8} = \frac{13}{8} = 1\frac{5}{8}.$$
$$3 + 4 = 7.$$
$$7 + 1\frac{5}{8} = 8\frac{5}{8}.$$

**Written Work**

Find the sums:

| | |
|---|---|
| **1.** $4\frac{1}{2} + 3\frac{1}{3}$. | **2.** $2\frac{1}{5} + 5\frac{1}{7}$. |
| **3.** $7\frac{1}{8} + 5\frac{2}{3}$. | **4.** $17\frac{1}{5} + 18\frac{2}{7}$. |
| **5.** $275\frac{2}{3} + 78\frac{1}{5}$. | **6.** $3\frac{7}{8} + 5\frac{1}{4}$. |
| **7.** $9\frac{1}{10} + 17\frac{2}{5}$. | **8.** $3\frac{1}{3} + 4\frac{7}{8} + 3\frac{5}{12}$. |
| **9.** $23\frac{1}{3} + 16\frac{5}{12} + 7\frac{3}{4}$. | **10.** $18\frac{5}{6} + 42\frac{1}{10} + 37\frac{3}{5}$. |

**52. Rule for Subtraction of Mixed Numbers.**

If the fraction of the minuend is larger than that of the subtrahend, the above rule for addition can be easily changed to apply to subtraction.

**Example.** Subtract $4\frac{2}{5}$ from $7\frac{5}{6}$.

SOLUTION.

$$\frac{5}{6} - \frac{2}{5} = \frac{13}{30}.$$
$$7 - 4 = 3.$$

*Result* $3\frac{13}{30}$.

If the fraction of the minuend is smaller than that of the subtrahend, we may proceed as follows:

**Example.** Subtract $4\frac{2}{3}$ from $9\frac{1}{5}$.

SOLUTION. $\frac{2}{3}$ cannot be subtracted from $\frac{1}{5}$, but $9\frac{1}{5}$ may be regarded as $8\frac{6}{5}$. $\frac{6}{5} - \frac{2}{3} = \frac{8}{15}$. Therefore, $9\frac{1}{5} - 4\frac{2}{3} = 4\frac{8}{15}$.

**Written Work**

Add as indicated:

1. $\frac{3}{7} + \frac{2}{5}$.
2. $\frac{6}{13} + \frac{8}{9}$.
3. $\frac{7}{9} + \frac{4}{15}$.
4. $\frac{2}{15} + \frac{7}{24}$.
5. $\frac{3}{5} + \frac{9}{25} + \frac{9}{16}$.
6. $\frac{15}{32} + \frac{7}{24} + \frac{11}{36}$.
7. $3\frac{5}{8} + \frac{1}{7}$.
8. $12\frac{9}{16} + \frac{7}{18}$.
9. $13\frac{5}{7} + 22\frac{9}{15}$.

Subtract as indicated:

10. $\frac{4}{7} - \frac{3}{11}$.
11. $\frac{15}{32} - \frac{7}{20}$.
12. $2\frac{9}{14} - \frac{3}{16}$.
13. $4\frac{7}{8} - \frac{6}{7}$.
14. $3\frac{2}{3} - 2\frac{1}{2}$.
15. $4\frac{7}{9} - 2\frac{1}{6}$.
16. $5\frac{9}{25} - 2\frac{8}{9}$.

Add upward and across:

17.
$$\begin{array}{l} \frac{1}{4} + \frac{1}{3} + \frac{1}{2} \\ \frac{2}{3} + \frac{3}{4} + \frac{5}{12} \\ \frac{5}{6} + \frac{7}{8} + \frac{7}{6} \\ \hline ? + ? + ? \end{array}$$

18.
$$\begin{array}{l} \frac{5}{8} + \frac{3}{16} + \frac{2}{3} \\ \frac{7}{24} + \frac{5}{6} + \frac{1}{24} \\ \frac{1}{8} + \frac{5}{6} + \frac{7}{4} \\ \hline ? + ? + ? \end{array}$$

19. Add upward; subtract across:

$$\begin{array}{l} 14\frac{2}{3} - 4\frac{1}{5} \\ 18\frac{5}{6} - 7\frac{2}{3} \\ 5\frac{1}{12} - 3\frac{1}{4} \\ 15\frac{3}{4} - 7\frac{5}{6} \\ \hline ? \quad - \quad ? \end{array}$$

## MULTIPLICATION AND DIVISION OF FRACTIONS

### 53. Rule for Multiplication of Fractions.

*Multiply the numerators to form the numerator of the product. Multiply the denominators to form the denominator of the product. Reduce the product to its simplest form.*

Thus, $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$.

NOTE. — The word "of" placed between fractions indicates multiplication.

Mixed numbers should be changed to improper fractions before multiplying.

**Example.** $8 \times 6\frac{3}{4} \times \frac{2}{3} \times 3\frac{4}{5} = ?$

SOLUTION. Reducing mixed numbers to improper fractions:

$$\frac{8}{1} \times \frac{27}{4} \times \frac{2}{3} \times \frac{19}{5} = ?$$

$$\frac{\overset{2}{\cancel{8}}}{1} \times \frac{27}{\underset{1}{\cancel{4}}} \times \frac{2}{\underset{3}{\cancel{3}}} \times \frac{19}{5}.$$

Multiplying the resulting numerators: $2 \times 9 \times 2 \times 19 = 684$, the numerator of the product.

Multiplying the resulting denominators: $1 \times 5 = 5$, the denominator of the product.

Changing to a mixed number: $\frac{684}{5} = 136\frac{4}{5}$, the product.

### Written Work

Perform the following multiplications:

1. $\frac{2}{3} \times \frac{4}{5}$.
2. $\frac{3}{8} \times \frac{5}{7}$.
3. $\frac{6}{5} \times \frac{8}{7}$.
4. $\frac{1}{2} \times \frac{5}{12}$.
5. $\frac{3}{4} \times \frac{7}{8}$.
6. $3\frac{1}{3} \times 4\frac{2}{3}$.
7. $\frac{8}{9} \times \frac{3}{4}$.
8. $\frac{2}{3}$ of $\frac{3}{4} \times \frac{1}{2}$.
9. $\frac{3}{7} \times \frac{3}{5}$ of $\frac{4}{3}$.
10. $\frac{8}{12} \times \frac{5}{7} \times \frac{4}{9} \times \frac{3}{10}$.
11. $5\frac{2}{3} \times 3\frac{1}{4} \times 4\frac{2}{5}$.
12. $\frac{3}{8} \times 4 \times \frac{7}{2} \times \frac{3}{15}$.

## 54. Rule for Division of Fractions.

*Invert the divisor, and multiply the fractions, using cancellation when possible.*

**Examples.** 1. $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

2. $\frac{8}{9} \div \frac{4}{5} = \frac{8}{9} \times \frac{5}{4} = \frac{10}{9} = 1\frac{1}{9}$.

3. $8\frac{2}{3} \div \frac{4}{9} = ?$

SOLUTION.

$$8\frac{2}{3} \div \frac{4}{9} = \frac{\overset{13}{\cancel{26}}}{\underset{1}{\cancel{3}}} \times \frac{9}{\underset{2}{\cancel{4}}} = \frac{39}{2} = 19\frac{1}{2}.$$

### Written Work

1. $\frac{3}{5} \div \frac{2}{3}$.
2. $\frac{4}{7} \times \frac{2}{3}$.
3. $\frac{2}{3}$ of $\frac{3}{4} \times \frac{1}{6}$.
4. $5\frac{1}{2} \div 4\frac{2}{3}$.
5. $7\frac{1}{8} \div 3\frac{1}{4}$.
6. $\frac{3}{8} + \frac{2}{3}$.
7. $\frac{4}{7} - \frac{2}{5}$.
8. $3\frac{1}{5} + 5\frac{2}{3}$.
9. $\frac{3}{4}$ of $\frac{3}{7}$.
10. $9\frac{1}{2} \times 12\frac{2}{3} \div 3\frac{1}{4}$.
11. $\frac{7}{8}$ of $\frac{2}{11} \div \frac{14}{22}$.
12. $\frac{3}{5} + \frac{2}{9}$.
13. $4\frac{2}{5} - 3\frac{1}{3} + 5\frac{3}{4}$.
14. $17\frac{2}{3} \times 5\frac{1}{2} \div 4\frac{5}{12}$.
15. $\frac{3}{8}$ of $\frac{2}{3} \div \frac{2}{5}$ of $\frac{4}{9}$.

### Review Exercises

**1.** Multiply each of the fractions in the column at the left by each of the fractions at the top. Enter the products in the spaces formed by the horizontal and vertical lines. (For example, the space marked "x" is to contain the product of $\frac{5}{8}$ and $\frac{7}{8}$.)

| Multiply | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{5}{6}$ | $\frac{7}{8}$ |
|---|---|---|---|---|
| $\frac{5}{6}$ | | | | |
| $3\frac{1}{3}$ | | | | |
| $\frac{5}{8}$ | | | | × |
| $\frac{7}{8}$ | | | | |

**2.** In the following form divide each of the fractions in the column at the left by the fractions in the upper row. Rule a blank similar to the form given and record the quotients in the proper spaces.

| | $\frac{3}{5}$ | $\frac{7}{8}$ |
|---|---|---|
| $\frac{5}{6}$ | | |
| $\frac{8}{3}$ | | |
| $\frac{3}{4}$ | | |
| $\frac{2}{5}$ | | |
| $\frac{4}{9}$ | | |
| $\frac{5}{4}$ | | |
| $\frac{3}{8}$ | | |
| $\frac{5}{12}$ | | |
| $\frac{8}{9}$ | | |

### Miscellaneous Problems

1. If $\frac{1}{3}$ of a number is 5, what is the number?
2. If $\frac{2}{3}$ of a number is 12, what is the number?
3. If $\frac{4}{7}$ of a number is 20, what is the number?
4. If $\frac{3}{5}$ of $\frac{2}{3}$ of a number is 15, what is the number?
5. After spending $\frac{1}{4}$ of his monthly salary for board and room, and $\frac{1}{10}$ of it for clothes, a man has \$78 left. What is his salary?
6. If 5 yards of cloth cost $47\frac{1}{2}$¢, what will 1 yard cost at the same rate?
7. A train runs $\frac{2}{3}$ of a mile in $\frac{5}{8}$ of a minute. What is the rate per hour?
8. Which will give the larger result, multiplying or dividing an integer by a proper fraction? Multiplying or dividing by an improper fraction?
9. If you are told the amount of a man's wages and the fractional part of his wages which he spent, how can you find the fractional part which he saved?
10. State two ways of finding the amount saved, and state which you think is the easier.
11. If you were told what $\frac{3}{4}$ of $\frac{2}{3}$ of a number is, how could you find the number?
12. A man owned $\frac{3}{4}$ of a farm and sold $\frac{1}{5}$ of what he owned. If he received \$3,000 for the land sold, what was the value of the entire farm at the same rate?
13. A mechanic works 8 hours a day. Two and one half hours are spent at a bench and the remainder at a machine. What fractional part of the day is spent at a machine?
14. In a certain family $\frac{1}{4}$ of the yearly income is spent for rent, $\frac{1}{5}$ for clothes, $\frac{1}{4}$ for food. and $\frac{1}{2}$ of the remainder for travel. If the yearly income is \$3,000, how much is spent for travel?
15. A man sold $\frac{2}{3}$ of an acre of land for \$76. At that rate what is his farm of 120 acres worth?
16. $\frac{4}{7}$ of the number of students in a high school are girls. The number of boys is 120. How many students are there in the school?
17. Mr. Jones bought some land and sold it so as to realize $\frac{1}{5}$ more than the cost. If the selling price was \$360, what did he pay for the land?
18. Mr. Williams sold some goods for $\frac{1}{6}$ less than the cost. If he received \$70 for the goods, what was the cost?

**19.** In making up a certain cake recipe for 6 people, $1\frac{1}{2}$ cups of sugar and $\frac{2}{3}$ of a cup of butter are used. How much sugar and butter should be used in making up the recipe for 4 people?

**20.** A boy works $3\frac{1}{2}$ days at the rate of \$6.50 a week of 6 days. How much does he earn?

**21.** Find the total cost of $8\frac{2}{3}$ yards of ribbon at 28 cents per yard, $3\frac{1}{2}$ yards of insertion at $12\frac{1}{2}$ cents per yard, and $5\frac{1}{3}$ yards of silk at \$1.35 per yard.

**22.** A dealer bought oranges at the rate of 4 for 10 cents and sold them at the rate of 3 for 10 cents. How much did he gain on each orange?

**23.** Mr. Rankin owned $\frac{2}{3}$ of a store and sold $\frac{2}{3}$ of his share to Mr. Johnson for \$3,600. At this rate what was the store worth?

**24.** If a man drives his automobile 18 miles in $\frac{2}{3}$ of an hour, how long would he require, at the same rate, to travel 60 miles?

**25.** When $\frac{3}{4}$ of a yard of cloth costs \$1.80, what is the price of $\frac{4}{5}$ of a yard?

**26.** A girl grew $1\frac{1}{4}$ inches during one year and $\frac{5}{8}$ of an inch during the next year. How much more did she grow during the first year than during the second?

**27.** The entire length of a skirt is to be $24\frac{3}{4}$ inches, and the ruffle at the bottom is to be $2\frac{7}{8}$ inches wide. What will be the length of the skirt above the ruffle?

**28.** The record for the hundred yard dash is $9\frac{3}{5}$ seconds. A boy can run the distance in $11\frac{1}{5}$ seconds. What is the difference between his time and the record time?

**29.** A farmer raised $296\frac{1}{3}$ bushels of wheat on $15\frac{1}{4}$ acres. What was the average yield per acre?

**30.** Three and one half bushels of seed were sown and the yield was $21\frac{1}{2}$ bushels. What was the average yield per bushel of seed?

## Short Methods Involving Fractions

**55. To add fractions when the numerators are the same.**

*Add the denominators and multiply this sum by the common numerator to obtain the numerator of the result. Multiply the denominators to obtain the denominator of the result. Reduce to simplest form.*

**Examples.** 1. Add $\frac{1}{3}$ and $\frac{1}{7}$.

Solution. $3 + 7 = 10$, the numerator of the result.
$3 \times 7 = 21$, the denominator of the result.
Hence, $\frac{1}{3} + \frac{1}{7} = \frac{10}{21}$.

**2.** Add $\frac{2}{5}$ and $\frac{2}{9}$.

SOLUTION. $5 + 9 = 14$. $14 \times 2 = 28$, the numerator of the result.
$5 \times 9 = 45$, the denominator of the result.
Hence, $\frac{2}{5} + \frac{2}{9} = \frac{28}{45}$.

### Oral Work

Add as indicated:

| | | | |
|---|---|---|---|
| **1.** $\frac{1}{5} + \frac{1}{3}$. | **2.** $\frac{1}{7} + \frac{1}{9}$. | **3.** $\frac{1}{5} + \frac{1}{8}$. | **4.** $\frac{1}{9} + \frac{1}{16}$. |
| **5.** $\frac{1}{7} + \frac{1}{8}$. | **6.** $\frac{1}{12} + \frac{1}{7}$. | **7.** $\frac{1}{9} + \frac{1}{14}$. | **8.** $\frac{2}{7} + \frac{2}{13}$. |
| **9.** $\frac{1}{3} + \frac{1}{6}$. | **10.** $\frac{2}{3} + \frac{2}{7}$. | **11.** $\frac{2}{11} + \frac{2}{7}$. | **12.** $\frac{3}{5} + \frac{3}{7}$. |
| **13.** $\frac{3}{8} + \frac{3}{16}$. | **14.** $\frac{3}{5} + \frac{3}{11}$. | **15.** $\frac{3}{8} + \frac{3}{5}$. | **16.** $\frac{2}{3} + \frac{2}{5}$. |

**56. To subtract fractions when the numerators are the same.**

*Subtract the denominators and multiply this difference by the common numerator to form the numerator of the result. Multiply the denominators to form the denominator of the result. Reduce to simplest form.*

**Example.** Subtract $\frac{2}{5}$ from $\frac{2}{3}$.

SOLUTION. $5 - 3 = 2$. $2 \times 2 = 4$, the numerator of the result.
$5 \times 3 = 15$, the denominator of the result.
Hence, $\frac{2}{3} - \frac{2}{5} = \frac{4}{15}$.

### Oral Work

Perform the following subtractions:

| | | |
|---|---|---|
| **1.** $\frac{1}{6} - \frac{1}{8}$. | **2.** $\frac{1}{7} - \frac{1}{13}$. | **3.** $\frac{1}{8} - \frac{1}{12}$. |
| **4.** $\frac{2}{7} - \frac{2}{15}$. | **5.** $\frac{3}{7} - \frac{3}{16}$. | **6.** $\frac{5}{9} - \frac{5}{14}$. |

**57. To add two fractions by "cross multiplication."**

*Multiply the numerator of each fraction by the denominator of the other fraction. Add these products to form the numerator. Multiply the denominators to form the denominator. Reduce to lowest terms.*

**Example.** Add $\frac{2}{3}$ and $\frac{3}{4}$.

SOLUTION. $2 \times 4 = 8$.
$3 \times 3 = 9$.
$9 + 8 = 17$, the numerator.
$3 \times 4 = 12$, the denominator.
Hence, $\frac{2}{3} + \frac{3}{4} = \frac{17}{12}$, $\frac{17}{12} = 1\frac{5}{12}$.

### Oral Work

Add as indicated:

1. $\frac{3}{5} + \frac{2}{7}$.
2. $\frac{2}{3} + \frac{3}{7}$.
3. $\frac{1}{2} + \frac{3}{5}$.
4. $\frac{3}{10} + \frac{7}{16}$.
5. $\frac{3}{5} + \frac{7}{8}$.
6. $\frac{3}{5} + \frac{7}{8}$.
7. $\frac{5}{12} + \frac{3}{8}$.
8. $\frac{6}{7} + \frac{6}{11}$
9. $\frac{5}{7} + \frac{9}{16}$.

**58. To subtract fractions by "cross multiplication."**

*Multiply the numerator of each fraction by the denominator of the other fraction. Subtract these products to form the numerator. Multiply the denominators to form the denominator. Reduce to lowest terms.*

**Example.** Subtract $\frac{2}{3}$ from $\frac{5}{8}$.

SOLUTION.

$5 \times 3 = 15$.
$6 \times 2 = 12$.
$15 - 12 = 3$, the numerator.
$6 \times 3 = 18$, the denominator.

Hence, $\frac{5}{8} - \frac{2}{3} = \frac{3}{18}$, $\frac{3}{18} = \frac{1}{6}$.

### Written Work

Subtract as indicated:

1. $\frac{4}{7} - \frac{1}{3}$.
2. $\frac{8}{11} - \frac{3}{5}$.
3. $\frac{5}{8} - \frac{3}{10}$.
4. $\frac{7}{8} - \frac{7}{11}$.
5. $\frac{8}{15} - \frac{4}{11}$.
6. $\frac{13}{25} - \frac{4}{11}$.
7. $\frac{5}{9} - \frac{6}{7}$.
8. $\frac{4}{5} - \frac{2}{7}$.

**59. To find the approximate product of mixed numbers.**

*Multiply the integers. Multiply each integer by the other fraction to the nearest unit. Add these three products.*

**Example.** Multiply $13\frac{1}{2}$ by $6\frac{2}{3}$.

SOLUTION.

$$\begin{array}{r} 13\frac{1}{2} \\ 6\frac{2}{3} \\ \hline \end{array}$$

$13 \times 6 = 78$
$\frac{1}{2}$ of $6 = 3$
$\frac{2}{3}$ of $13 = 9$ (to the nearest unit).
$90$

### Written Work

Find approximate products:

| | | | | |
|---|---|---|---|---|
| 1. $43\frac{1}{3}$ | 2. $89\frac{1}{4}$ | 3. $76\frac{1}{4}$ | 4. $15\frac{2}{5}$ | 5. $98\frac{1}{6}$ |
| $62\frac{1}{5}$ | $7\frac{1}{2}$ | $82\frac{1}{7}$ | $40\frac{2}{7}$ | $47\frac{1}{3}$ |
| 6. $17\frac{3}{8}$ | 7. $13\frac{2}{7}$ | 8. $79\frac{2}{3}$ | 9. $38\frac{2}{5}$ | 10. $73\frac{5}{8}$ |
| $46\frac{1}{3}$ | $16\frac{1}{2}$ | $63\frac{1}{3}$ | $71\frac{1}{6}$ | $96\frac{7}{8}$ |

**60. To find the product of any two numbers ending in ½.**

(*a*) When the sum of the integers is an even number.

*To the product of the integers, add one half of their sum and annex ¼ to the result.*

**Example.** Multiply $39\frac{1}{2}$ by $3\frac{1}{2}$.

SOLUTION.

$$\begin{array}{r} 39\frac{1}{2} \\ 3\frac{1}{2} \\ \hline 39 \times 3 = 117 \\ \frac{1}{2} \text{ the sum of } 39 \text{ and } 3 = \underline{\ 21} \\ 138\frac{1}{4} \textit{ Result.} \end{array}$$

**Written Work**

Multiply:

| | | | | |
|---|---|---|---|---|
| **1.** $44\frac{1}{2}$ | **2.** $18\frac{1}{2}$ | **3.** $38\frac{1}{2}$ | **4.** $75\frac{1}{2}$ | **5.** $29\frac{1}{2}$ | **6.** $81\frac{1}{2}$ |
| $64\frac{1}{2}$ | $12\frac{1}{2}$ | $14\frac{1}{2}$ | $35\frac{1}{2}$ | $33\frac{1}{2}$ | $75\frac{1}{2}$ |

(*b*) When the sum of the integers is an odd number.

*To the product of the integers, add the result obtained by taking half of one less than their sum. To this result annex ¾.*

**Example.** Multiply $39\frac{1}{2}$ by $6\frac{1}{2}$.

SOLUTION.

$$\begin{array}{r} 39\frac{1}{2} \\ 6\frac{1}{2} \\ \hline 234 \\ 22 \\ \hline 256\frac{3}{4} \end{array}$$

**Written Work**

Multiply:

| | | | | |
|---|---|---|---|---|
| **1.** $73\frac{1}{2}$ | **2.** $57\frac{1}{2}$ | **3.** $92\frac{1}{2}$ | **4.** $35\frac{1}{2}$ | **5.** $98\frac{1}{2}$ |
| $42\frac{1}{2}$ | $24\frac{1}{2}$ | $47\frac{1}{2}$ | $58\frac{1}{2}$ | $17\frac{1}{2}$ |
| **6.** $29\frac{1}{2}$ | **7.** $83\frac{1}{2}$ | **8.** $76\frac{1}{2}$ | **9.** $43\frac{1}{2}$ | **10.** $98\frac{1}{2}$ |
| $36\frac{1}{2}$ | $96\frac{1}{2}$ | $19\frac{1}{2}$ | $46\frac{1}{2}$ | $23\frac{1}{2}$ |
| **11.** $48\frac{1}{2}$ | **12.** $95\frac{1}{2}$ | **13.** $73\frac{1}{2}$ | **14.** $47\frac{1}{2}$ | **15.** $27\frac{1}{2}$ |
| $72\frac{1}{2}$ | $81\frac{1}{2}$ | $71\frac{1}{2}$ | $33\frac{1}{2}$ | $71\frac{1}{2}$ |

## General Review of Common Fractions and the Fundamental Operations

Add the following:

| | | | | |
|---|---|---|---|---|
| **1.** | $\frac{1}{5} + \frac{1}{6}$ | $\frac{1}{7} + \frac{1}{8}$ | $32\frac{1}{2} + 17\frac{1}{3}$ | $43\frac{3}{4} + 9\frac{2}{5}$ |
| **2.** | $\frac{1}{2} + \frac{1}{3}$ | $\frac{1}{6} + \frac{3}{4}$ | $37\frac{2}{3} + 24\frac{1}{2}$ | $82\frac{2}{5} + 37\frac{7}{10}$ |
| **3.** | $\frac{4}{5} + \frac{2}{3}$ | $\frac{1}{2} + \frac{1}{5}$ | $42\frac{4}{5} + 31\frac{3}{4}$ | $46\frac{5}{8} + 17\frac{2}{3}$ |

Subtract the following:

| | | | | |
|---|---|---|---|---|
| **4.** | $\frac{3}{5} - \frac{1}{2}$ | $47\frac{4}{5} - 31\frac{1}{2}$ | $198\frac{4}{5} - 74\frac{2}{3}$ | $92\frac{3}{4} - 74\frac{1}{8}$ |
| **5.** | $\frac{2}{5} - \frac{1}{8}$ | $26\frac{3}{4} - 18\frac{2}{5}$ | $149\frac{3}{4} - 34\frac{7}{8}$ | $35\frac{2}{3} - 18\frac{1}{5}$ |
| **6.** | $\frac{1}{3} - \frac{1}{5}$ | $83\frac{2}{3} - 27\frac{1}{2}$ | $187\frac{2}{3} - 83\frac{5}{8}$ | $43\frac{5}{8} - 29\frac{2}{3}$ |

Multiply the following:

| | | | | |
|---|---|---|---|---|
| **7.** | $\frac{7}{8} \times \frac{3}{16}$ | $3\frac{1}{3} \times 62\frac{1}{2}$ | $72\frac{1}{5} \times 45$ | $142\frac{1}{2} \times 408\frac{2}{3}$ |
| **8.** | $\frac{2}{3} \times \frac{4}{15}$ | $5\frac{1}{2} \times 48$ | $38 \times 17\frac{5}{8}$ | $36\frac{1}{3} \times 60\frac{1}{2}$ |
| **9.** | $\frac{5}{6} \times 12\frac{1}{2}$ | $6\frac{1}{3} \times 7\frac{1}{4}$ | $24\frac{1}{5} \times 4\frac{1}{4}$ | $16\frac{2}{3} \times 37\frac{1}{2}$ |

Divide the following:

| | | | | |
|---|---|---|---|---|
| **10.** | $\frac{3}{4} \div \frac{7}{8}$ | $2\frac{1}{2} \div \frac{7}{8}$ | $16\frac{2}{3} \div \frac{2}{3}$ | $18\frac{2}{3} \div 4$ |
| **11.** | $\frac{5}{8} \div \frac{2}{3}$ | $3\frac{3}{4} \div 5\frac{1}{2}$ | $87\frac{1}{2} \div \frac{5}{4}$ | $\frac{2}{3} \div 4\frac{2}{3}$ |
| **12.** | $\frac{7}{8} \div \frac{4}{5}$ | $\frac{1}{3} \div 2\frac{2}{5}$ | $160 \div \frac{2}{5}$ | $160 \div 3\frac{1}{3}$ |

**13.** A woman ordered the following amounts of meat during a week: $1\frac{3}{4}$ lb., $1\frac{1}{2}$ lb., $2\frac{3}{4}$ lb., 1 lb., $1\frac{7}{8}$ lb., $1\frac{2}{3}$ lb., $2\frac{1}{3}$ lb. How many pounds of meat did she buy?

**14.** How many pieces of cloth each $3\frac{1}{3}$ yards long can be cut from a piece of goods containing 40 yards?

**15.** In drawing plans for a house an architect wishes to draw a line that shall be $\frac{2}{3}$ as long as one that is $\frac{7}{8}$ in. long. How long shall he draw the line?

**16.** A fourpenny nail is $1\frac{3}{8}$ in. long and a sixpenny nail is $\frac{5}{8}$ in. longer. How much shorter is the sixpenny nail than a tenpenny nail the length of which is $2\frac{3}{4}$ in.?

**17.** A turkey weighing $9\frac{3}{4}$ lb. was bought at $31\frac{1}{2}$ cents per pound. What did the turkey cost?

Add and check:

| **18.** | **19.** | **20.** | **21.** |
|---|---|---|---|
| $473\frac{1}{2}$ | $7,098\frac{2}{5}$ | $478\frac{8}{9}$ | $8,372\frac{1}{4}$ |
| $289\frac{1}{3}$ | $4,860\frac{1}{3}$ | $362\frac{2}{3}$ | $4,326\frac{1}{5}$ |
| $706\frac{2}{5}$ | $9,287\frac{5}{8}$ | $748\frac{5}{12}$ | $7,029\frac{3}{4}$ |
| $483\frac{5}{6}$ | $746\frac{5}{16}$ | $392\frac{4}{5}$ | $8,270\frac{5}{6}$ |
| $729\frac{3}{4}$ | $982\frac{1}{6}$ | $807\frac{1}{6}$ | $4,207\frac{1}{3}$ |

**22.** What is the effect on the value of a proper fraction when each term is multiplied or divided by the same number? Illustrate.

What is the effect when the numerator only is multiplied by a number? Illustrate.

**23.** Make a blank form, record the following purchases, and compute the total amount:

| | | | | | |
|---|---|---|---|---|---|
| $17\frac{1}{2}$ yd. muslin | at | 17 | cents | per | yard |
| $15\frac{3}{4}$ yd. lace | at | 48 | " | " | " |
| $5\frac{2}{3}$ yd. silk | at | $1.68 | " | " | " |
| $6\frac{1}{2}$ yd. serge | at | 1.56 | " | " | " |
| $3\frac{2}{3}$ yd. ribbon | at | 65 | " | " | " |

**24.** For a period of four years beginning with his thirteenth birthday a boy grew $1\frac{3}{4}$ inches the first year, $1\frac{1}{2}$ inches the second year, $1\frac{3}{4}$ inches the third year, and $1\frac{7}{8}$ in. the fourth year. How many inches did he grow during the four years?

**25.** A gallon of water weighs $8\frac{1}{3}$ pounds and a gallon of mercury is $13\frac{3}{5}$ times as heavy. What is the weight of a gallon of mercury?

**26.** An auto truck travelled $102\frac{5}{6}$ miles on Monday, $104\frac{2}{3}$ miles on Tuesday, $83\frac{1}{2}$ miles on Wednesday, $94\frac{5}{8}$ miles on Thursday, $71\frac{5}{8}$ miles on Friday, and $83\frac{1}{4}$ miles on Saturday. Compute the total distance travelled during the six days.

**27.** The rear wheel of a certain bicycle is $7\frac{1}{3}$ feet in circumference, and revolves $2\frac{1}{2}$ times as often as the pedals. How many miles are travelled in 2,000 revolutions of the pedals? (1 mile is 5,280 feet.)

**28.** How many acres are there in five tracts of land, the first of which contains $135\frac{2}{3}$ acres, the second, $82\frac{5}{8}$ acres, the third, $253\frac{1}{2}$ acres, the fourth, $48\frac{7}{8}$ acres, and the fifth, $76\frac{1}{5}$ acres?

**29.** Add across and down. Do the results check?

*a.*
$$47\tfrac{1}{2} + 23\tfrac{1}{3} + 15\tfrac{1}{6}$$
$$23\tfrac{2}{3} + 39\tfrac{5}{6} + 18\tfrac{1}{3}$$
$$37\tfrac{5}{6} + 82\tfrac{2}{3} + 54\tfrac{2}{3}$$
$$46\tfrac{1}{2} + 87\tfrac{2}{3} + 42\tfrac{5}{6}$$

*b.*
$$37\tfrac{3}{4} + 18\tfrac{2}{3} + 19\tfrac{5}{12}$$
$$42\tfrac{2}{3} + 37\tfrac{1}{6} + 72\tfrac{3}{4}$$
$$17\tfrac{5}{8} + 29\tfrac{3}{4} + 47\tfrac{1}{6}$$
$$31\tfrac{1}{4} + 92\tfrac{5}{6} + 35\tfrac{2}{3}$$

Copy, perform the multiplication or division and check the following:

**30.** $4{,}786 \times 392$

**31.** $8{,}409 \times 756$

**32.** $3{,}892 \times 708$

**33.** $42{,}873 \times 5{,}496$

**34.** $287\overline{)18{,}476}$

**35.** $429\overline{)30{,}871}$

**36.** $403\overline{)72{,}876}$

**37.** $921\overline{)341{,}278}$

**38.** A plate of glass $22\frac{3}{4}$ inches by $28\frac{1}{2}$ inches is set in a picture frame that covers it $\frac{3}{8}$ inches from each edge. What are the dimensions of the glass not covered by the frame?

**39.** A certain recipe calls for 1 cup molasses, $\frac{1}{2}$ cup shortening, $2\frac{1}{2}$ cups flour, 1 tablespoon ginger, 1 tablespoon soda, 2 tablespoons milk, 1 teaspoon salt. What quantities of each should be used for one and one-half portions of this recipe?

**40.** The following is a recipe for spiced currants:

| | |
|---|---|
| 7 lbs. currants | 4 tablespoons cinnamon |
| 5 lbs. brown sugar | 3 tablespoons cloves |
| | 1 pint vinegar |

Determine the quantities to be used in making a one-third portion of this recipe; also for one and one-half portions.

**41.** A cubic foot of pure ice weighs 56 pounds and a cubic foot of water weighs $62\frac{1}{2}$ pounds. Each weight is what fractional part of the other?

**42.** By how much does the thickness of a $1\frac{1}{2}$ inch board exceed the length of a $\frac{7}{8}$ inch screw?

**43.** If you know $\frac{3}{4}$ of the cost of a lot, what multiplication would you perform to find the entire cost?

**44.** Which is the larger of each of the following pairs of fractions? What is the difference between the two fractions?

| | | |
|---|---|---|
| *a.* $\frac{1}{2}$ or $\frac{1}{3}$ | *e.* $\frac{5}{8}$ or $\frac{3}{4}$ | *i.* $\frac{7}{8}$ or $1\frac{3}{16}$ |
| *b.* $\frac{1}{3}$ or $\frac{1}{7}$ | *f.* $\frac{9}{16}$ or $\frac{2}{3}$ | *j.* $\frac{5}{8}$ or $\frac{7}{12}$ |
| *c.* $\frac{1}{5}$ or $\frac{1}{10}$ | *g.* $\frac{7}{10}$ or $\frac{2}{3}$ | *k.* $\frac{3}{8}$ or $\frac{4}{9}$ |
| *d.* $\frac{3}{5}$ or $\frac{3}{4}$ | *h.* $\frac{3}{5}$ or $\frac{5}{8}$ | *l.* $\frac{5}{12}$ or $\frac{3}{8}$ |

**45.** Under what conditions will the quotient be larger than the dividend? Give several illustrations.

**46.** Under what conditions will the quotient be smaller than the dividend? Give several illustrations.

### Problems without Numbers

**1.** If you know the quotient of two fractions and one of the fractions, how can you find the other fraction?

**2.** If you know the product of two fractions and one of the fractions, how can you find the other fraction?

**3.** What must you know and what must you do to find the average size of several tracts of land?

**4.** If you know the total cost of some land and the cost per acre, how do you find the number of acres?

**5.** If you know the quotient, remainder, and dividend, how would you find the divisor?

**6.** If you add the same number to both terms of a proper fraction, what is the effect on the value of the fraction? Illustrate.

**7.** If you know the amount that each of three persons invests in a certain partnership, how could you find the part of the profits to which each is entitled on the basis of investment?

**8.** If you know the circumference of an automobile wheel, how could you find how many times the wheel will turn in going half a mile?

**9.** If you know what part of an hour it takes a train to go a mile, how could you find how far the train would go in an hour at the same rate?

**10.** What are the advantages of cancellation? On what principle does it depend? Illustrate.

**11.** If you know the number of cubic feet of water in a tank and the weight of a cubic foot of water, how could you find the weight of the water in the tank?

# CHAPTER VIII

## DECIMAL FRACTIONS

**61. Importance of Decimal Fractions.** Decimal Fractions, because of their frequent use in business, are really the common fractions of today. It is of the greatest importance that the student of business arithmetic acquire speed and accuracy in computations involving decimal fractions.

**62. Comparison of Common and Decimal Fractions.**

The denominator of a common fraction may be any number, and it is always expressed.

The denominator of a decimal fraction is always some power of 10, and the power is indicated by the number of figures to the right of the decimal point.

Thus, $.7 = \frac{7}{10}$. $.0007 = \frac{7}{10,000}$.

$.07 = \frac{7}{100}$. $.423 = \frac{423}{1,000}$.

$.007 = \frac{7}{1,000}$. $.027 = \frac{27}{1,000}$.

**63. Reading and Writing Decimals.**

The names of the various decimal orders are stated in the following table:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Tenths | Hundredths | Thousandths | Ten-thousandths | Hundred-thousandths | Millionths | Ten-millionths |

The names of the various orders should be learned. You should know that the *name* of the third decimal order is thousandths; that of the fifth is hundred-thousandths; that of the second is hundredths, etc.

*To read a decimal fraction, read the number as if it were an integer and then state the name of the last decimal order which a digit of the number occupies.*

Pronounce the word " and " at the decimal point and omit it elsewhere, except in reading complex decimals, as noted below:

.045 is read forty-five thousandths.
.505 is read five hundred five thousandths.
500.005 is read five hundred *and* five thousandths.
.4206 is read four thousand two hundred six ten-thousandths.
$.0\frac{2}{3}$ is read two thirds of a tenth.
$2.0\frac{2}{3}$ is read two *and* two thirds of a tenth.
$.2\frac{2}{3}$ is read two and two thirds tenths (a complex decimal).

Read:

| | | |
|---|---|---|
| **1.** .043. | **2.** .1764. | **3.** 2.04. |
| **4.** 500.17. | **5.** 400.004. | **6.** $.00\frac{2}{3}$. |
| **7.** 214.0014. | **8.** .000562. | **9.** 64.3. |
| **10.** 14.003. | **11.** .003726. | **12.** 542.54. |
| **13.** 3,005.016. | **14.** $.00024\frac{1}{3}$. | **15.** 7.00707. |

Write decimally:

**1.** Seven tenths; forty-two thousandths; five hundred four thousandths.

**2.** Four hundred and three thousandths; eight hundred two ten-thousandths.

**3.** Three fourths of a tenth; two and one seventh tenths.

**4.** Twenty-four ten-thousandths; five millionths; seventeen hundred-thousandths; two hundred and three hundredths.

**5.** Five hundred seventeen millionths; sixteen ten-thousandths; eighteen hundred and fifteen ten-thousandths.

**6.** Five thousand four hundred-millionths; eighteen and four thousandths.

**64. Addition of Decimal Fractions.**

*Place the decimals so that the decimal points fall vertically. Add as with integers. Place the decimal point of the sum directly below those in the numbers added.*

**Example.** Find the sum of 4.023, .507, 2.003, and 1.125.

SOLUTION.

$$\begin{array}{r} 4.023 \\ .507 \\ 2.003 \\ 1.125 \\ \hline 7.658 \end{array}$$

### Written Work

1. Add 2,569.327, 1,462.978, 47.9634, 2,693.072, and .019.
2. Add 27.9548, 91.0005, 37.427, and 27,563.974.
3. Add 2,752.9374, .0003, 23.247, and 259.6347.

Write the following common fractions as decimal fractions, and add:

4. $\frac{24}{100}, \frac{372}{1{,}000}, \frac{19}{10}, \frac{254}{100}$.
5. $\frac{3}{1{,}000}, \frac{24}{10{,}000}, \frac{675}{10}, \frac{170}{10{,}000}$.
6. $\frac{36.5479}{100}, \frac{762}{1{,}000}, \frac{7642}{10{,}000}, \frac{271}{1{,}000}$.
7. Copy, find the totals required, and check:

| | | | |
|---|---|---|---|
| \$ 324.80 | \$3,764.20 | \$7,436.08 | ? |
| 17.04 | 762.83 | 427.36 | ? |
| 9,320.87 | 9,402.03 | 92.18 | ? |
| 473.26 | 1.18 | 4,724.63 | ? |
| 791.14 | 13,420.03 | 924.56 | ? |
| ? | ? | ? | ? |

8. The receipts in a department store are shown in the table below. What were the total receipts for each day, for each department, and for the week?

| | DEP'T. I | DEP'T. II | DEP'T. III | DEP'T. IV | DEP'T. V | TOTAL |
|---|---|---|---|---|---|---|
| Monday | \$418.30 | \$ 67.08 | \$492.70 | \$702.30 | \$280.40 | |
| Tuesday | 214.06 | 43.90 | 386.40 | 347.20 | 165.90 | |
| Wednesday | 185.30 | 120.85 | 297.80 | 436.10 | 380.45 | |
| Thursday | 542.65 | 106.92 | 380.70 | 290.18 | 260.92 | |
| Friday | 320.80 | 231.40 | 920.40 | 482.70 | 192.73 | |
| Saturday | 435.90 | 192.60 | 870.60 | 395.17 | 243.65 | |
| Total | | | | | | |

**65. Subtraction of Decimal Fractions.**

*Place the decimals so that the points fall vertically. Subtract as with integers, then place the decimal point in the remainder immediately below the decimal points above.*

**Example.** Subtract 23.213 from 34.047.

SOLUTION.

$$\begin{array}{r} 34.047 \\ 23.213 \\ \hline 10.834 \end{array}$$

Subtract as indicated. Check the results.

1. 394.237 − 1.027.
2. .47 − .003.
3. .394237 − .15.
4. 7,763.421 − 28.796.
5. 3.027 − .489
6. 87.5932 − 2.3579.

Write as decimal fractions and subtract:

7. $\frac{465}{1{,}000} - \frac{23}{100}$.
8. $\frac{7{,}6943}{100} - \frac{235}{1{,}000}$.
9. $\frac{8{,}732}{10} - \frac{236{,}547}{100{,}000}$.

10. The sum of two numbers is 342.086. One of the numbers is 206.78. What is the other number?

11. A man deposited \$5,764.80 in a bank. He later drew out \$3,780.92 and then deposited \$4,814.60. How much had he then?

12. The daily bank deposits and withdrawals of a merchant during a week were as follows:

| DAYS | DEPOSITS | WITHDRAWALS |
|---|---|---|
| Monday | \$486.30 | \$211.85 |
| Tuesday | 718.90 | 392.18 |
| Wednesday | 143.65 | 417.21 |
| Thursday | 638.95 | 243.62 |
| Friday | 531.80 | 428.30 |
| Saturday | 782.45 | 1,243.90 |

Compute the difference between the deposits and the withdrawals for the week.

**66. Multiplication of Decimal Fractions.**

*Multiply as with integers. From the right of the product, point off as many places as the sum of the number of decimal places in the multiplicand and the multiplier.*

**Examples.** 1. Multiply 4.625 by .05.

SOLUTION.

$$\begin{array}{r} 4.625 \\ .05 \\ \hline .23125 \end{array}$$

2. Multiply .00362 by .06.

SOLUTION.

$$\begin{array}{r} .00362 \\ .06 \\ \hline .0002172 \end{array}$$

**Oral Work**

| | | |
|---|---|---|
| 1. $.7 \times .03$. | 2. $.08 \times .4$. | 3. $.06 \times .09$. |
| 4. $.25 \times .06$. | 5. $.026 \times .5$. | 6. $4.5 \times 4$. |
| 7. $.009 \times .8$. | 8. $.025 \times .10$. | 9. $.25 \times .008$. |
| 10. $.030 \times .09$. | 11. $.08 \times .018$. | 12. $.0125 \times .4$. |

**Written Work**

Multiply as indicated. Check the results.

| | |
|---|---|
| 1. $25.763 \times .1463$. | 2. $75.46 \times .03$. |
| 3. $.2462 \times .347$. | 4. $.083 \times .0462$. |
| 5. $78.23 \times .000007$. | 6. $17.13 \times .042$. |
| 7. $570.04 \times .00326$. | 8. $\$374.26 \times .055$. |
| 9. $\$9{,}037.28 \times .035$. | 10. $\$473.54 \times .335$. |

11. Find the cost of 46 lots at an average cost of $847.60.

12. Find the cost of 143 barrels of apples at an average price of $5.64 per barrel.

**67. Division of Decimal Fractions.**

*a.* When the divisor is an integer.

**Examples.** 1. $.12 \div 4 = .03$; just as $\$12 \div 4 = \$3$.

2. $.426 \div 3 = .142$; just as $\$426 \div 3 = \$142$.

In making the division the decimal point should be written in the quotient when it is reached in the dividend.

3. Divide 157.25 by 37.

SOLUTION.

$$\begin{array}{r} 4.25 \\ 37\overline{)157.25} \\ 148 \\ \hline 92 \\ 74 \\ \hline 185 \\ 185 \\ \hline \end{array}$$

*b.* When the divisor is not an integer.

*Multiply both dividend and divisor by the least power of ten that will make the divisor an integer.* Divide as in case *a*.

**Example.** Divide .028 by .4.

SOLUTION. First multiply both dividend and divisor by 10, to make the divisor an integer. Then divide the resulting dividend, .28, by 4. The result is .07.

Similarly, $.0456 \div .03 = 4.56 \div 3 = 1.52$.
Similarly, $5.64 \div .0004 = 56{,}400 \div 4 = 14{,}100$.

If there is a remainder after all of the decimal places in the dividend have been used, zeros may be annexed to the dividend and the division may be carried as far as is desired.

### Oral Work

| | | |
|---|---|---|
| **1.** .08 by .4. | **2.** .014 by .2. | **3.** .364 by .04. |
| **4.** 3,300 by .11. | **5.** 8.48 by .008. | **6.** .220 by .0011. |
| **7.** 1 by .01. | **8.** .10 by .1. | **9.** .018 by 18. |
| **10.** .0001 by .00001. | **11.** .042 by 2.1. | **12.** 1.1 by .011. |

### Written Work

| | |
|---|---|
| **1.** 268,632 ÷ 36.9. | **2.** 26,863.2 ÷ 36.9. |
| **3.** 26,863.2 ÷ 369. | **4.** 2,686.32 ÷ 36.9. |
| **5.** 26.8632 ÷ 36.9. | **6.** 2,686.32 ÷ .369. |
| **7.** 26.8632 ÷ .369. | **8.** 26.8632 ÷ .0369. |
| **9.** 2.68632 ÷ .369. | **10.** 2.68632 ÷ 3.69. |
| **11.** 2.68632 ÷ 36.9. | **12.** 2.68632 ÷ 369. |
| **13.** .268632 ÷ 369. | **14.** 2.68632 ÷ 3690. |
| **15.** 293.45 ÷ 14.24. | **16.** 2.734 ÷ 1.32. |
| **17.** .73469 ÷ 127.9638. | **18.** 762.397 ÷ 36,947.28. |
| **19.** .1479376 ÷ 293.4798. | **20.** .0097 ÷ 12.34692. |

**21.** The multiplier is .045, the product is .01665. What is the multiplicand?

**22.** At \$.44 per dozen, how many eggs can be bought for \$3.52?

**23.** If a man's annual income is \$5,420 and his annual expenses are \$4,262, what are his average weekly savings?

**24.** What number is $\frac{3}{7}$ as large as .0427?

**25.** What number divided by 4.28 gives a quotient of .07 and a remainder of .04?

**26.** In a certain factory 30 men are employed at \$2.15 per day, 10 men at \$3.60 per day, 28 men at \$2.90 a day, and 12 men at \$1.80 a day. Find the average daily wage.

**27.** Using the current market price find the cost of the following: 1 pk. of apples, $\frac{1}{2}$ bu. of sweet potatoes, $\frac{1}{2}$ lb. of tea, 6 bars of laundry soap, $1\frac{1}{2}$ lb. porterhouse steak.

**28.** A train ran at the rate of .87 mile a minute. At this rate how many miles would it travel in $3\frac{1}{2}$ minutes? In 10 minutes? In 1 hour?

**29.** An automobile traveled 72.25 miles in $2\frac{1}{3}$ hours. What was the average rate of speed per hour?

### 68. Changing Decimal Fractions to Equivalent Common Fractions.

*Omit the decimal point; write for the denominator* 1 *with as many zeros annexed as there are places in the decimal. Reduce to lowest terms.*

**Examples.** 1. Change .75 to a common fraction.

SOLUTION. $.75 = \frac{75}{100} = \frac{3}{4}$.

2. Change .00864 to a common fraction.

SOLUTION. $.00864 = \frac{864}{100000} = \frac{27}{3125}$.

#### Written Work

Change the following decimals to equivalent common fractions and reduce to lowest terms:

| | | | | |
|---|---|---|---|---|
| **1.** .48. | **2.** .095. | **3.** .3705. | **4.** .0012. | **5.** 3.6425. |
| **6.** .125. | **7.** .9825. | **8.** .625. | **9.** .1875. | **10.** .0015. |
| **11.** $.06\frac{1}{4}$. | **12.** $.00\dot{2}\frac{1}{2}$. | **13.** $.37\frac{1}{2}$. | **14.** .333. | **15.** $.012\frac{1}{2}$. |
| **16.** $.06\frac{2}{3}$. | **17.** .00625. | **18.** .0036. | **19.** $.008\frac{1}{3}$. | **20.** $.009\frac{1}{11}$. |
| **21.** $.01\frac{1}{4}$. | **22.** $.08\frac{1}{3}$. | **23.** $.83\frac{1}{3}$. | **24.** $.87\frac{1}{2}$. | **25.** $.11\frac{1}{9}$. |

### 69. Changing Common Fractions to Equivalent Decimal Fractions.

It is frequently desirable to reduce common to decimal fractions in order that the required operations may be performed more easily and in order that the values may be compared more easily.

Since a common fraction may be regarded as an indicated division,

the reduction may be made by the methods of division previously explained. Although most common fractions do not have an exact decimal equivalent, it is possible by continuing the division to find the equivalent as approximately as necessity requires.

**Example.** Change $\frac{4}{5}$ to a decimal fraction.

SOLUTION. $\frac{4}{5} = \frac{1}{5}$ of 4. Place a decimal point to the right of the 4, annex zeros, and divide.

$$5\overline{)4.0} \quad .8$$

Similarly, $\frac{7}{8} = \frac{1}{8}$ of 7.

$$8\overline{)7.000} \quad .875$$

Similarly, $\frac{2}{375} = \frac{1}{375}$ of 2.

$$375\overline{)2.0000} \quad .0053+$$

1875

1250

1125

125

It is usually not necessary to carry the division more than three or four places.

For example, $\frac{1}{7}$ may be expressed decimally as $.142\frac{8}{7}$ or as .1429–.

There are some common fractions whose decimal equivalents should be memorized. These are listed below and are further discussed in Chapter IX.

IMPORTANT EQUIVALENTS

| | | | |
|---|---|---|---|
| $.50 = \frac{1}{2}$. | $.16\frac{2}{3} = \frac{1}{6}$. | $.06\frac{2}{3} = \frac{1}{15}$. | $.37\frac{1}{2} = \frac{3}{8}$. |
| $.33 = \frac{1}{3}$. | $.12\frac{1}{2} = \frac{1}{8}$. | $.06\frac{1}{4} = \frac{1}{16}$. | $.62\frac{1}{2} = \frac{5}{8}$. |
| $.25 = \frac{1}{4}$. | $.10 = \frac{1}{10}$. | $.05 = \frac{1}{20}$. | $.87\frac{1}{2} = \frac{7}{8}$. |
| $.20 = \frac{1}{5}$. | $.08\frac{1}{3} = \frac{1}{12}$. | $.25 = \frac{1}{25}$. | $.83\frac{1}{3} = \frac{5}{6}$. |

### Written Work

Considering the figures in the top row of the following form as numerators, and those at the left as denominators, change the common fractions thus formed to decimals, entering the results in the proper places. Carry the results to three decimal places.

Prepare the work on a carefully ruled form.

| Denominators | Numerators | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 3 | 7 | 6 | 26 | 47 | 126 | 79 | 814 | 1216 |
| 18 | | | | | | | | | |
| 27 | | | | | | | | | |
| 94 | | | | | | | | | |
| 38 | | | | | | | | | |
| 172 | | | | | | | | | |
| 9 | | | | | | | | | |
| 66 | | | | | | | | | |
| 213 | | | | | | | | | |
| 793 | | | | | | | | | |
| 2361 | | | | | | | | | |
| 43 | | | | | | | | | |
| 109 | | | | | | | | | |

**70. Approximate Results.** For most practical purposes it is unnecessary to carry a result to more than three decimal places. Thus, \$25.86347 is stated with sufficient accuracy as \$25.86.

In finding approximate decimal fractions, follow these directions.

*Carry the division at least one more place than the number of digits desired in the result. If the digit at the right of the last place desired in the result is less than* 5, *drop it; if it is* 5 *or larger than* 5, *increase the preceding digit by* 1.

## Review of Decimal Fractions

### Written Work

**1.** If a boy's average gain in weight is 8.25 lb. per year, how much will he gain in weight in $2\frac{1}{2}$ yr.?

**2.** If one turn of a screw advances the point .14 in., how far will 7 turns advance it?

**3.** A gallon of milk weighs 8.622 lb. and a gallon of water weighs 8.355 lb. How much more does $\frac{1}{2}$ gal. of milk weigh than $\frac{1}{2}$ gal. of water?

**4.** A boy can run 100 yd. in 11.2 sec. At this rate, how long would it take him to run 80 yd.?

**5.** In a certain city the rainfall during March was 2.46 in., during April 3.15 in., and during May 1.09 in. Find the average for these months.

**6.** A man sold 24 dozen eggs, which was .4 of all that he had. How many did he have?

**7.** If .375 of an acre of land sells for $48, what should $2\frac{1}{2}$ acres sell for, at the same rate?

**8.** An English pound contains 113.00001 grains of pure gold, and an American gold dollar contains 23.22 grains of pure gold. Find the par value of an English pound in American money.

**9.** A nautical mile is 6,080 feet. Express this as a decimal of the statute mile (5,280 feet).

**10.** The record for the 100 yard dash is $9\frac{3}{5}$ seconds. How many yards per second is this?

**11.** An American bushel contains 2,150.42 cubic inches. What decimal part of a bushel is a cubic foot (1,728 cubic inches)?

**12.** If your school team played 11 games during the season and won 8 of them, what part of the games played did the team win? (State the result to the nearest hundredth.)

**13.** The heating surface of a certain boiler is 4,195.25 sq. feet and the grate surface is 86.5 sq. feet. How many square feet of heating surface are there to each square foot of grate surface?

**14.** The distance which a freely falling object will fall from any height in any number of seconds may be found by multiplying 16.08 feet by the square of the number of seconds. (The square of a number is the number multiplied by itself.) Compute the distance that an object will fall in 2 seconds; in 5 seconds; in 12 seconds?

Compute the distance that an object will fall during the third second.

**15.** A man paid $34.50 for an automobile tire. At the end of 5,480 miles the tire is worthless. Compute the cost per mile for this tire.

**16.** The total area of the United States is 3,026,788 square miles and the population in 1920 was 105,708,771. What was the population per square mile?

**17.** The land area of Massachusetts is 8,039 square miles and the population in 1920 was 3,852,356. What was the population per square mile? (Carry results to decimals.)

The land area of Texas is 262,398 square miles. If Texas were as densely populated as Massachusetts what would its population be?

What would the population of your state be, if it were as densely populated as Massachusetts?

**18.** A gallon contains 231 cubic inches; a cubic foot, 1,728 cubic inches. How many gallons of water can be placed in a vessel which has a capacity of 1 cubic foot?

**19.** Compute the weight of a gallon of water, if a cubic foot of water weighs 62.5 pounds.

**20.** The following table shows the number of games won and the number lost by each of the teams of a certain school during a season. Which made the best record? (Express the fractions as decimals.)

| TEAMS | GAMES WON | GAMES LOST |
|---|---|---|
| Reds | 7 | 5 |
| Whites | 6 | 4 |
| Blues | 2 | 7 |
| Blacks | 5 | 7 |
| Greens | 7 | 4 |

## CHAPTER IX

## SHORT METHODS INVOLVING ALIQUOT PARTS

An *aliquot part* of a number is contained in the given number an integral number of times. Thus, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10. Similarly, 10, $12\frac{1}{2}$, $16\frac{2}{3}$, 20, and 50, are aliquot parts of 100.

### 71. Equivalent Common and Decimal Fractions in Frequent Use.

| | | | |
|---|---|---|---|
| $.50 = \frac{1}{2}$. | $.12\frac{1}{2} = \frac{1}{8}$. | $.06\frac{1}{4} = \frac{1}{16}$. | $.60 = \frac{3}{5}$. |
| $.33\frac{1}{3} = \frac{1}{3}$. | $.11\frac{1}{9} = \frac{1}{9}$. | $.05 = \frac{1}{20}$. | $.62\frac{1}{2} = \frac{5}{8}$. |
| $.25 = \frac{1}{4}$. | $.10 = \frac{1}{10}$. | $.04 = \frac{1}{25}$. | $.75 = \frac{3}{4}$. |
| $.20 = \frac{1}{5}$. | $.09\frac{1}{11} = \frac{1}{11}$. | $.03\frac{1}{3} = \frac{1}{30}$. | $.80 = \frac{4}{5}$. |
| $.16\frac{2}{3} = \frac{1}{6}$. | $.08\frac{1}{3} = \frac{1}{12}$. | $.40 = \frac{2}{5}$. | $.83\frac{1}{3} = \frac{5}{6}$. |
| $.14\frac{2}{7} = \frac{1}{7}$. | $.06\frac{2}{3} = \frac{1}{15}$. | $.37\frac{1}{2} = \frac{3}{8}$. | $.87\frac{1}{2} = \frac{7}{8}$. |

These equivalents should be memorized, as they can be used to shorten many computations.

### 72. A Short Method of Multiplying by These Fractions.

Which is easier, $1{,}248 \times .16\frac{2}{3} = 208$; or $\frac{1}{6}$ of $1{,}248 = 208$?

State a short method of multiplying by each of the decimal fractions in the table above.

#### Exercise

Multiply:

1. $642 \times .50$.
2. $936 \times .33\frac{1}{3}$.
3. $488 \times .25$.
4. $650 \times .20$.
5. $366 \times .16\frac{2}{3}$.
6. $840 \times .14\frac{2}{7}$.
7. $720 \times .12\frac{1}{2}$.
8. $456 \times .08\frac{1}{3}$.
9. $930 \times .06\frac{2}{3}$.
10. $320 \times .06\frac{1}{4}$.

The results in the preceding exercise are integers. The same method may be used when the result is a fraction.

**Example.** Multiply 4,634 by .25.

SOLUTION. $.25 = \frac{1}{4}$.

$4634 \div 4 = 1158.5$.

**Exercise**

Without copying, state the results of the following:

1. $8{,}735 \times .50$.
2. $3{,}940 \times .33\frac{1}{3}$.
3. $267 \times .25$.
4. $379.3 \times .16\frac{2}{3}$.
5. $842 \times .20$.
6. $3{,}824 \times .14\frac{2}{7}$.
7. $5{,}316 \times .12\frac{1}{2}$.
8. $4{,}026 \times .08\frac{1}{3}$.
9. $1{,}848 \times .06\frac{2}{3}$.
10. $1{,}940 \times .06\frac{1}{4}$.
11. $35.69 \times .33\frac{1}{3}$.
12. $649.3 \times .25$.
13. $.8347 \times .50$.
14. $723.68 \times .20$.
15. $36.92 \times .14\frac{2}{7}$.
16. $56.32 \times .12\frac{1}{2}$.
17. $7{,}836 \times .06\frac{1}{4}$.
18. $1{,}223 \times .16\frac{2}{3}$.
19. $3{,}380 \times .06\frac{2}{3}$.
20. $290 \times .06\frac{2}{3}$.

21. Find the value of each of the following:

48 lb. at $37\frac{1}{2}¢$ per pound.
32 lb. at $12\frac{1}{2}¢$ per pound.
21 lb. at $87\frac{1}{2}¢$ per pound.
24 yd. at $12\frac{1}{2}¢$ per yard.
30 yd. at $62\frac{1}{2}¢$ per yard.
21 lb. at $14\frac{2}{7}¢$ per pound.
48 yd. at $6\frac{2}{3}¢$ per yard.
96 lb. at $83\frac{1}{3}¢$ per pound.

**73. A Short Method of Dividing by These Fractions.**

Which is easier, $6 \div .25 = 24$; or $6 \times 4 = 24$?

State a short method of dividing by each of the decimal fractions in the table on page 91.

**Exercise**

Without copying, state the results of the following:

1. $7 \div .12\frac{1}{2}$.
2. $49 \div .50$.
3. $126 \div .33\frac{1}{3}$.
4. $63 \div .14\frac{2}{7}$.
5. $8 \div .06\frac{1}{4}$.
6. $10 \div .06\frac{2}{3}$.
7. $15 \div .25$.
8. $1{,}627 \div .20$.
9. $13.63 \div .16\frac{2}{3}$.
10. $1.43 \div .25$.
11. $3.249 \div .50$.
12. $.023 \div .08\frac{1}{3}$.
13. $63.98 \div .2$.
14. $.043 \div .16\frac{2}{3}$.

**74. Aliquot Parts which are not Unit Fractions.**

State a short method of multiplying by $.37\frac{1}{2}$; by $.66\frac{2}{3}$; by .75; by $.87\frac{1}{2}$.

State a short method of dividing by $.37\frac{1}{2}$; by $.66\frac{2}{3}$; by .75; by $.87\frac{1}{2}$.

When dividing by one number and multiplying by another, observe the following directions:

*Divide first if the division results in an integer. If the division would result in a mixed number, multiply first.*

**Examples.** 1. Multiply 32 by $.37\frac{1}{2}$.

SOLUTION. $.37\frac{1}{2} = \frac{3}{8}$.
$32 \div 8 = 4$.
$4 \times 3 = 12$. Since 8 is a factor of 32, the division is performed first.

2. Divide 16 by $.37\frac{1}{2}$.

SOLUTION. Substituting the equivalent common fraction, we have
$16 \div \frac{3}{8} = ?$
or $16 \times \frac{8}{3} = ?$ Inverting the divisor.
$16 \times 8 = 128$.
$128 \div 3 = 42\frac{2}{3}$. Since 3 is not a factor of 16, the multiplication is performed first.

What advantage is there in observing these directions?

**Exercise**

Without recopying, find the results of the following:

| | | |
|---|---|---|
| **1.** $64 \times .375$. | **2.** $96 \times .66\frac{2}{3}$. | **3.** $480 \times .75$. |
| **4.** $168 \times .875$. | **5.** $248 \times .87\frac{1}{2}$. | **6.** $3{,}260 \times .75$. |
| **7.** $4{,}864 \times .37\frac{1}{2}$. | **8.** $639 \times .66\frac{2}{3}$. | **9.** $42 \div .37\frac{1}{2}$. |
| **10.** $56 \div .87\frac{1}{2}$. | **11.** $40 \div .66\frac{2}{3}$. | **12.** $36 \div .75$. |
| **13.** $1{,}462 \div .66\frac{2}{3}$. | **14.** $465 \div .375$. | **15.** $11{,}322 \div .75$. |
| **16.** $994 \div .875$. | **17.** $426 \times .875$. | **18.** $143 \times .37\frac{1}{2}$. |
| **19.** $1{,}347 \times .75$. | **20.** $539 \times .66\frac{2}{3}$. | **21.** $3.29 \times .75$. |
| **22.** $1.267 \times .875$. | **23.** $139 \div .87\frac{1}{2}$. | **24.** $1{,}426 \div .37\frac{1}{2}$. |
| **25.** $139 \div .66\frac{2}{3}$. | **26.** $46 \div .75$. | **27.** $46.9 \div .37\frac{1}{2}$. |
| **28.** $.429 \div .66\frac{2}{3}$. | | |

**75. General Table of Aliquot Parts.** The following table shows the most frequently used decimal parts of 1, 10, 100, and 1,000.

| Fractions | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{6}$ | $\frac{5}{6}$ |
|---|---|---|---|---|---|---|---|
| of 1 | .50 | $.33\frac{1}{3}$ | $.66\frac{2}{3}$ | .25 | .75 | $.16\frac{2}{3}$ | $.83\frac{1}{3}$ |
| of 10 | 5. | $3.33\frac{1}{3}$ | $6.66\frac{2}{3}$ | 2.5 | 7.5 | $1.6\frac{2}{3}$ | $8.33\frac{1}{3}$ |
| of 100 | 50. | $33.33\frac{1}{3}$ | $66.66\frac{2}{3}$ | 25. | 75. | $16\frac{2}{3}$ | $83.3\frac{1}{3}$ |
| of 1,000 | 500. | $333\frac{1}{3}$ | $666\frac{2}{3}$ | 250. | 750. | $166\frac{2}{3}$ | $833.3\frac{1}{3}$ |

| Fractions | $\frac{1}{7}$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{5}{8}$ | $\frac{7}{8}$ | $\frac{1}{12}$ | $\frac{1}{15}$ | $\frac{1}{16}$ |
|---|---|---|---|---|---|---|---|---|
| of 1 | $.14\frac{2}{7}$ | .125 | .375 | .625 | .875 | $.08\frac{1}{3}$ | $.06\frac{2}{3}$ | $.06\frac{1}{4}$ |
| of 10 | 1.42 | 1.25 | 3.75 | 6.25 | 8.75 | $.83\frac{1}{3}$ | $.66\frac{2}{3}$ | .625 |
| of 100 | 14.28 | 12.5 | 37.5 | 62.5 | 87.5 | $8.3\frac{1}{3}$ | $6.6\frac{2}{3}$ | 6.25 |
| of 1,000 | 142.8 | 125. | 375. | 625. | 875. | $83\frac{1}{3}$ | $66\frac{2}{3}$ | 62.5 |

Study this table until you can recognize quickly these fractional parts of 1, 10, 100, and 1,000.

**76. Rules for Multiplication by Aliquot Parts of 1, 10, 100, and 1,000.**

*a.* To multiply by fractional parts of 10.

*Multiply by the equivalent fraction; multiply this product by* 10, *by moving the decimal point one place to the right, annexing a zero, if necessary.*

**Examples.** 1. Multiply 16 by $2\frac{1}{2}$.

SOLUTION. $2\frac{1}{2}$ is $\frac{1}{4}$ of 10. $\frac{1}{4}$ of 16 = 4. Multiply by 10, by annexing a zero; the result is 40.

2. Multiply 26 by 1.25.

SOLUTION. 1.25 is $\frac{1}{8}$ of 10. $\frac{1}{8}$ of 26 = 3.25. Move the decimal point one place to the right; the result is 32.5.

*b.* To multiply by fractional parts of 100.

*Multiply by the equivalent common fraction; multiply this product by* 100.

**Examples.** 1. Multiply 24 by $87\frac{1}{2}$.

SOLUTION.
$87\frac{1}{2}$ is $\frac{7}{8}$ of 100.
$\frac{7}{8}$ of 24 = 21.
$21 \times 100 = 2100$.

2. Multiply 4,252 by 12.5.

SOLUTION.
12.5 is $\frac{1}{8}$ of 100.
$\frac{1}{8}$ of 4,252 = 531.5.
$531.5 \times 100 = 53150$.

### Exercise

State a short method of multiplying by each of the following.

| | | | | | | |
|---|---|---|---|---|---|---|
| $.06\frac{1}{4}$; | 2.50; | $6\frac{2}{3}$; | $33\frac{1}{3}$; | 250; | $87\frac{1}{2}$; | 6.25; |
| $37\frac{1}{2}$; | 7.50; | 5; | $333\frac{1}{3}$; | 75; | 125; | $8\frac{1}{3}$; |
| $2\frac{1}{2}$; | 12.50; | $6\frac{1}{4}$; | $62\frac{1}{2}$; | $1\frac{1}{4}$; | $8\frac{3}{4}$; | 25; |
| $66\frac{2}{3}$; | 375; | 750; | 875; | 37.50; | 7500; | 12,500. |

Find the products of the following. Do the work orally when possible. Be prepared to state how the short method was applied in each example.

| | | |
|---|---|---|
| **1.** $32 \times 25$. | **2.** $111 \times 3\frac{1}{3}$. | **3.** $390 \times 66\frac{2}{3}$. |
| **4.** $648 \times 250$. | **5.** $724 \times 75$. | **6.** $832 \times 1\frac{1}{8}$. |
| **7.** $96 \times 87\frac{1}{2}$. | **8.** $108 \times 83\frac{1}{3}$. | **9.** $3{,}264 \times .625$. |
| **10.** $144 \times 8\frac{1}{3}$. | **11.** $80 \times 875$. | **12.** $48 \times 375$. |
| **13.** $464 \times 12.50$. | **14.** $24 \times 7\frac{1}{2}$. | **15.** $3.20 \times 25$. |
| **16.** $.27 \times 6\frac{2}{3}$. | **17.** $8.4 \times 3.75$. | **18.** $640 \times 6.25$. |
| **19.** $3.612 \times .83\frac{1}{3}$. | **20.** $208 \times 3\frac{3}{4}$. | **21.** $143 \times 250$. |
| **22.** $1{,}936 \times 62.50$. | **23.** $1{,}721 \times 7.50$. | **24.** $1{,}239 \times 125$. |
| **25.** $426 \times 750$. | **26.** $420 \times .87\frac{1}{2}$. | **27.** $369 \times 33\frac{1}{3}$. |

**77. Interchanging Multiplicand and Multiplier.** When the multiplicand and multiplier are abstract numbers, they may be interchanged.

**Example.** Multiply 25 by 428.

SOLUTION. Interchanging, $428 \times 25$
$\frac{1}{4}$ of 428 = 107.
$107 \times 100 = 10{,}700$.

### Exercise

Perform the following multiplications as indicated. State how the short method was applied in each example.

| | | |
|---|---|---|
| **1.** $924 \times 12.50$. | **2.** $864 \times 750$. | **3.** $125 \times 488$. |
| **4.** $264 \times .12\frac{1}{2}$. | **5.** $45 \times .06\frac{2}{3}$. | **6.** $250 \times 3{,}288$. |
| **7.** $62.50 \times 648$. | **8.** $8\frac{1}{3} \times 3.60$. | **9.** $342 \times 87.50$. |
| **10.** $375 \times 112$. | **11.** $6\frac{2}{3} \times 456$. | **12.** $1{,}468 \times 250$. |
| **13.** $875 \times 88$. | **14.** $1{,}250 \times 96$. | **15.** $1{,}776 \times .625$ |
| **16.** $7.2 \times 8\frac{3}{4}$. | **17.** $1\frac{1}{4} \times 104$. | **18.** $125 \times 1.248$. |
| **19.** $66 \times 875$. | **20.** $83\frac{1}{3} \times 9.6$. | **21.** $62\frac{1}{2} \times 20$. |

Find the cost of the following:

**22.** 12 articles at \$.50 each
16 articles at .25 each
18 articles at .33⅓ each
30 articles at .40 each
16 articles at .75 each
Total ____

**23.** 64 doz. articles at \$ .62½ per dozen
76 doz. articles at 8.75 per dozen
58 doz. articles at .75 per dozen
125 doz. articles at .16 per dozen
6¼ doz. articles at .32 per dozen
Total ____

**24.** 72 lb. at \$ .83⅓ per pound
52 lb. at 1.25 per pound
44 lb. at .37½ per pound
90 lb. at .12½ per pound
112 lb. at .06¼ per pound
Total ____

**25.** 80 yd. at \$ .62½ per yard
66⅔ yd. at 1.20 per yard
86 yd. at 1.25 per yard
25 yd. at .78 per yard
75 yd. at .66⅔ per yard
Total ____

**26.** 83⅓ yd. at \$1.80 per yard
12½ doz. at 1.60 per dozen
8¾ lb. at 1.60 per pound
6⅔ yd. at 2.10 per yard
3⅓ yd. at 1.80 per yard
Total ____

**27.** 135 yd. at \$.06¼ per yard
56 yd. at .08¾ per yard
96 yd. at .16⅔ per yard
152 yd. at .12½ per yard
24 yd. at .07½ per yard
Total ____

**78. Rules for Division by Aliquot Parts of 1, 10, 100, 1,000.**

*a.* To divide by fractional parts of 1.

*Divide by the equivalent common fraction.*

**Example.** Divide 13 by $.12\frac{1}{2}$. SOLUTION. $.12\frac{1}{2} = \frac{1}{8}$.
$13 \div \frac{1}{8} = 13 \times \frac{8}{1} = 104.$

*b.* To divide by fractional parts of 10.
*Divide by* 10; *divide this quotient by the equivalent common fraction.*

**Example.** Divide 80 by 2.50. SOLUTION. 2.50 is $\frac{1}{4}$ of 10.
$80 \div 10 = 8.$
$8 \div \frac{1}{4} = 8 \times \frac{4}{1} = 32.$

*c.* To divide by fractional parts of 100.

*Divide by* 100; *divide this quotient by the equivalent common fraction.*

**Exercise**

1. State a rule for dividing by fractional parts of 1,000.
2. \$100 will pay for how many articles at $\$.33\frac{1}{3}$ each?
3. \$12 will pay for how many yards of cloth at \$.25 per yard?
4. Which is easier —

$14 \div .75 = ?$ or $14 \div \frac{3}{4} = ?$

5. State how to find the total cost of some articles, when the number of articles bought is given and the cost of each is $8\frac{1}{3}¢$; $14\frac{2}{7}¢$; $87\frac{1}{2}¢$; $16\frac{2}{3}¢$; $33\frac{1}{3}¢$; $62\frac{1}{2}¢$; $37\frac{1}{2}¢$.

Without copying, state results for the following examples. State the method of operation in each.

6. $15 \div .12\frac{1}{2}$.
7. $14 \div .06\frac{1}{4}$.
8. $9 \div .25$.
9. \$6 ÷ \$.75.
10. \$14 ÷ \$.87½.
11. \$8 ÷ \$1.33⅓.

State the quotients of each of the following:

| | DIVIDEND | DIVISOR | QUOTIENT | | DIVIDEND | DIVISOR | QUOTIENT |
|---|---|---|---|---|---|---|---|
| 12. | 42 | $.33\frac{1}{3}$ | | 13. | 13 | .25 | |
| 14. | 22 | $.66\frac{2}{3}$ | | 15. | 36 | .75 | |
| 16. | 5 | $.06\frac{1}{4}$ | | 17. | 45 | $.37\frac{1}{2}$ | |
| 18. | .17 | .125 | | 19. | 28 | 87.5 | |
| 20. | 15 | 1.25 | | 21. | 900 | 6.25 | |
| 22. | 400 | $8.33\frac{1}{3}$ | | 23. | 150 | $.06\frac{2}{3}$ | |

### Exercise

Complete the following:

| | Total Cost | Price Each | Number Purchased |
|---|---|---|---|
| 1. | $ 5.25 | $ .37½ | |
| 2. | 20.25 | .75 | |
| 3. | 56.00 | .66⅔ | |
| 4. | 40.00 | 1.25 | |
| 5. | 77.00 | .87½ | |
| 6. | 212.50 | 2.50 | |
| 7. | 97.50 | 3.75 | |

8. Divide each of the following by .12½:

.37½; .33⅓; .625; .87½; .25; 1.14⅔.

9. Find the sum of ¼, ⅛, ⅜, ⅝, ⅞, and ½ and express the sum decimally.

10. Divide 36 by each of the following:

.37½; .12½; .14⅔; .50; .06¼.

11. How many yards can be bought in each case?

| Total Cost | Cost per Yard | Number of Yards |
|---|---|---|
| $16 | $ .25 | |
| 24 | .33⅓ | |
| 36 | .37½ | |
| 20 | .62½ | |
| 32 | .50 | |
| 26 | .66⅔ | |
| 56 | 1.25 | |
| 44 | 1.33⅓ | |
| 24 | 2.66⅔ | |

### Review of Short Methods

Perform the following operations, stating what short method was used in each case:

| | | |
|---|---|---|
| 1. $360 \div .37\frac{1}{2}$. | 2. $\frac{2}{7} + \frac{3}{8}$. | 3. $436 \times 11$. |
| 4. $65 \times 25$. | 5. $463 \times 287$. | 6. $1{,}016 \times 1{,}005$. |
| 7. $3{,}654 \times 2{,}700$. | 8. $105 \times 126$. | 9. $95 \times 87$. |
| 10. $\frac{1}{3} - \frac{1}{5}$. | 11. $48\frac{1}{4} \times 63\frac{7}{8}$. | 12. $61\frac{1}{2} \times 12\frac{1}{2}$. |
| 13. $109 \times 113$. | 14. $\frac{9}{16} + \frac{8}{11}$. | 15. $463 \times 416$. |

| | | |
|---|---|---|
| **16.** $96 \times 94$. | **17.** $1{,}015 \times 1{,}014$. | **18.** $13 \times 18$. |
| **19.** $6\frac{1}{2} \times 4\frac{1}{2}$. | **20.** $825 \times 927$. | **21.** $.000825 \times 100$. |
| **22.** $17 \times 12$. | **23.** $724 \times 999$. | **24.** $116 \times 104$. |
| **25.** $\frac{3}{5} \div \frac{7}{8}$. | **26.** $113 \times 102$. | **27.** $1{,}026 \times 1{,}003$. |
| **28.** $\frac{1}{5} + \frac{1}{3}$. | **29.** $5{,}362 \times 111$. | **30.** $433 \times 99$. |
| **31.** $109 \times 112$. | **32.** $976 \times 990$. | **33.** $333 \times 12\frac{1}{2}$. |
| **34.** $17 \times 19$. | **35.** $13\frac{1}{2} \times 14\frac{2}{5}$. | **36.** $\frac{3}{7} + \frac{3}{8}$. |
| **37.** $\frac{3}{7} - \frac{3}{8}$. | **38.** $15\frac{1}{2} \times 18\frac{1}{2}$. | **39.** $96 \times 88$. |
| **40.** $905.387 \times 3{,}000$. | **41.** $986 \times 993$. | **42.** $16 \times 18$. |

### General Review Exercises

**1.**

| | | |
|---|---|---|
| $7{,}205 \div 87\frac{1}{2}$. | $360 \times 16\frac{2}{3}$. | $648 \times 12\frac{1}{2}$. |
| $1{,}240 \div 83\frac{1}{3}$. | $48 \times .66\frac{2}{3}$. | $246 \div 37\frac{1}{2}$. |
| $1{,}940 \times 62\frac{1}{2}$. | $189 \times .33\frac{1}{3}$. | $147 \div .87\frac{1}{2}$. |

**2.** Find the cost of:

| | | | |
|---|---|---|---|
| 156 articles at | \$12.50 each | 420 articles at | $\$.37\frac{1}{2}$ each |
| 116 articles at | 2.50 each | 96 articles at | $.66\frac{2}{3}$ each |
| 48 articles at | 3.33 each | 360 articles at | $.62\frac{1}{2}$ each |

**3.** At $12\frac{1}{2}$ cents per yard how many yards of cloth can be bought for each of the following amounts: \$4? \$8.50? \$.50? \$10? \$6.50?

**4.** Copy from dictation, multiply, and check:

| | | | |
|---|---|---|---|
| *a.* 140.78 | *b.* 70.926 | *c.* 3.6409 | *d.* 428.5 |
| 32.5 | 1.248 | 20.8 | 7.48 |
| *e.* 87.34 | *f.* 9.284 | *g.* 409.63 | *h.* 246.3 |
| .005 | 4.56 | 5.09 | .258 |

**5.** The monthly rainfall by inches in a certain place was as follows: 2.1, 1.9, 1.8, 2.3, 2, 2.1, 1.4, 1.5, 1.3, 1.5, 1.4, 1.1. What was the average monthly rainfall?

**6.** At $66\frac{2}{3}$ cents each how many articles will \$8 buy? \$14? \$126? \$180?

**7.** A girl used seven balls of worsted yarn in making a sweater. Each ball weighed $1\frac{1}{2}$ ounces. What was the weight of the sweater and what did the material cost, the price of yarn being 65 cents a ball?

8. A certain piece of beef lost $\frac{1}{8}$ of its weight in roasting. The roasted piece weighs $3\frac{3}{4}$ lb. What was the weight before roasting?

9. Add and check:

| *a.* | *b.* | *c.* | *d.* |
|---|---|---|---|
| 4,876 | 2,089 | 9,076 | 4,807 |
| 3,294 | 7,648 | 5,492 | 3,928 |
| 7,386 | 3,975 | 398 | 7,096 |
| 9,275 | 8,437 | 7,863 | 4,279 |
| 8,143 | 2,978 | 47 | 847 |

## CHAPTER X

### DENOMINATE NUMBERS

MANY of the purchases and sales in modern business life involve measurement. We find it necessary to use frequently the various units to measure weight, distance, area, volume, value, and time. A mastery of the most commonly used tables is necessary.

In the tables the units which are most frequently used are printed in heavy type.

**79. Linear Measure.**

| | |
|---|---|
| 12 **inches** (in.) | = 1 **foot** (ft.) |
| 3 feet | = 1 **yard** (yd.) |
| 5½ yards, or 16½ feet | = 1 **rod** (rd.) |
| 320 rods, or 5280 feet | = 1 **mile** (mi.) |
| 1,760 yards | = 1 mile |

Architects, carpenters, and mechanics frequently write ′ for foot, and ″ for inch. Thus 8′ 7″ means 8 ft. 7 in.

In engineering it is customary to divide the foot and the inch into tenths and hundredths, instead of into halves and fourths.

Other measures of length are:

1 hand = 4 in. Used in measuring the height of horses.
1 fathom = 6 ft. Used in measuring depths at sea.
1 knot, nautical or geographical mile = 1.1526⅔ mi. or 6,089 ft.

The knot is used in measuring distances at sea. It is equivalent to 1 min. of longitude at the equator.

**80. Square Measure.**

| | |
|---|---|
| 144 **square inches** (sq. in.) | = 1 **square foot** (sq. ft.) |
| 9 square feet | = 1 **square yard** (sq. yd.) |
| 30¼ square yards | = 1 **square rod** (sq. rd.) |
| 160 square rods | = 1 **acre** (A.) |
| 640 acres | = 1 **square mile** (sq. mi.) |

Sq. ′ and sq. ″ are frequently used for square foot and square inch. Thus, 15 sq. ′ 6 sq. ″ means 15 sq. ft. 6 sq. in.

An acre contains 43.560 sq. ft.

A *square* is 100 sq. ft. It is used in measuring roofing.

**81. Cubic Measure.**

| | | |
|---|---|---|
| 1,728 **cubic inches** (cu. in.) | = 1 | **cubic foot** (cu. ft.) |
| 27 cubic feet | = 1 | **cubic yard** (cu. yd.) |
| 128 cubic feet | = 1 | cord |

A cubic yard (of earth) is considered a load.

| | | |
|---|---|---|
| $24\frac{3}{4}$ cubic feet | = 1 | perch (P.) |

A perch of stone or masonry is $16\frac{1}{2}$ ft. (1 rd.) long, $1\frac{1}{2}$ ft. wide, and 1 ft. high.

**82. Avoirdupois Weight.**

| | | |
|---|---|---|
| 16 **ounces** (oz.) | = 1 | **pound** (lb.) |
| 100 pounds | = 1 | hundredweight (cwt.) |
| 2,000 pounds | = 1 | **short ton** (T.) |
| 2,240 pounds | = 1 | long ton (T.) |

Avoirdupois weight is used in weighing all ordinary substances except precious metals, jewels, and drugs at retail.

The long ton is used in measuring coal and iron and in the United States customhouse.

**83. Liquid Measure.**

| | | |
|---|---|---|
| 4 gills (gi.) | = 1 | **pint** (pt.) |
| 2 pints | = 1 | **quart** (qt.) |
| 4 quarts | = 1 | **gallon** (gal.) |
| $31\frac{1}{2}$ gallons | = 1 | **barrel** (bbl.) |

A gallon contains 231 cu. in.

63 gal. = 1 hogshead (hhd.).

In practice hogsheads and barrels differ in size. The standard gasoline barrel has a capacity of 56 gallons.

One cubic foot is $7\frac{1}{2}$ gal. approximately.

**84. Dry Measure.**

| | | |
|---|---|---|
| 2 **pints** (pt.) | = 1 | **quart** (qt.) |
| 8 quarts | = 1 | **peck** (pk.) |
| 4 pecks | = 1 | **bushel** (bu.) |

A bushel contains 2,150.42 cu. in.

A heaped bushel contains 2,747.71 cu. in.

A dry quart contains 67.2 cu. in.

A liquid quart contains 57.75 cu. in.

Many of the states have fixed by statute the weight of a bushel of certain grains and seeds and these must be used in buying and selling unless specific agreements to the contrary are made. Determine these specifications for your state. See Appendix.

**85. Measures of Time.**

| | | |
|---|---|---|
| 60 | **seconds** (sec.) | = 1 **minute** (min.) |
| 60 | minutes | = 1 **hour** (hr.) |
| 24 | hours | = 1 **day** (da.) |
| 7 | days | = 1 **week** (wk.) |
| 30 | days | = 1 commercial **month** (mo.) |
| 52 | weeks | = 1 **year** (yr.) |
| 12 | months | = 1 year |
| 360 | days | = 1 commercial year |
| 365 | days | = 1 common year |
| 366 | days | = 1 leap year |
| 100 | years | = 1 century |

**86. Counting.**

| | | |
|---|---|---|
| 12 | units | = 1 **dozen** (doz.) |
| 12 | dozen | = 1 **gross** (gro.) |
| 12 | gross | = 1 great gross (gr. gro.) |
| 20 | units | = 1 score |

**87. Wood Measure.**

A straight pile of wood, 4 ft. × 4 ft. × 1 ft., contains 1 cord foot. 8 cord feet, or 128 cubic feet = 1 cord.

**88. United States Money.**

The principal units of United States money are the cent and the dollar.

| | | |
|---|---|---|
| 10 | mills | = 1 **cent** |
| 5 | cents | = 1 **nickel** |
| 10 | cents | = 1 **dime** |
| 25 | cents | = 1 **quarter** |
| 50 | cents | = 1 **half dollar** |
| 100 | cents | = 1 **dollar** |
| 10 | dollars | = 1 eagle |

The mill is not a coin, but the term is frequently used in computations such as the levying of taxes. The term " eagle " is not common in business.

**89. English Money.**

4 farthings (far.) = 1 **penny** (*d.*), (plural, pence)
12 pence = 1 **shilling** (*s.*)
20 shillings = 1 **pound sterling** (£)

1 pound sterling at par is $4.8665. The actual exchange value fluctuates.

**90. French Money.**

10 millimes (m.) = 1 **centime** (c.)
10 centimes = 1 decime (d.)
10 decimes = 1 **franc** (F.)

The par value of a franc is $.193.

**91. German Money.**

100 **pfennigs** (pf.) = 1 **mark** (M.)

The par value of a mark is $.238.

### Units of Money in Other Countries

The following table gives the principal units of value in several other countries with their approximate par value in United States money.

| Country | Principal Unit | Par Value |
|---|---|---|
| Argentine Republic | Peso | $ .96 |
| Brazil | Milrei | .55 |
| Denmark | Krone | .27 |
| Ecuador | Libra | 4.86 |
| Japan | Yen | .50 |
| Mexico | Peso (Dollar) | .49 |
| Netherlands | Guilder | .40 |
| Peru | Libra | 4.86 |
| Russia | Ruble | .52 |
| Turkey | Lira | 4.40 |

Belgium, Greece, Italy, Roumania, Servia, and Spain use the same monetary system as France. Norway and Sweden use the same system as Denmark.

**Written Work**

**1.** Draw a line on the blackboard 1 foot long. Try to form a definite mental picture of this length so that you may be able to estimate lengths with a fair degree of accuracy.

**2.** Draw lines of the following lengths without aid, then check each estimate.

$\frac{1}{2}$ ft., 2 ft., 3 ft., $\frac{2}{3}$ ft., $1\frac{1}{2}$ ft., 7 ft., 3 in., 9 in., 15 in.

**3.** Estimate the length and width of your desk; the length and width and height of the classroom; the dimensions of the school building and grounds. Check your estimates.

**4.** Practice estimating and stepping off various distances until you can do so with a fair degree of accuracy.

**5.** Lay off on the school grounds a square rod and, if possible, an acre. Estimate the areas of various lots, parks, or fields, and check your results, when possible.

By lifting various units of weights and then checking your own estimates of the weights of numerous objects, you can acquire the ability to estimate certain weights with but a small error.

**92. Reduction of Denominate Numbers.** A denominate number is reduced by changing the units in which it is expressed, without changing the value. When the units are reduced to those of lower denomination the process is called descending reduction. When they are reduced to units of a higher denomination the process is called ascending reduction.

**93. Changing to Lower Denominations.**

**Example.** How many quarts in 3 bu. 3 pk. 5 qt.?

SOLUTION.

3 bu. = 12 pk.
12 pk. + 3 pk. = 15 pk.
15 pk. = 120 qt.
120 qt. + 5 qt. = 125 qt.
Therefore, 3 bu. 3 pk. 5 qt. = 125 qt.

After studying this solution, state a method for reducing measured quantities to lower denominations.

### Oral Work

Reduce as indicated; do as much of the work orally as possible.

1. $15\frac{1}{3}$ yd. to feet.
2. $4\frac{2}{3}$ ft. to inches.
3. 5 gal. 3 qt. 1 pt. to pints.
4. 4 bu. 3 pk. 5 qt. to quarts.
5. 5 lb. 9 oz. to ounces (avoir.).

**Example.** Reduce .345 mi. to lower denominations.

SOLUTION.

1 mi. = 320 rd.
.345 mi. = 110.4 rd
1 rd. = 5.5 yd.
.4 rd. = 2.2 yd.
1 yd. = 3 ft.
.2 yd. = .6 ft.
1 ft. = 12 in.
.6 ft. = 7.2 in.

Therefore, .345 mi. = 110 rd. 2 yd. 7.2 in.

After studying this solution, state a method for reducing fractions of a large denomination to units of lower denominations.

### Written Work

Reduce:

1. 8 mi. to rods.
2. .236 mi. to feet.
3. .89 bu. to quarts.
4. $\frac{1}{4}\frac{3}{4}$ sq. mi. to square rods.
5. 62 rds. to feet.
6. .85 gal. to pints.
7. 4.3 mi. to feet.
8. 43 A. to square feet.

**94. Changing to Higher Denominations.** When a quantity is expressed in a low denomination, it may be desired to express it in higher denominations.

**Examples.** 1. Express 1,316 pints in higher denominations.

SOLUTION.

Since 2 pints = 1 quart.
1316 pints = 658 quarts.
Since 4 quarts = 1 gallon,
658 quarts = 164 gals., and 2 qt. remaining.
Therefore, 1316 pints = 164 gal., 2 qt.

After this studying solution, state a method for reducing quantities to units of higher denominations.

2. Reduce 3 qt. 1 pt. to a decimal part of a peck.

SOLUTION. 3 qt. 1 pt. = 7 pt.
1 pk. = 16 pt.
Therefore, 3 qt. 1 pt. = $\frac{7}{16}$ pk.
$\frac{7}{16}$ pk. = .4375 pk.

**Written Work**

Reduce to units of higher denominations:

1. 367 pints (dry measure).
2. 133 in.
3. 47 ft.
4. 12 yd.
5. 216 oz. (avoir.).
6. 31 pt (liquid measure).
7. 2,369 sq. in.
8. Reduce 4 sq. ft. 68 sq. in. to a decimal part of a square rod.
9. Reduce 1 qt. 2 pt. to a decimal part of a gallon.
10. Reduce 7 oz. to a decimal part of a pound (avoir.).
11. Reduce $\frac{2}{3}$ yd. to a decimal part of a rod.
12. Reduce $2\frac{1}{2}$ ft. to a decimal part of a yard.
13. Reduce 47 sq. rd. to a decimal part of an acre.
14. Reduce $8\frac{1}{2}$ in. to a decimal part of a foot.
15. Reduce $2\frac{1}{2}$ qt. to a decimal part of a gallon.
16. Reduce 4 ft. 7 in. to a decimal part of a rod.

**95. Addition of Denominate Numbers.**

**Example.** Find the sum of 2 yd. 2 ft. 9 in. and 4 yd. 1 ft. 7 in.

SOLUTION.

| | | |
|---|---|---|
| 2 yd. | 2 ft. | 9 in. |
| 4 " | 1 " | 7 " |
| 6 yd. | 3 ft. | 16 in. |
| 7 " | 1 " | 4 " |

or

State a method for adding denominate numbers.

**Written Work**

Add:

1.

| | | |
|---|---|---|
| 2 yr. | 5 mo. | 18 da. |
| 5 " | 8 " | 19 " |
| 6 " | 11 " | 13 " |

2.

| | | |
|---|---|---|
| 5 gal. | 3 qt. | 1 pt. |
| 4 " | 2 " | 1 " |
| 5 " | 1 " | 1 " |

3.

| | | |
|---|---|---|
| 2 lb. | 5 oz. | 17 pwt. |
| 3 " | 8 " | 19 " |
| 4 " | 9 " | 12 " |

4.

| | | |
|---|---|---|
| 5 A. | 126 sq. rd. | |
| 16 " | 249 " | " |
| 13 " | 168 " | " |

5. 7 mo. 12 da.
5 " 5 "

6. 12 mi. 4 rd.
8 " 9 "

7. A rectangular field is 18 rd. 2 yd. 2 ft. long, and 13 rd. 1 yd. $2\frac{1}{2}$ ft. wide. What length of wire will be required for a fence 6 wires high?

8. A dealer bought 17 gal. 3 qt. of milk from each of two men, and 29 gal. 3 qt. 1 pt. from a third man. What was the entire cost at 24 cents a gallon?

**96. Subtraction of Denominate Numbers.**

If it is required to subtract 1 mi. 42 rd. 13 ft. from 3 mi. 25 rd. 14 ft., we write the numbers as follows:

| | | |
|---|---|---|
| 3 mi. | 25 rd. | 14 ft. |
| 1 " | 42 " | 13 " |

When, as in this problem, the number of units of some denomination of the minuend is smaller than the number of corresponding units of the subtrahend, combine one unit of the next larger denomination with the number of units of the minuend. The problem thus becomes:

| | | |
|---|---|---|
| 2 mi. | 345 rd. | 14 ft. |
| 1 " | 42 " | 13 " |
| 1 " | 303 " | 1 " |

**Written Work**

Subtract as indicated:

1. 13 bu. 3 pk. 2 qt.
6 " 2 " 7 "

2. 7 A. 5 sq. rd. 6 sq. ft.
3 " 7 " " 2 " "

3. 27 rd. 4 yd. 1 ft.
13 " 5 " 2 "

4. 14 gal. 2 qt. 1 pt.
5 " 3 "

5. Daniel Webster was born Jan. 18, 1782, and lived to the age of 70 yr. 9 mo. 6 da. On what date did he die?

6. A man owned a field containing $1\frac{2}{5}$ A. From it he sold two lots each having an area of $11\frac{1}{2}$ sq. rd. What was the area of the part remaining?

**7.** From a cask which contained 31 gal. 3 qt. 1 pt. of vinegar, 19 gal. 3 pt. were drawn out. How much remained?

### 97. Multiplication Involving Denominate Numbers.

**Example.** Multiply 3 yd. 2 ft. 7 in. by 4.

SOLUTION.

| | | | |
|---|---|---|---|
| | 3 yd. | 2 ft. | 7 in. |
| | | | 4 |
| | 12 yd. | 8 ft. | 28 in. |
| or | 15 yd. | 1 ft. | 4 in. |

#### Written Work

Multiply as indicated:

**1.** 5 gal. 3 qt. 1 pt. by 7. **2.** 7 lb. 5 oz. by 12.

**3.** 5 cu. yd. 19 cu. ft. 364 cu. in. by 13.

**4.** 4 mi. 19 rd. 3 yd. by 18.

### 98. Division Involving Denominate Numbers.

**Examples.** 1. Divide 4,568 inches by 8.

SOLUTION. 4568 in. ÷ 8 = 571 in.
571 in. = 2 rd. 8 yd. 2 ft. 7 in.

2. Divide 17 gal. 2 qt. 1 pt. by 3.

SOLUTION. 17 gal. 2 qt. 1 pt. = 141 pt.
141 pt. ÷ 3 = 47 pt.
47 pt. = 5 gal. 3 qt. 1 pt.

3. How many times is 4 gal. 2 qt. contained in 1 barrel?

SOLUTION. 4 gal. 2 qt. = 18 qt.
1 bbl. = 126 qt.
126 qt. contains 18 qt. 7 times.

Explain the method used in this illustration.

#### Written Work

**1.** Find out at what time the sun will rise and set tomorrow at the place where you live and compute the time from sunrise to sunset.

**2.** How many times are 4 yd. 2 ft. 7 in. contained in 29 yd. 6 in.?

3. Reduce to the decimal part of a mile: 5 rd.; $7\frac{1}{2}$ rd.; 5 ft.; 13 yd; 5 ft. 9 in.

4. An automobile traveled 1 mi. in $3\frac{1}{2}$ min. What was the rate per hour?

5. If a man sells 17 loads of wheat, each containing 53 bu. 3 pk., at $97\frac{1}{2}$ cents a bushel, how much should he receive?

6. Fifteen cans hold an average of 10 gal. 2 qt. 1 pt. How much do they all hold?

7. Change .427 sq. mi. to units of lower denominations.

8. A cubic foot of water weighs 62.5 lb., cast iron weighs about 7.2 as much as water; what is the weight of $2\frac{3}{8}$ cu. ft. of cast iron?

9. Determine whether the window space in your school room is as much as $\frac{1}{6}$ of the floor space. It should be at least $\frac{1}{6}$ in a well-lighted room.

10. A train leaves a city at 4:10 P.M., and reaches a certain station $87\frac{1}{2}$ mi. distant at 6:50 P.M. Allowing 12 min. for station stops, what is the rate of travel per hour?

11. There are twenty-four students in a class and each needs $2\frac{1}{2}$ cups of hot water for a cooking lesson. How much water is needed to supply all the class at one time, allowing 4 cups to a quart?

### General Exercises — Denominate Numbers

1. Lots in a certain section of a city are worth 32 cents a square foot. What is the value of a plot containing .6 acres?

2. A bin contains 2,360 cubic feet. What is its capacity in bushels when level full?

3. A coal dealer bought 1,890 tons of coal by the long ton and sold it all by the short ton. When he sold the coal it had shrunk in weight 40 pounds per long ton. How many short tons of coal did the dealer sell from this purchase?

4. From a barrel which contained 50 gallons of lubricating oil a dealer sold the following quantities: 4 gal. 2 qt., 5 gal. 1 qt., 6 gal. 3 qt. What quantity is left in the barrel?

5. A projectile leaves a certain gun with a velocity of 2,060 feet per second. At this rate how far would it travel in 42 seconds?

6. In a contest an aviator flew at the rate of 130 miles an hour. What was the time per mile? How many feet did he travel per second?

7. How many tons of steel rails will be required to lay a mile of railroad if the rails weigh 63 lb. per yard?

8. Abraham Lincoln was born February 12, 1809. How many years, months, and days ago was that?

9. How many times will a wheel with a circumference of 14 feet revolve in going one mile?

10. A dairyman delivered the following quantities of milk to a hotel during a week. Compute the total quantity.

| | GAL. | QT. | PT. |
|---|---|---|---|
| Mon. | 19 | 2 | 1 |
| Tues. | 28 | 1 | |
| Wed. | 27 | 3 | 1 |
| Thurs. | 19 | 1 | 1 |
| Fri. | 26 | 3 | |
| Sat. | 27 | 2 | 1 |
| Sun. | 28 | 3 | 1 |

11. A man can row across a lake, a distance of $2\frac{3}{5}$ mi. in 55 min. At this rate, how long will it take him to row one mile? How far could he row in 1 hour, at this rate?

12. If a cubic foot of ice weighs 57 lb. 8 oz., what will be the weight in tons of 840 cubic feet of ice?

13. Estimate the following distances then, check your estimates by measurement: 100 ft., 10 rd., 50 ft., 30 yd.

14. What place is approximately a mile from your school?

15. Certain flower seeds are to be planted 4 inches apart. How many such seeds should be planted in a row 3 yd. 2 ft. 9 in. long?

16. When the wind is blowing with a pressure of .07 lb. per sq. in. what is the total pressure on a surface of 920 sq. ft. 30 sq. in.?

17. How many cubic feet of air will be required to supply 75 cu. yd. of fresh air per hour for each student in your class?

18. A boy sold $\frac{1}{2}$ bu. of raspberries in pint boxes at 15 cents a box. How much did he receive for them?

19. How many years, months, and days ago was George Washington born?

20. Why is the date, July 4, 1776, of great interest to us? How many years, months, and days have passed since that date?

21. If a watch gains 4 seconds an hour, how much will it gain in two days?

22. At the beginning of a trip the indicator on a man's automobile registered 1,784.7 mi. At the end of the trip it registered 1,949.1 mi.

What was the distance traveled per hour if the trip was made in 6 hours?

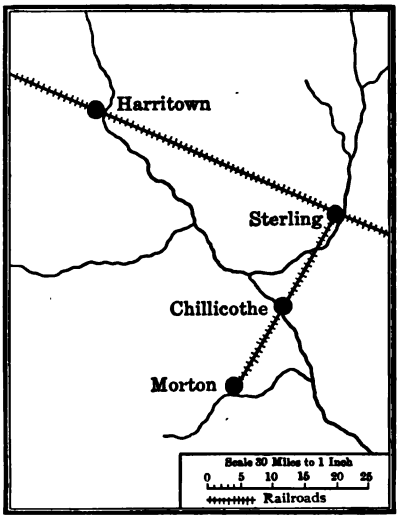
**23.** Milk is usually shipped in 10 gal. cans to stations in cities where it is bottled and distributed to customers. How many quarts in a shipment of 5 such cans?

**24.** A farmer living near a lake cut his own ice. If he cut 165 cakes averaging 70 lb. each, how much per ton did the ice cost him, the total cost being $9.40?

# CHAPTER XI

## MAPS, DRAWINGS, AND GRAPHS

NEWSPAPERS, magazines, and trade journals make frequent use of drawings and graphs in order to make clear the relations between magnitudes. It is often possible to make values and comparisons more clearly understood in this way than by merely printing a table of statistics. Drawings may be divided into three classes: maps, plans, and other dimension drawings and graphs.




### MAPS

**99. Scale.** Distances on maps are usually indicated by a scale similar to the one in the lower right hand corner of the accompanying illustration.

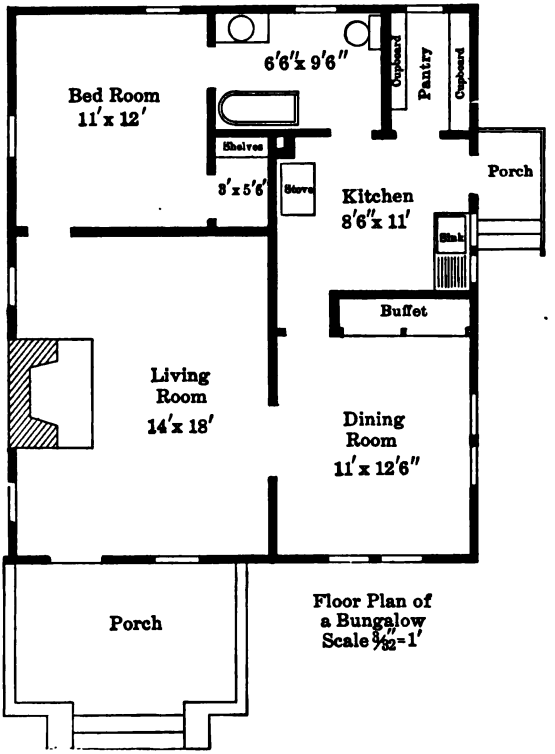
**Oral Work**

1. The map on this page is drawn to the scale 1″ = ? miles.
2. $\frac{1}{8}$″ on this map represents what true distance?
3. What is the true distance between each of the towns on the railroad? Measure distances from centers of towns.
4. An electric railroad is in process of construction between Morton and Harritown. It is surveyed in a direct line. How much shorter will this road be than the one via Sterling?
5. It is planned to locate a power house halfway between Sterling and Harritown. Indicate its position on the map and determine its distance from the two towns. How far will it be from the nearest point on the new electric line?

## Dimension Drawings

**100. Purpose.** Dimension drawings are used to show quickly and conveniently the shape and dimensions of fields, rooms, buildings, furniture, machinery, etc.

Drawings similar to the illustration are of great value. An architect designs a building and the contractor builds it in accordance with the plans shown in the drawings. Tools and machinery are usually constructed in accordance with drawings prepared by draftsmen and engineers.




**101. Drawing to Scale.** The ability to read drawings and graphic representations intelligently depends somewhat upon a knowledge of how to draw figures to scale.

The values represented by the lines in drawings and graphs can be determined easily because the lines are drawn to scale; that is, a certain distance on the drawing represents a specified amount of the quantity represented.

In the floor plan on this page one foot of true distance is represented by $\frac{3}{32}$ of an inch plotted distance on the drawing. All lines in the drawing correspond to this scale.

**102. Types of Scales.** The scale may be stated in various ways, the most common of which are:

*a.* A fraction.

For example, a scale of $\frac{1}{4}$ means that the true distances or values are four times as great as the plotted distances or values.

*b.* Statement of equivalent dimensions or values.

For example, 1″ = 1′. (Read 1 in. = 1 ft.) One inch of the drawing represents one foot of the real object. In the graph showing immigration into the United States, page 117, the scale is 1″ = 440,000.

*c.* A diagram or scaled rule accompanying the drawing. This is the method commonly used in map drawing.

*d.* Graphs may be drawn on ruled paper, the distance between the ruled lines representing a specified value.

**103. Determining the Scale.** When a drawing or a graph is to be made, it is first necessary to determine the scale. The scale depends, in general, upon three things:

*a.* The size of the paper to be used.
*b.* The largest dimension to be shown on the graph.
*c.* Convenience in showing fractional parts of the scale.

**Problem.** A room is 15′ 6″ long by 10′ 3″ wide. What scale should be used to show a diagram of this room on a sheet of paper 8″ by 10″, leaving a margin of at least one inch around the drawing?

SOLUTION. The largest dimension of the room is 15′ 6″, or 186″.

The largest dimension of the paper is 10 inches. After deducting 2 inches for margins, there is a space of 8″ available for the drawing.

The largest scale possible, therefore, is 8″ represents 186″.

This is equivalent to a scale of 1″ = 23¼″. (How was this determined?)

For convenience in showing fractional parts of a foot, it will be better to use the scale of 1″ = 24″.

### Written Work

**1.** A scale of 1″ = 1′ is equivalent to what fractional scale?

**2.** A scale of 1″ = 5″ is equivalent to what fractional scale?

**3.** A scale of 1″ = 5′ is equivalent to what fractional scale?

**4.** If the scale is $\frac{1}{12}$, what distance on the drawing would represent 6′ 3″? 4′ 9″?

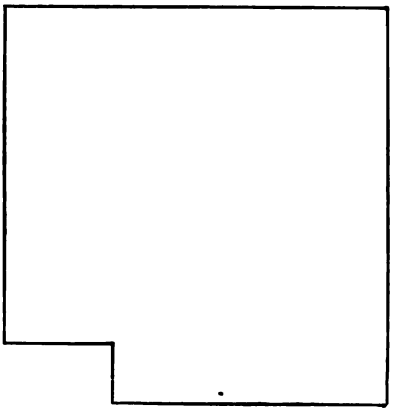
**5.** If the scale is $\frac{1}{300}$, what plotted distance would represent 750 feet? 750 inches?

**6.** If the scale is 1″ = 1,000 bushels, what distance on a graph would represent 5,125 bushels? 8,500 bushels? 1,750 bushels?

7. The following floor plan is drawn to a scale of $\frac{1}{128}$.

One inch of plotted distance on the floor plan is equivalent to how many feet of true distance?

8. One foot of true distance on the floor plan is represented by what part of an inch on the drawing?



9. What are the true dimensions of this room?

10. Assume that this drawing is made to the scale, $\frac{1}{8}$ in. = 1 ft. ($\frac{1}{8}'' = 1'$). What are the true dimensions of the room?

11. With the scale $1'' = 2'$, what length lines would represent the dimensions of a room $15'\ 6'' \times 10'\ 3''$?

12. Use this scale to make a drawing of a room $12'\ 6''$ by $9'\ 9''$.

13. A field is 120 rods by 80 rods. Scale, $1'' = 1$ rod. How large a paper is needed to diagram the field, leaving a margin of 2 inches all around? Would this be a reasonable scale to use?

14. If a scale 1 inch = 20 rods is used, what size paper is necessary in order to diagram this field and leave a margin of one inch on all sides?

15. What is the largest scale that could be used to plot this field on a paper the size of this page, leaving margins of at least $\frac{3}{4}''$?

16. Draw a diagram of your schoolroom, entering the dimensions as illustrated in the drawing on page 114.

17. Draw a diagram of a room in your home. Do not enter the dimensions but state the fractional scale used, and draw all lines very carefully in accordance with this scale.

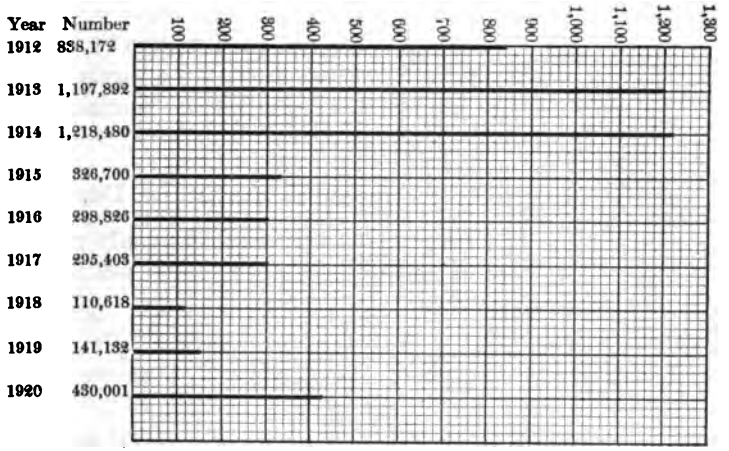
NOTE TO TEACHER. When these drawings are brought to class, they may be exchanged, and the students required to compute the dimensions in accordance with the scale stated.

## GRAPHS

**104. Forms of Graphs.** Since graphs are used to display many kinds of statistics, it is to be expected that the graphs will be drawn in a variety of forms. The following illustrations are intended to suggest the uses which may be made of different types of bar, line, and circle graphs.

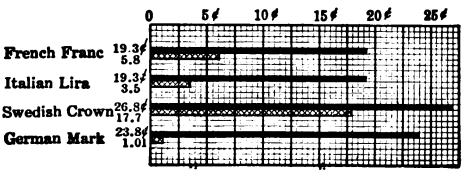
**105. Comparison by Straight Lines.** The following graph illustrates the use of straight lines for the comparison of quantities, and is based on the following immigration statistics:

| Year (Ending June 30) | Number |
|---|---|
| 1912 | 838,172 |
| 1913 | 1,197,892 |
| 1914 | 1,218,480 |
| 1915 | 326,700 |
| 1916 | 298,826 |
| 1917 | 295,403 |
| 1918 | 110,618 |
| 1919 | 141,132 |
| 1920 | 430,001 |



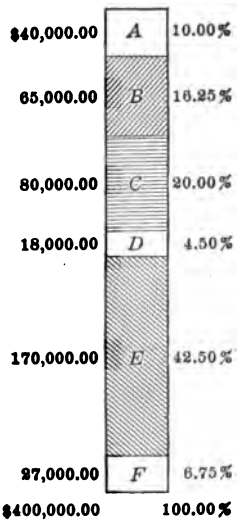

IMMIGRATION INTO THE UNITED STATES 1912 TO 1920

**106. Cross-Hatching.** More extensive comparisons are possible if cross-hatched bars are used. In the following graph, the solid black bars represent the par value in cents of certain foreign coins; the cross-hatched bars represent their lowest exchange value during the year 1920.



**107. Component Parts Shown by Cross-Hatching.** A bar may be cross-hatched in various ways to show different parts of a total, in the manner described below.

TOTAL SALES OF A DEPARTMENT STORE

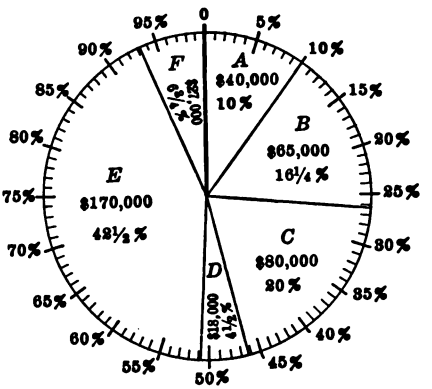



| DEPARTMENT | SALES | PER CENT |
|---|---|---|
| A | $ 40,000.00 | 10.00 |
| B | 65,000.00 | 16.25 |
| C | 80,000.00 | 20.00 |
| D | 18,000.00 | 4.50 |
| E | 170,000.00 | 42.50 |
| F | 27,000.00 | 6.75 |
| Total | $400,000.00 | 100.00 |

**108. Component Parts Shown by Circle.** The same facts may be shown by a circle graph, but this form is more difficult to use and is not as easily read as a bar.

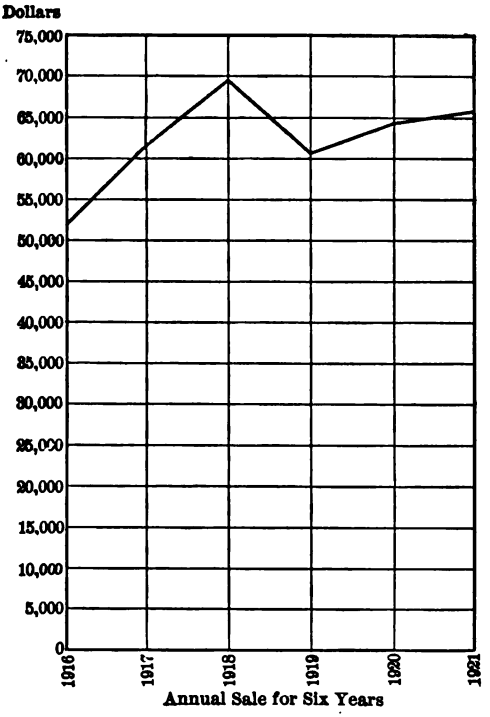
COMPARISON OF SALES IN SIX DEPARTMENTS

This type of graph may be easily prepared by obtaining paper with the circle printed thereon, the circumference being divided by graduations on a percentage basis. If such paper can not be obtained, a protractor may sometimes be obtained scaled in per cents. If necessary, an ordinary protractor may be used. The protractor shown in the illustration at the right is divided on the basis of degrees of arc. Since 6here are 360° in a circle, 100 % is represented by 360°; 10 % by t3°; 16.25 % by 58.5°; etc.



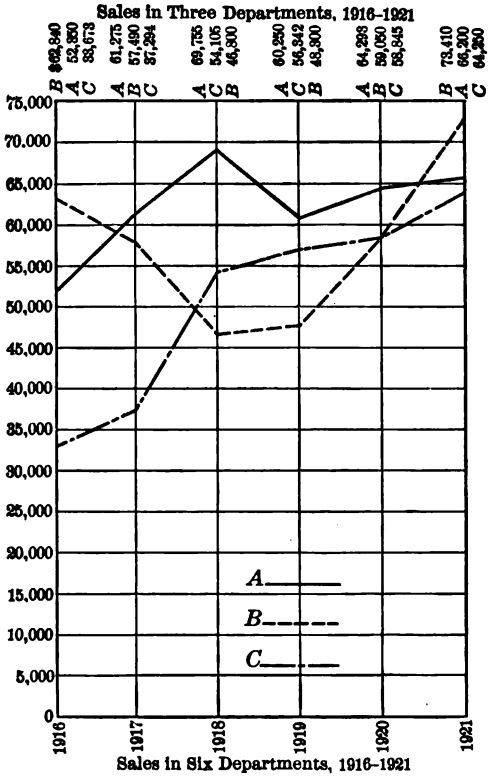

**109. Single Curve Showing Quantities.** Plotted curves are especially adapted to showing changes in quantities at different dates or during a series of periods. The paper should be ruled both horizontally and vertically. The horizontal lines should represent a quantity scale,

and the vertical lines should represent dates or time periods. To draw such a curve, dots are placed on the vertical lines at the proper points to represent the desired quantities, and the dots are connected by a line called a curve.




**110. Several Curves in One Graph.** By using different colors of ink or different forms of lines, it is possible to plot several curves together, as in the following illustration, which shows the sales in three departments of a store during six successive years. The graph is prepared from the following statistics:

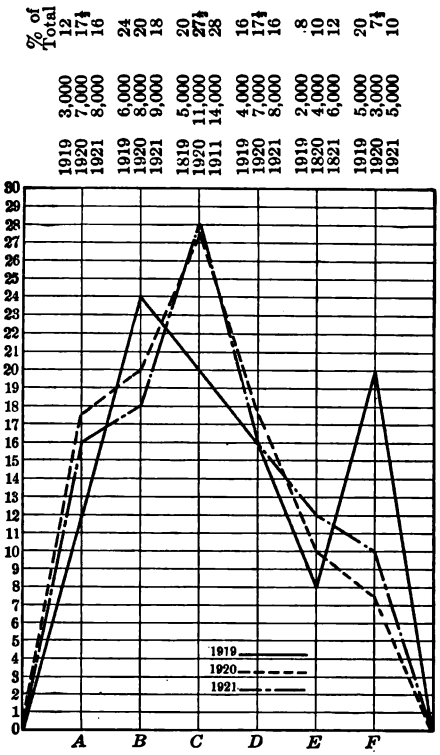
| Year | Dept. A | Dept. B | Dept. C |
|---|---|---|---|
| 1916 | $52,350 | $62,840 | $33,673 |
| 1917 | 61,275 | 57,490 | 37,294 |
| 1918 | 69,755 | 46,800 | 54,105 |
| 1919 | 60,250 | 48,300 | 56,342 |
| 1920 | 64,293 | 59,050 | 58,845 |
| 1921 | 66,200 | 73,410 | 64,250 |




**111. Comparative Percentage Curves.** Several curves may be plotted in one graph as shown in the following illustration.

SALES IN SIX DEPARTMENTS

| DEPARTMENT | 1919 | | 1920 | | 1921 | |
|---|---|---|---|---|---|---|
| | $ | % | $ | % | $ | % |
| A | 3,000 | 12 | 7,000 | 17.5 | 8,000 | 16 |
| B | 6,000 | 24 | 8,000 | 20 | 9,000 | 18 |
| C | 5,000 | 20 | 11,000 | 27.5 | 14,000 | 28 |
| D | 4,000 | 16 | 7,000 | 17.5 | 8,000 | 16 |
| E | 2,000 | 8 | 4,000 | 10 | 6,000 | 12 |
| F | 5,000 | 20 | 3,000 | 7.5 | 5,000 | 10 |
| | 25,000 | 100 | 40,000 | 100 | 50,000 | 100 |




Per Cent of Total Annual Sales in each of Six Departments

## Written Work

Prepare graphs from the following statistics.

1. The figures in the following table are known as "index numbers." The basis is 100, and represents the average for the years 1909 to 1913. Then if the value of land in 1919 is represented by 202, farm land more than doubled in value.

| Year | Land Values | Farm Wages | Crop Prices | Articles Farmers Buy |
|---|---|---|---|---|
| 1919 | 202 | 207 | 221 | 212 |
| 1918 | 167 | 172 | 206 | 202 |
| 1917 | 153 | 142 | 188 | 153 |
| 1916 | 136 | 114 | 117 | 125 |
| 1915 | 123 | 105 | 101 | 112 |
| 1914 | 111 | 104 | 100 | 103 |

2. Bradstreet reports the following analysis of the reasons why business men fail, and the per cent of total failures due to each cause.

| | |
|---|---|
| Incompetence | 38.2% |
| Inexperience | 5.6 |
| Lack of capital | 30.3 |
| Unwise credits | 1.3 |
| Fraud | 7.0 |
| Failures of others | 1.7 |
| Extravagance | 1.1 |
| Neglect | 1.7 |
| Competition | 1.1 |
| Specific conditions | 11.3 |
| Speculation | .7 |
| | 100.0 |

3. The following statistics are taken from Lloyd's annual returns showing the merchant vessels of over 100 gross tons launched during a recent year.

| Country | Number | Tons |
|---|---|---|
| United Kingdom | 612 | 1,620,442 |
| Canada | 201 | 271,264 |
| Denmark | 46 | 37,766 |
| France | 34 | 32,633 |
| Holland | 100 | 137,086 |
| Italy | 32 | 82,713 |
| Japan | 133 | 611,883 |
| Norway | 82 | 57,578 |
| Spain | 41 | 52,609 |
| Sweden | 53 | 50,971 |
| United States | 1,051 | 4,075,385 |

4. Five departments of a wholesale store produced profits in one year as follows:

| Department | Profits |
|---|---|
| I | $2,165.00 |
| II | 3,297.00 |
| III | 794.00 |
| IV | 1,719.00 |
| V | 4,625.00 |

Show the profits of each department and the per cent of the total made by each department.

5. Show by a graph the data for the attendance at your school for a school year. (The principal can supply the data.)

6. Graph the temperature for a certain hour for several successive days.

7. Make a graph showing your progress in speed and accuracy in addition, subtraction, multiplication, and division for a given set of examples.

8. Devise a convenient form for showing the following data graphically.

Great ship canals of the world:

| Canal | Length (miles) | Width (feet) |
|---|---|---|
| Cape Cod | 8 | 150 |
| Corinth (Greece) | 4 | 72 |
| Kronstadt-Petrograd (Russia) | 16 | 220 |
| Elbe and Trave (Germany) | 41 | 72 |
| Kiel (Germany) | 61 | 150 |
| Manchester | 35.5 | 120 |
| New Orleans Industrial | 6 | 150 |
| Panama | 50.5 | 300 |
| Suez (Egypt) | 90 | 108 |
| Welland (Canada) | 26.75 | 100 |

9. Graph the following:

Population of the world in 1920·

| | |
|---|---|
| Africa | 142,872,000 |
| Asia | 830,108,000 |
| Europe | 484,118,000 |
| North America | 145,441,000 |
| Oceania | 63,998,000 |
| South America | 63,754,000 |

# PERCENTAGE

## CHAPTER XII

### PERCENTAGE

A KNOWLEDGE of percentage is necessary in many lines of business. The topic does not involve any mathematical principles with which the student is not familiar.

**112. Relation of Percentage to Common and Decimal Fractions. Percentage** is the process of computing by hundredths. The symbol % stands for per cent.

Thus, $15\,\%$ means $\frac{15}{100}$ or .15.

$8\,\%$ means $\frac{8}{100}$ or .08.

$300\,\%$ means $\frac{300}{100}$ or 3.00.

$\frac{1}{2}\,\%$ means $\frac{\frac{1}{2}}{100}$ or $.00\frac{1}{2}$ or .005.

$\frac{2}{3}\,\%$ means $\frac{\frac{2}{3}}{100}$ or $.00\frac{2}{3}$.

$7\frac{1}{2}\,\%$ means $\frac{7\frac{1}{2}}{100}$ or $.07\frac{1}{2}$ or .075.

From the illustrations above it is clear that common fractions, decimal fractions, and per cents are different forms of expression, and that any per cent may be expressed as either a common or a decimal fraction.

**Oral Work**

Express each of the following as a common fraction and as a decimal fraction:

| | | | |
|---|---|---|---|
| **1.** $7\,\%$ | **2.** $4\,\%$ | **3.** $19\,\%$ | **4.** $24\,\%$ |
| **5.** $2\frac{1}{3}\,\%$ | **6.** $\frac{1}{4}\,\%$ | **7.** $\frac{1}{10}\,\%$ | **8.** $200\,\%$ |
| **9.** $184\,\%$ | **10.** $5\frac{1}{6}\,\%$ | **11.** $\frac{1}{100}\,\%$ | **12.** $16\frac{2}{3}\,\%$ |

Express with the symbol %:

| | | | |
|---|---|---|---|
| **13.** .74 | **14.** .03. | **15.** $.07\frac{1}{4}$ | **16.** 2.00 |
| **17.** $.12\frac{1}{2}$ | **18.** .001 | **19.** 1.00 | **20.** .0045 |
| **21.** .5 | **22.** .005 | **23.** $.16\frac{2}{3}$ | **24.** $.14\frac{2}{7}$ |
| **25.** $\frac{1}{2}$ | **26.** $\frac{1}{4}$ | **27.** $\frac{1}{5}$ | **28.** $\frac{3}{2}$ |
| **29.** $\frac{3}{4}$ | **30.** $\frac{3}{5}$ | **31.** $\frac{4}{5}$ | **32.** $\frac{1}{3}$ |

**33.** 40 % of a quantity is how many hundredths of it? What fractional part of it?

**34.** 100 % of a number is how many times the number?

**35.** 300 % of a number is how many times the number?

**36.** 140 % of a number is how many times the number?

**37.** 2 times a number is what per cent of the number?

**38.** 5 times a number is what per cent of the number?

**39.** What fractional part of a number is:

| | | |
|---|---|---|
| 20 % of it? | 12 % of it? | 13 % of it? |
| 80 % of it? | 75 % of it? | $\frac{1}{8}$ % of it? |
| 100 % of it? | $\frac{1}{2}$ % of it? | $12\frac{1}{2}$ % of it? |
| $37\frac{1}{2}$ % of it? | $87\frac{1}{2}$ % of it? | 50 % of it? |
| $6\frac{2}{3}$ % of it? | $62\frac{1}{2}$ % of it? | $66\frac{2}{3}$ % of it? |

**40.** Express each of the following as a decimal fraction and with the symbol %.

$\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{1}{4}$, $\frac{3}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{10}$, $\frac{5}{12}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$

### 113. Terms Used in Percentage.

**Base.** The number of which a per cent is to be taken is called the **base.**

**Rate.** The per cent of the base to be taken is called the **rate.**

**Percentage.** The result obtained by taking a certain per cent of the base is called the **percentage.** Thus,

$$64 \times .25 = 16;$$

or $$\textbf{base} \times \textbf{rate} = \textbf{percentage.}$$

#### Fundamental Processes of Percentage

Computations in percentage are based on the following:

$$\text{base} \times \text{rate} = \text{percentage.}$$
$$64 \times .25 = 16.$$

Since the percentage is the product of the base and rate, it is evident that,

*a.* The percentage divided by the base equals the rate. For example: $16 \div 64 = .25 = 25\%$.

*b.* The percentage divided by the rate equals the base. For example: $16 \div 25\% = 16 \div .25 = 64$.

From these statements it is evident that if any two of the terms are known, the other one can be found.

In the chapter on Decimal Fractions you learned how to multiply and divide by any number expressed decimally. Percentage involves the mathematical principles of decimal fractions. The exercises in decimal fractions involved computations with tenths, hundredths, thousandths, etc. In percentage, computation is by hundredths.

**114.** All mathematical computations involved in percentage may be grouped under three headings:

*a.* To find a given per cent of a number.
*b.* To find what per cent one number is of another.
*c.* To find a number when a certain per cent of it is known.

**115. To find a given per cent of a number; that is, to find the percentage.**

**Problems.** 1. Find 1% of 428.

SOLUTION. $1\% = .01$ .01 of 428 = 4.28.

2. Find 7% of 378.

SOLUTION. $.07 \times 378 = 26.46$.

3. Find 14% of \$360.

SOLUTION.

$$\begin{array}{r} \$\ 360 \\ .14 \\ \hline 1440 \\ 360 \\ \hline \$50.40 \end{array}$$

*To find the percentage, multiply the base by the rate.*

**Oral Work**

1. Find 1% of each of the following: 18; 146; 198; 7; .03; 876; .045.

**2.** Find 5% of each of the following: 80; 60; 25; 42.

**3.** Find 10% of \$18.20; of 478 mi.; of 32 yd.; of 648 A.

**4.** How many places should be pointed off in finding each of the following:

| | | |
|---|---|---|
| 17% of 4762? | 17% of 476.2? | 17% of 47.62? |
| 1.7% of 4762? | .17% of 4762? | 17% of .4762? |

**5.** Find 10% of 400; 560; 8.46.

**6.** Find 3% of 40; 60; 15.

**7.** Find 100% of 15; 40; 3.

**8.** Find 20% of 80; 70; 60.

**9.** Find 50% of 40; 2; 10.

**10.** Find 30% of 30; 40; 80.

**11.** Find $12\frac{1}{2}$% of 480; 96; 640.

**12.** Find $37\frac{1}{2}$% of 24; 80; 72.

**13.** Find 200% of 9; 32; $18\frac{1}{2}$.

**14.** Find $14\frac{2}{7}$% of 21; 35; 350.

**15.** Find 25% of 12; 17; 96.

**16.** Find $\frac{1}{2}$% of 120; 34; 18.

**17.** Find $\frac{1}{2}$% of 30; 50; 24.

### Written Work

Find:

**1.** 17% of 472.

**2.** 92% of \$537.

**3.** $16\frac{2}{3}$% of \$420.

**4.** 3% of 48,729.

**5.** 200% of 37 ft.

**6.** 1.8% of 42,690.

**7.** 17.5% of 479,362.

**8.** 250% of \$142.

**9.** 128% of 346 yd.

**10.** 5% of 83.6.

**11.** $7\frac{1}{2}$% of 42.736.

**12.** $\frac{1}{2}$% of 8476.

**13.** 32% of \$4.76.

**14.** 19% of .084.

**15.** 25% of 264 sq. mi.

**16.** $37\frac{1}{2}$% of 24 A.

**17.** $12\frac{1}{2}$% of \$488.

**18.** $87\frac{1}{2}$% of 648 mi.

**19.** $33\frac{1}{3}$% of $\frac{1}{8}$.

**20.** 20% of $\frac{7}{8}$.

**21.** $6\frac{2}{3}$% of 32 cu. ft.

**22.** 40% of 85 sq. rd.

**23.** $62\frac{1}{2}$% of 40 A.

**24.** $33\frac{1}{3}$% of \$36.

**25.** $66\frac{2}{3}$% of 120 ft.

**26.** $12\frac{1}{2}$% of 64 yd.

**27.** A school has an enrollment of 450 students. How many students are absent when 4% are absent?

**28.** What is \$200 increased by 4% of itself?

**29.** In 1910 the population of a certain city was 52,840. In 1920 the population was 15% larger than in 1910. What was the population in 1920?

**30.** A boy made a journey of 48 miles. He rode 85% of the distance in an automobile and walked the remaining distance. How far did he walk?

**31.** An architect's fees for designing a house are 4% of its cost. The cost is \$7360. What is the fee?

**32.** A man whose salary is \$3,200 spends 19 % of it for rent. How much does he spend for rent?

**33.** A clerk receives 2 % of the amount of his sales. If his sales for Monday were \$160.40, how much did he receive?

**34.** The report of a school with an attendance of 400 showed the per cent of tardiness to be $1\frac{1}{2}$. How many were tardy?

**35.** A man earns \$1,400 a year and saves 18 % of it. How long will it take him to save \$756?

**116. To find what per cent one number is of another; that is, to find the rate.**

**Method.** Since the percentage is the product of the base and the rate, and since the product may be divided by one factor to find the other factor, we have the following:

**percentage ÷ base = rate.**

**Problems.** 1. 6 is what per cent of 12?

SOLUTION. $\frac{6}{12} = \frac{1}{2} = .50$ or 50 %.

2. A farmer had 72 sheep and sold 18 of them. What per cent of his flock did he sell?

SOLUTION. $18 \div 72 = .25$ or 25 %.

3. George had one brother and three sisters. What per cent of the children of the family were boys?

SOLUTION. There were 5 children in the family.
$\frac{2}{5}$ of the children were boys.
$2 \div 5 = .4$ or 40 %.

**Oral Work**

1. What per cent of 8 is 4?
2. What per cent of 8 is 2?
3. What per cent of 24 is 3?
4. What per cent of 12 is 18?
5. What per cent of 9 is 3?
6. What per cent of 25 is 5?
7. What per cent of 25 is 20?
8. What per cent of 15 is 30?
9. What per cent of 10 is 8?
10. What per cent of 50 is 20?

**11.** What per cent of 60 is 40?

**12.** 15 is what per cent of 20?

**13.** 20 is what per cent of 40?

**14.** 18 is what per cent of 24?

**15.** 1 ft. is what per cent of 1 yd?

**16.** 20 bu. are what per cent of 60 bu.?

**17.** 4 mo. are what per cent of 1 yr.?

**18.** 3 da. are what per cent of 1 wk.?

**19.** If you sleep 8 hr. out of 24, what per cent of the time do you sleep?

**20.** If a man earns $24 a week and saves $4, what per cent of his earnings does he save?

**21.** An agent sold goods amounting to $1,800 and received $200 for his services. What per cent did he receive on the sales?

### Written Work

**1.** A baseball team played 60 games and won 42. What per cent of the games played did the team win?

**2.** In a school of 250 students 45 were seniors. What per cent of the students were seniors?

**3.** In a school auditorium there are 650 seats. Four hundred eighty of the seats are reserved for the students. What per cent of the seats are not reserved?

**4.** A building worth $14,500 is insured for $10,150. For what per cent of its value is it insured?

**5.** In a spelling lesson a boy spelled all but 2 of the 30 words correctly. What per cent of the words did he spell correctly?

**6.** A corporation has a capital stock of $164,000. Dunham owns $41,000 of the stock. What per cent of the stock does he own?

**7.** A business man invested $3,000. His income from the investment the first year was $420. What per cent did he earn?

**8.** In a certain school there are 224 boys and 312 girls. What per cent of the total enrollment is boys? girls?

**9.** Eighteen of the boys in this school failed to pass in at least one study. What per cent of the boys failed?

**10.** Twenty-three of the girls failed. What per cent of the girls failed?

**11.** What per cent of the entire school failed?

**12.** The total population of the United States in 1910 was 91,972,266. The number of foreign-born inhabitants was 13,345,545. What per cent of the total population was foreign born?

**13.** The largest number of foreign-born people were Germans. They numbered 2,501,181. What per cent of the entire population was of German birth?

**14.** What per cent of the foreign-born population was of German birth?

**15.** The forests of the United States originally contained about 5,200 billion board feet of timber. Cutting, clearing, and fire have reduced the stand to about 2,500 billion board feet. What per cent of the timber remains?

**16.** The following table is similar to one used by a clothing store to determine the tailoring firm which furnishes suits having the best sale. Record is kept of the number of suits purchased from each firm, and of the number of these suits sold.

Find what per cent of the suits purchased from each of the manufacturers was sold, and enter all statistics on a ruled form similar to the model.

Of what value is this table of statistics to the merchant?

PER CENT OF SALES FROM STOCK PURCHASED FROM CLOTHING MANUFACTURERS

| MANUFACTURER | NUMBER BOUGHT | NUMBER SOLD | PER CENT SOLD |
|---|---|---|---|
| TAYLOR, BRIGHT & CO | 368 | 293 | |
| WELLS & BAILEY | 493 | 316 | |
| F.H BAINUM | 639 | 582 | |
| DUDLEY & COLE | 523 | 416 | |
| WILLIAMS, ELDRIDGE & CO. | 863 | 719 | |
| M.J HARTMAN & SONS | 1392 | 1202 | |
| W.C DENNISTON & CO. | 1020 | 905 | |
| | | | |

**17.** The following table shows one of the ways in which merchants determine which clerk is selling the most goods.

Rule a form similar to the model; enter on the blank the facts which are given in the model; find the total sales made by all of the salesmen. Find what per cent of the total sales was made by each salesman. Enter all of these facts on the blank.

TABLE SHOWING PER CENT OF MONTH'S SALES MADE BY VARIOUS SALESMEN

| SALESMAN'S NAME | SALES | | PER CENT OF TOTAL |
|---|---|---|---|
| J B. DENNY | $ 1264 | 90 | |
| O.F. ALBRIGHT | 1913 | 52 | |
| C.F. SLATER | 1732 | 69 | |
| WILLIAM WARNER | 2213 | 72 | |
| C.B. WILEY | 1963 | 45 | |
| H. WALKER | 1627 | 32 | |
| FRANK McGEE | 1692 | 18 | |
| F.L. MONTGOMERY | 2138 | 45 | |
| C.H. HENRY | 1728 | 46 | |
| TOTAL | | | |

**18.** The following tabulation is a record of a week's sales in four departments of a store.

Rule a blank similar to the model.

Enter the statistics on the blank.

Find the total sales for each day made in all departments; enter these totals in the "total" column at the right.

Find the total sales made during the week in each department; enter these totals on the line at the bottom marked "total."

Find (in two ways) the total sales made in all of the departments for the entire week. Enter in the space marked Grand Total.

Find what per cent of the grand total sales was made each day; enter these per cents in the column at the right.

Find what per cent of the grand total sales was made in each department. Enter these per cents on the line at the bottom marked per cent.

One of the qualities which a business man desires in his employees is the ability to follow directions. This exercise will give you an opportunity to prove your ability along this line. Get a clear understanding of the instructions, and follow them by preparing the exercise without asking for any further directions.

A Week's Sales in a Department Store

| Day | Dept. I | Dept. II | Dept. III | Dept. IV | Total | Per Cent |
|---|---|---|---|---|---|---|
| Monday . . . . | $362 50 | $562 83 | $862 94 | $126 39 | $ | |
| Tuesday . . . . | 415 75 | 475 92 | 732 83 | 143 62 | | |
| Wednesday . . . | 396 21 | 415 60 | 769 42 | 129 38 | | |
| Thursday . . . | 472 96 | 516 29 | 640 20 | 145 17 | | |
| Friday . . . . . | 387 29 | 429 36 | 721 32 | 96 27 | | |
| Saturday . . . . | 493 89 | 562 64 | 816 25 | 163 92 | | |
| Total . . . . | | | | | Grand Total | |
| Per Cent . . . | | | | | | |

**117. To find a number when a certain per cent of it is known; that is, to find the base.**

**Method.** Since the *percentage* is the product of two factors, the base and the rate, and since any product can be divided by one of its factors to find the other factor, it is evident that,

**percentage ÷ rate = base.**

Therefore, *to find the base, divide the percentage by the rate expressed as a decimal.*

**Problems.** 1. 18 is 25 % of what number?

Solution. 18 is the percentage, 25 % is the rate; the base is not known.
18 ÷ .25 = the required number, or base.
18 ÷ .25 = 72.

2. 45 % of the number of students in a school are boys. If there are 135 boys, how many students are in the school?

Solution. 135 ÷ .45 = 300, the total number of students.

This may be explained as follows:

Since 45 % of the number of students = 135
1 % of the number of students = $\frac{1}{45}$ of 135 = 3
and 100 % of the number of students = 100 × 3 = 300.

### Oral Work

Find the number of which:

| | | |
|---|---|---|
| 1. 40 is 20 %. | 2. 12 is 4 %. | 3. 10 is 5 %. |
| 4. 86 is 2 %. | 5. \$45 is 15 %. | 6. 40 is 80 %. |
| 7. 12 is $\frac{1}{2}$ %. | 8. 42 is 100 %. | 9. 60 is 200 %. |
| 10. 12 is $33\frac{1}{3}$ %. | 11. 35 is 7 %. | 12. 19 is $\frac{1}{2}$ %. |
| 13. 8 is $\frac{2}{5}$ %. | 14. 2,700 is $66\frac{2}{3}$ %. | 15. 24 is $\frac{1}{3}$ %. |

16. A boy lost 40 cents, which was 10 % of what he had. How much did he have?

17. A man sold 32 acres of land, which was 80 % of all that he owned. How many acres did he own?

### Written Work

1. In a certain school 5 % of the pupils are absent and 475 pupils are present. What is the enrollment?

2. By saving 28 % of his salary a man saved \$2,240 in 5 years. What was his salary?

3. 18.75 % of a class failed. If 26 passed, how many were in the class?

4. The distance from New York to San Francisco via the Panama Canal is 5,278 miles. This distance is 60.44 % less than the distance via Cape Horn. What is the distance via Cape Horn?

5. A boy paid \$3 for a pair of shoes and had 70 % of his money left. How much money had he at first?

6. Four partners engage in business. The first invests \$4,000; the second invests \$3,000; each of the others invests 25 % of the total capital. What is the total capital?

7. The Washington Monument is 555 ft. and the Eiffel Tower is 984 ft. high. The height of the monument is what per cent of the height of the tower?

8. The average velocity of wind is 18 mi. per hour and that of sound is 1,090 ft. per second. The velocity of wind is what per cent of the velocity of sound?

9. The mean annual rainfall in Denver is 14 inches, and that in New York City is 44.8 inches. Each is what per cent of the other?

10. The length of the Hudson River is 280 miles; the length of the Ohio is 950 miles; and the length of the Mississippi is 3,160 miles. The length of each is what per cent of the length of the others?

2. The deposits of a certain bank on June 30, 1921, were $365,894.56. On June 30, 1922, the deposits had increased to $396,268.55. What was the increase and the per cent of increase?

3. A factory produced 240 articles of a certain kind per day. By installing new machinery, the output was increased 12½ %. How many articles were produced per day after the new machinery was installed?

4. Williams paid $16.87 more taxes this year than he did last year; an increase of 7.8 %. What was the amount of his taxes each year?

5. The payroll of the Maynard Manufacturing Company for the week ending April 4, 1921, was $428.62. The payroll for the week ending April 3, 1922, was 13 % larger. What was the pay roll for the week in 1922, and what was the increase?

6. A merchant built an addition to his store which increased the floor space 28 %. After the addition was completed, the floor space was 8,960 square feet. What was the number of square feet of floor space in the original store, and what was the amount of the addition?

### 119. Per Cent of Decrease.

To find the per cent of decrease, *divide the decrease by the base.*

**Problem.** Mr. Smith's coal bill in 1920 was $80.00. In 1921 his bill was $12.00 less. What was the per cent of decrease?

SOLUTION. Decrease ÷ base = per cent of decrease.
Thus, $12.00 ÷ $80.00 = 15 %.
The decrease was 15 %.

#### Written Work

1. The proprietor of a retail grocery employed five clerks at a total annual expense of $5,000.00 for wages. In order to reduce the expense, he released one clerk whose wages were $60.00 per month. How much did he decrease the cost of the clerk hire per year? What was the per cent of decrease? What was the annual cost of the clerk hire after one clerk was dismissed?

2. A paper mill which burned 2,400 tons of soft coal per year remodeled its furnaces and boilers and now burns 2,000 tons of coal per year. How many tons less does this mill consume per year than formerly, and what is the per cent of decrease?

3. Previous to 1921, Mr. Waterbury owned the only drug store in the town of X. When a new drug store was opened, the competition decreased Mr. Waterbury's annual sales $1,067.60, or 17 %. What was

the amount of Mr. Waterbury's sales the year before the new store opened?

4. In 1920 Mr. Snowden, a grocer, lost $327.00 from bad debts. A credit-rating association was formed in the town; the merchants informed each other of persons who did not pay their bills; and the following year Mr. Snowden decreased his loss from bad debts 18%. How much did the credit-rating association save Mr. Snowden, and what was his loss from bad debts in 1921?

5. Robbins moved his store into a new building where the rent was $5.00 per month less than the rent of the building which he formerly occupied. The rent in the new location was $125.00 per month. What was the per cent saved on rent?

6. Barnes said to his partner, " We cut down our delivery expenses 30% when we sold the horses and bought an auto truck. Our annual cost for making deliveries after we made the change was only $1,645.00. When we delivered with horses, the cost per year was $ ———, so we are saving $ ———."

7. The following table shows one of the uses which business men make of per cent of increase and decrease.

COMPARATIVE SALES SHEET

| DEPARTMENT NUMBER | SALES, FRIDAY, JUNE 1, 1921 | SALES, FRIDAY, JUNE 2, 1922 | INCREASE OR DECREASE | PER CENT INCREASE OR DECREASE |
|---|---|---|---|---|
| 1 | $ 826 95 | $ 914 32 | $ | |
| 2 | 1,034 78 | 1,231 64 | | |
| 3 | 1,237 62 | 1,196 14 | | |
| 4 | 2,643 80 | 2,843 27 | | |
| 5 | 1,413 80 | 1,376 29 | | |
| 6 | 962 40 | 1,235 96 | | |
| 7 | 2,642 16 | 2,927 92 | | |
| 8 | 1,964 39 | 2,129 80 | | |
| 9 | 1,636 48 | 1,596 27 | | |
| 10 | 1,213 42 | 1,723 96 | | |
| 11 | 1,394 29 | 1,146 92 | | |
| 12 | 415 75 | 496 25 | | |
| 13 | 3,460 00 | 3,246 29 | | |
| 14 | 2,690 70 | 2,889 00 | | |
| 15 | 878 25 | 794 60 | | |
| Total | | | | |

This table compares the sales on the first Friday in June, 1921, with the sales on the first Friday in June, 1922. The merchant assumes that trade conditions were about the same on the two days. A table of this kind is prepared, comparing the business for the day with the business of the corresponding day of the previous year.

Prepare a table similar to the model, and enter the facts given. Show increases and per cents of increase in black ink; decreases and per cents of decrease in red ink.

What was the increase in business in department 1? What was the per cent of increase?

Complete the table.

**8.** The following table shows the value of irrigating arid land. For example, the government records show that, in the regions where rainfall is insufficient, the potato crop on an acre of irrigated land was recently worth $15.37 more than the crop grown on an acre of unirrigated land. This is an increase of 34.4% due to irrigation. What is the average value per acre of the potato crops grown on unirrigated land and on irrigated land?

Complete the table.

COMPARATIVE VALUE OF CROPS GROWN ON IRRIGATED AND UNIRRIGATED LAND

| CROP | EXCESS OF AVERAGE VALUE OF CROP PER ACRE GROWN ON IRRIGATED LAND, OVER THAT GROWN ON UNIRRIGATED LAND | | | AVERAGE VALUE OF CROP PER ACRE ON UNIRRIGATED LAND | | AVERAGE VALUE OF CROP PER ACRE ON IRRIGATED LAND | |
|---|---|---|---|---|---|---|---|
| Potatoes . . . . | $15 | 37 | 34.4% | | | | |
| Sugar Beets . . | 5 | 39 | 10.4 | | | | |
| Wheat . . . . . | 8 | 65 | 58.6 | | | | |
| Alfalfa . . . . . | 5 | 97 | 35.2 | | | | |
| Oats . . . . . | 7 | 36 | 63.2 | | | | |
| Barley . . . . . | 6 | 51 | 55.1 | | | | |
| Corn . . . . . | 3 | 51 | 24.0 | | | | |
| Timothy and Clover, mixed . . . | 3 | 63 | 27.6 | | | | |
| Timothy alone. . | 3 | 08 | 24.1 | | | | |

**120. Per Cent of Maximum.** It is sometimes desired to compare a certain number with a larger number. For example, a certain packing plant is equipped to slaughter and pack a maximum of 2,000 hogs daily. On October 22, this plant packed 1,832 hogs.

The day's pack was $\frac{1832}{2000}$ of the maximum capacity, or 91.6%. State a rule for finding per cent of maximum.

## MISCELLANEOUS PROBLEMS — PERCENTAGE

### Written Work

**1.** Compute the per cent of increase in the population of each of the following cities for each of the ten year periods.

| Cities | 1900 | 1910 | 1920 | Cities | 1900 | 1910 | 1920 |
|---|---|---|---|---|---|---|---|
| New York .. | 3,437,202 | 4,766,883 | 5,620,048 | Pittsburg .... | 462,801 | 553,905 | 588,343 |
| Chicago .... | 1,698,575 | 2,185,283 | 2,701,705 | Los Angeles.. | 104,266 | 319,198 | 576,673 |
| Philadelphia. | 1,293,697 | 1,549,008 | 1,823,779 | San Francisco. | 342,782 | 416,912 | 506,676 |
| Detroit ..... | 290,277 | 465,766 | 993,678 | Buffalo..... | 352,387 | 423,715 | 506,775 |
| Cleveland .. | 384,111 | 560,663 | 796,841 | Milwaukee ... | 285,315 | 373,857 | 457,147 |
| St. Louis.... | 575,238 | 687,029 | 772,897 | Washington .. | 278,718 | 331,069 | 437,571 |
| Boston ..... | 560,892 | 570,585 | 748,060 | Newark ..... | 248,849 | 347,469 | 414,524 |
| Baltimore .. | 508,957 | 558,485 | 733,826 | Cincinnati ... | 331,866 | 363,591 | 401,247 |

**2.** The total value of a man's property is $28,400. Of this amount 45% is invested in city real estate, 12½% in government bonds, and the remainder in a farm. Compute the amount invested in each.

**3.** The following table gives the number of farms in the United States and in several of the states in 1920 and the percentage of increase since 1910. Compute the number of farms in each in 1910.

Number of Farms in 1920, and Percentage of Increase Since 1910

| | Number | Per Cent |
|---|---|---|
| United States | 98,496 | 1.5 |
| Arizona | 1,589 | 17.2 |
| California | 29,493 | 33.4 |
| Colorado | 13,821 | 29.9 |
| Idaho | 11,302 | 36.7 |
| Montana | 31,227 | 119.1 |
| Nevada | 475 | 17.7 |
| Oregon | 4,686 | 10.3 |
| Utah | 3,988 | 18.4 |
| Washington | 10,096 | 18.0 |
| Wyoming | 4,624 | 42.1 |

**121. Per Cent of Average.** Percentage is also used to show the relation between different numbers by comparing them with their average.

### Written Work

The following table shows the monthly sales of a number of clerks:

| Clerk No. | Sales | Per Cent | Clerk No. | Sales | Per Cent |
|---|---|---|---|---|---|
| 201 | $1,412.32 | | 205 | $1,836.24 | |
| 202 | 1,218.16 | | 206 | 1,216.26 | |
| 203 | 967.32 | | 207 | 1,375.85 | |
| 204 | 1,046.89 | | 208 | 1,493.85 | |

Find the average sales.

The sales of Clerk No. 201 are what per cent of the average?

Complete the table.

What is the value of such a table for the manager of a business?

### Miscellaneous Exercises

**1.** If a merchant sells all goods at an advance of 10% of the cost, what will be the selling price of an article which cost $3.40? What will be the profit on this article? If this merchant's annual sales are $24,126.35, what is his gross profit?

He marked all goods at 25% increase on the cost the following year and his sales dropped to $15,625.90. What was the cost of the goods sold, and the gross profit?

**2.** In the following table is recorded the number of games won and lost by the teams in a certain school. What per cent of the games played did each team win?

| Team | Games won | Games lost |
|---|---|---|
| Invincibles . . . | 18 | 7 |
| Tigers . . . . . | 15 | 10 |
| Midgets . . . . | 9 | 16 |
| Lions . . . . . | 12 | 13 |
| Giants . . . . . | 7 | 18 |

**3.** Compute the per cent of increase in population from 1910 to 1920 for each of the largest five cities in your state.

**4.** Six salesmen reported the annual sales and expenses given in the table below. Compute the per cent which each man's expenses were of his sales. Compute also the average sales and average expenses.

| | Total Sales | Total Expenses |
|---|---|---|
| George Robbins. . . | $18,428.50 | $2,148.65 |
| Arthur Smith . . . | 16,302.90 | 1,943.90 |
| Robert Adams . . . | 21,408.50 | 2,290.50 |
| Martin Evans . . . | 11,390.00 | 2,380.40 |
| Carl Moore . . . . | 17,365.40 | 2,406.80 |
| Joseph Carter . . . | 9,438.80 | 1,841.50 |

**5.** When a man's daily wages are increased from $4.80 to $5.60, what is the per cent of increase? When his daily wages are reduced from $5.60 to $4.80, what is the per cent of decrease?

**6.** The purity of gold is measured in carats, or twenty-fourths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in a ring that is 12 carats fine? What is the per cent of pure gold in a watch chain that is 18 carats fine?

**7.** What was the per cent of advance in price when sugar increased from 8 cents to 27 cents a pound?

**8.** What is the difference in pounds between $\frac{3}{4}$ of a ton and $\frac{3}{4}$% of a ton?

**9.** A certain kind of cloth loses $1\frac{7}{8}$% of its length when it is sponged. What will be the length of a piece of this cloth after it is sponged if the length before sponging is 8 yards?

**10.** An ice dealer finds that on the average the ice which is placed in his ice houses loses 15% of its weight before it is sold. How much ice should he store in order to have 38,250 tons to sell?

**11.** A certain poultry dealer finds that chickens lose 27% in weight when dressed. He has an order from a hotel for 219 pounds of dressed chickens. How many pounds of live chickens will be required to fill the order?

**12.** A dairyman decides to test two cows. One cow gives 32 pounds of milk a day, which tests 3.4% of butter fat; another gives 30.5 pounds, testing 3.7% of butter fat. Butter fat being worth 38 cents a pound, which cow yields the larger daily return and how much larger?

**13.** A poultry man finds by experience that on the average 84% of the eggs which he places in an incubator hatch out chicks. If he wishes to obtain about 700 chicks, how many eggs must be placed in the incubators?

**14.** The following table gives a summary of the government appropriations for the fiscal year 1920. Compute the per cent of the total appropriated for each.

RECAPITULATION OF APPROPRIATIONS FOR THE FISCAL YEAR 1920

| | |
|---|---|
| Legislative | $ 18,899,687.03 |
| Executive Proper | 221,080.00 |
| Department of State | 12,313,743.16 |
| Department of War | 861,465,699.64 |
| Department of the Treasury | 1,423,921,594.35 |
| Department of the Navy | 637,544,113.54 |
| Department of the Interior | 268,398,697.13 |
| Department of Justice | 17,229,282.73 |
| Post Office Department | 471,750,370.26 |
| Department of Agriculture | 143,187,541.50 |
| Department of Commerce | 34,532,720.03 |
| Department of Labor | 8,278,062.03 |
| Independent Establishments | 1,969,664,423.98 |
| District of Columbia | 17,721,049.94 |
| Miscellaneous Objects | 1,134,260,047.67 |
| Total | |

**15.** It is estimated that for every $10 spent by the average wage earner's family in a certain city, $3.78 is spent for food; $1.60 for clothing; $1.49 for rent; $.60 for light and fuel; $.44 for furnishings; and $2.09 for miscellaneous items. Compute the per cent of the total spent for each purpose.

**16.** In the following table is recorded the total imports and exports of the United States for certain years. Compute the per cent of increase or decrease for each year, using the year 1914 as a basis of comparison.

FOREIGN COMMERCE OF UNITED STATES

| YEAR ENDED | EXPORTS | IMPORTS |
|---|---|---|
| June 30 | | |
| 1880 | $ 835,638,658 | $ 667,954,746 |
| 1890 | 857,828,684 | 789,310,409 |
| 1900 | 1,394,483,082 | 849,941,184 |
| 1907 | 1,880,851,078 | 1,434,421,425 |
| 1912 | 2,204,322,409 | 1,653,264,934 |
| 1914 | 2,364,579,148 | 1,893,925,657 |
| 1916 | 4,333,482,885 | 2,197,883,510 |
| 1919 | 7,920,425,990 | 3,904,364,932 |
| 1920 | 8,228,016,307 | 5,278,481,490 |

**122. Percentage Analysis of a Business.** Mr. E. C. Barton is the proprietor of a wholesale store. At the end of each month he analyzes the records of his business. The following analysis was made June 30, 1922. Results should be approximate to the nearest hundredth of a per cent.

1. The purchases during June, 1921, were \$8,146.90. Some of the goods purchased were defective, and were returned. The value of goods returned was \$123.60. The returned goods were what per cent of the purchases?

2. Gross sales for June, 1921, were \$11,216.29. Goods returned to the store by dissatisfied customers, \$97.65. The returned sales were what per cent of the gross sales?

3. Gross sales for June, 1921, were \$9,862.15. Returned sales, \$101.80. What per cent of the sales were returned in June, 1921?

4. What was the increase in the gross sales, and what was the per cent of increase?

5. Gross Sales – Returned Sales = Net Sales.

What were the net sales for June, 1921? For June, 1922? What was the increase in net sales, and the per cent of increase?

6. The merchandise on hand June 1, 1922, was worth \$14,162.45 at selling prices. The net sales for the month were found in problem 5. What per cent of the stock on hand at the beginning of the month was sold during the month?

7. Net Sales – Cost of Goods Sold = Gross Profit.

The gross profit for June, 1921, was \$1,688.54. The gross profit was what per cent of the net sales for the month? (For net sales see results to problem 5.)

8. The gross profit for June, 1922, was \$2,001.36. The gross profit was what per cent of the net sales for that month?

9. The expenses of the business in June, 1921, were \$1,015.60. The expenses had increased in June, 1922, to \$1,140.26. What was the per cent of increase in the expenses?

10. Net Sales – Gross Profit = Cost of Goods Sold.

What was the cost of goods sold in June, 1921?

What was the cost of goods sold in June, 1922?

11. Gross Profit – Expenses = Net Profit.

What was the net profit for June, 1921?

What was the net profit for June, 1922?

**12.** What was the per cent of increase or decrease in the net profit?

**13.** Net Profit ÷ Net Sales = Per Cent of Net Profit on Sales.

What was the per cent of net profit for June, 1921?

What was the per cent of net profit for June, 1922?

# TRADING ACTIVITIES. PROFIT AND LOSS

## CHAPTER XIII

### BUYING AND SELLING MERCHANDISE

**123. The Invoice.** When a merchant sells goods, he usually gives the customer a bill or invoice. The invoice states the following facts:

The date of the sale.

The name and address of the seller.

The name and address of the purchaser.

The terms of the sale.

A detailed list of the articles sold, including the price of each item and the total of the invoice.

If prepaid freight is to be charged to the purchaser, the amount of freight is added to the invoice.

If the invoice has been paid, a statement of the receipt of payment is made by the seller.

Study the following invoice and state as many as possible of the facts enumerated above.

TELEPHONE MAIN, 1340
PRIVATE EXCHANGE

A. B. HUGHES

DEALER IN

STAPLE AND FANCY GROCERIES

211-213-215 Lake Street,

Chicago, Illinois

**TERMS:** 2% Cash in 10 days, Net 30 days.

June 10, 1922

**SOLD TO** Scobey & Company, Fayette, Iowa

| | | | | | | |
|---|---|---|---|---|---|---|
| | 2 Cases Tomato Catsup 4 doz. | 1.90 | 7 | 60 | | |
| | 6 Cases Elgin Canned Corn 12 doz. | 1.80 | 21 | 60 | | |
| | 5 Cases Echo Peas 10 doz. | 1.85 | 18 | 50 | 47 | 70 |

**124. What the Purchaser Does.** After the purchaser receives both the goods and the invoice, he inspects the merchandise to see whether it agrees with the items for which he has been charged on the invoice. If stock of the correct kind and quantity has been received, a check mark is entered on the invoice opposite each item in the column at the left.

The invoice is next checked to ascertain:

*a.* Whether the correct prices have been charged.

*b.* Whether an error has been made in the extension of the cost of any item, or in the total of the invoice.

**125. Credit Memorandum.** In case an error is found, notice is sent to the firm from which the goods were purchased. When credit is to be given by the seller on account of an error in the invoice, or for imperfect goods, or for any other cause, the purchaser will usually receive a credit memorandum, notifying him of the amount of credit entered to his account.

CREDIT MEMORANDUM

A. B. HUGHES
CHICAGO.

June 17, 1922

TO Williams & Co.,
Albany, New York

Your account has been credited $1.25
On account of overcharge on invoice of June 10, 19

**Written Work**

When an invoice has been checked and found to be correct, the extensions and footings are checked as shown in the illustration.

| | | |
|---|---|---|
| $16 | 50✓ | |
| 23 | 00✓ | |
| 4 | 80✓ | $44 30✓ |

Check the invoice on page 145. If the multiplications are found to be correct, enter a check mark at the right of each item. If the total is correct, check it also.

### Written Work

Find the totals of each of the following sales:

Apply short methods when possible.

**1.**

| | | | Price |
|---|---|---|---|
| 4 doz. | Men's Irish Linen Handkerchiefs | doz. | $6.25 |
| 5 doz. | Men's Hemstitched Handkerchiefs | doz. | 7.35 |
| 14 doz. | Men's White Initial Handkerchiefs | doz. | 4.87 |
| 9 doz. | Women's Imported Swiss Embroidered Handkerchiefs | doz. | 3.75 |
| 22 doz. | Children's Linen Handkerchiefs | doz. | 1.66⅔ |
| 7 doz. | Women's Swiss Lawn Handkerchiefs | doz. | .95 |
| 13 doz. | Children's Mull Handkerchiefs | doz. | 1.05 |

**2.**

| | | | |
|---|---|---|---|
| 17 doz. | Men's Leather Gauntlets | doz. | $11.85 |
| 11 doz. | Men's Buckskin Gloves, sizes 8 to 10 | doz. | 12.50 |
| 16 pairs | Men's Leather Automobile Mittens | pair | 2.37½ |
| 4½ doz. | Men's Suede Gloves, sizes 7¾ to 10 | doz. | 7.50 |
| 3½ doz. | Men's No. J 264 Kid Gloves | doz. | 17.50 |
| 5½ doz. | Imported Kasan Cape Gloves | doz. | 19.75 |

**3.**

| | | | |
|---|---|---|---|
| 16 | Silk Taffeta Umbrellas | each | $ 3.65 |
| 28 | 8 Rib Pure Silk Taffeta Umbrellas | each | 6.15 |
| 32 | Men's Suit Case Umbrellas | each | 4.05 |
| 12 | Extra Size Umbrellas | each | 1.12½ |

**4.**

| | | | |
|---|---|---|---|
| 3 doz. pairs | Walton Cotton Blankets | pair | $ 1.15 |
| 1 doz. pairs | Cotton Crib Blankets | pair | .92½ |
| 8 doz. pairs | Wool Filled Gray Blankets | pair | 5.15 |
| 7 doz. pairs | Douglass Wool Finish Plaids | pair | 2.67½ |
| 4 doz. pairs | All Wool Plaids | pair | 9.00 |
| 10 doz. pairs | "Canada Camp" Blankets | pair | 1.87½ |

**5.**

| | | | |
|---|---|---|---|
| 20 | Assorted Pattern Smyrna Rugs, 36 × 72 | each | $ 5.75 |
| 80 | Scotch Jute Rugs | each | .95 |
| 3 doz. | Cotton Bathroom Rugs, 27 × 54 | each | 1.87½ |

**6.**

| | | | Price |
|---|---|---|---|
| 3 doz. Size 6 | Children's Fast Black Hose | doz. | $ 1.67½ |
| 2 doz. Size 6½ | Children's Fast Black Hose | doz. | 1.75 |
| 4 doz. Size 7 | Children's Fast Black Hose | doz. | 1.83½ |
| 5 doz. Size 7½ | Children's Fast Black Hose | doz. | 1.57½ |
| 6 doz. Size 8 | Children's Fast Black Hose | doz. | 1.62½ |
| 8 doz. Size 8½ | Children's Fast Black Hose | doz. | 1.77½ |
| 7 doz. Size 9 | Children's Fast Black Hose | doz. | 1.83½ |
| 6 doz. Size 9½ | Children's Fast Black Hose | doz. | 2.07½ |
| 5 doz. Size 10 | Children's Fast Black Hose | doz. | 2.12½ |

**7.**

| | | | |
|---|---|---|---|
| 20 No. B 645 | Silk Woven Waist Patterns (each pattern 3½ yd.) | yd. | $ 1.50 |
| 25 No. B 493 | Dresden Silk Mull Waist Patterns (3½ yd. pieces) | yd. | 1.17½ |
| 18 bolts | Irish Linette, 10, $11^2$, 12, $10^3$, $11^1$, $11^3$, $12^1$, $11^3$, $10^3$, 12, 10, $10^3$, $10^1$, $11^2$, 12, $11^3$, $10^2$, $10^3$ | yd. | .92½ |
| 20 bolts | French Gauze Chiffon, 14, $13^3$, $12^3$, $14^1$, 14, $13^2$, 13, $12^3$, 13, $13^1$, $14^2$, $13^3$, $14^2$, $14^1$, $13^3$, $12^3$, 14, $13^3$, $13^2$, $12^3$ | yd. | 1.45½ |

NOTE. The small figures mean quarter yards. Thus $11^3$ means 11¾ yards.

### EXPLANATION OF GROCERY ORDERS

The sign # if placed before figures means "number"; if placed after figures it means pounds.

Articles sold in cases [cs.] are usually priced by the dozen. The total number of dozens in the cases is therefore given after the name of the commodity, and the price is the price per dozen.

Some articles sold in barrels and cases are priced by the pound; in such instances the total number of pounds in all the barrels or cases appears immediately before the price.

Sugar is sold by the hundred pounds (cwt.). The net weight of each barrel is given. Find the total number of pounds; point off two places to the left to find the number of hundred pounds, and multiply by the price per hundred.

**8.**

| | | Price |
|---|---|---|
| 7 cases | Acme Peas | 14 doz. $1.85 |
| 3 boxes | Peona Soap | 6.05 |
| 7 bbl. | Northern Salt | 2.80 |
| 15 bbl. | Winter Wheat Flour ⅛ sacks | 8.85 |

| | | | |
|---|---|---|---|
| 12 bbl. | H. & E. Granulated Sugar 329, 335, 347, 351, 344, 347, 350, 331, 342, 355, 349, 333 | | 8.20 |
| 5 sacks | H. & E. Granulated Sugar | 500# | 8.15 |
| 9 cases | Acme Peas | 18 doz. | 1.85 |
| 17 cases | Acme Corn | 34 doz. | 2.25 |
| 6 boxes | Dried Apples | 150# | .12¾ |
| 2 bbl. | 10# sacks Northern Salt | | 2.65 |
| 6 cases | Algonac Tomatoes | 12 doz. | 1.87½ |

9.

| | | | |
|---|---|---|---|
| 1 cs. | 25# Macaroni | | $2.80 |
| 5# | Cream of Tartar | | .60 |
| 1 cs. | Acme Peas | 2 doz. | 1.85 |
| 5 cases | Hawthorne Pears | 10 doz. | 3.45 |
| 11 cases | Hawthorne Peaches | 22 doz. | 3.20 |
| 12 cans | Pimento | | .23 |
| 15 cs. | H. L. Shredded Pineapple | | 3.15 |
| 10 cases | Tall Salmon | 40 doz. | 2.85 |
| 12 cases | F. & B. Tomatoes | 24 doz. | 3.95 |
| 85 sacks | Yellow Corn Meal | 850# | .05 |
| 8 cases | 25# Macaroni | | 2.80 |
| 5 cases | 15# Spaghetti | | 2.80 |
| 7½ doz. | pkgs. Seeded Raisins | | 1.95 |
| 125# | Rice | | .11 |
| 25 quarts | Olive Oil | | 1.35 gal. |
| 18¾ gal. | Sweet Pickles | | 1.29¼ |
| 6½ gal. | Olives $\frac{70}{80}$ | | 1.45 |
| 25 cases | 4 oz. Grd. Pepper | 125# | .40 |
| 6 cans | Paprika | | 2.65 doz. |
| 14 cases | G. M. Soap | | 3.35 |
| 12 cases | Soda Crackers | 93¾# | .13 |
| 12 | Cracker cans | | .50 |
| 1 tub | Peanut Butter | 48½# | .17½ |
| 62½ bbl. | ⅛ sacks Spring Wheat Flour | | 8.40 |
| 49¾ bbl. | ⅛ sacks Winter Wheat Flour | | 8.85 |
| 16 cases | #2 Algonac Lima Beans | 32 doz. | 1.85 |
| 14 cases | #2 Algonac String Beans | 28 doz. | 1.75 |

**10.**

| | | Price | |
|---|---|---|---|
| 10 doz. | W/W 6 in. Plates | $1.37½ | doz. |
| 13½ doz. | W/W 4 in. Plates | 1.12½ | " |
| 5 doz. | /S/ 2½ in. Butter Chips Tk. | .33 | " |
| 8¼ doz. | #13 Cups | 1.25 | " |
| 6 doz. | W/W 4 in. Ice Creams | .67½ | " |
| 15 doz. | #64 /G/ Nappies | 1.67½ | " |
| 8 doz. | A.C. ½ Tk. 30's L/F Bowls | 1.92 | " |
| ¾ doz. | #211 Tea Pots | 3.65 | " |
| 2½ doz. | #0 /S/ Custard Pots | 1.75 | " |
| 17 doz. | #1,352 Tumblers | .98 | " |
| 12 doz. | #1,296 Finger Bowls | 2.33⅓ | " |
| 1¾ doz. | #3,234 Oil Bottles | 3.50 | " |
| 3½ doz. | 12–199 Salts | 2.25 | " |
| 3½ doz. | 12–200 Peppers | 2.25 | " |
| 2⅙ doz. | 444 A Pitchers | 13.00 | " |
| ⅓ doz. | Punch Glasses | 1.25 | each |
| 12½ doz. | 3,404 Dessert Forks | 4.25 | doz. |
| 12½ doz. | 3,405 Dessert Knives | 4.20 | " |
| 16 doz. | 3,413 Tea Spoons | 2.95 | " |
| 8 doz. | 3,406 Dessert Spoons | 3.40 | " |

Rule invoices and enter the following sales, assuming that you are the selling merchant.

**11.** To W. K. Sears,
Elgin, Illinois.
Sept. 10.

| | | |
|---|---|---|
| 45 | No. R 721 Body Brussels Rugs 6 × 9 ft. each | $31.75 |
| 32 | No. R 731 Body Brussels Rugs 8 × 10 ft. each | 37.50 |
| 36 | No. R 741 Body Brussels Rugs 9 × 12 ft. each | 41.25 |
| 25 | No. R 751 Body Brussels Rugs 11 × 13 ft. each | 48.80 |

**12.** To C. B. Perkins,
Amherst, Minn.
Sept. 12.

| | | | | |
|---|---|---|---|---|
| 20 | | H 20 Family Coffee Mills | each | $ 1.95 |
| 16 | sets | H 33 Sad Irons | set | 1.23 |
| 12 | doz. | H 545 Flour Sifters | doz. | 2.17⅞ |

| | | |
|---|---|---|
| 25 doz. H 463 Carpet Beaters | doz. | 1.11 |
| 8 doz. H 6,646 Galvanized Clothes Lines, 20 gauge wire, | doz. | 1.29⅜ |
| 10 doz. H 6,546 Galvanized Clothes Lines, 18 gauge wire, | doz. | 1.88½ |

13. To Owen & Hendersen,
Cedar Rapids, Iowa.
Sept. 13.

| | | |
|---|---|---|
| 35 F 384 Plush Library Chairs | each | $26.30 |
| 28 F 376 Reed Rockers | each | 6.80 |
| 18 F 394 Bedroom Rockers | each | 8.15 |
| 26 F 465 Roman Chairs | each | 13.75 |
| 19 F 414 Morris Chairs | each | 27.75 |

# CHAPTER XIV

## COMMERCIAL DISCOUNTS

**Classes of Commercial Discount.** Commercial discounts are of three general classes; namely, Cash Discount, Trade Discount, and Quantity Discount.

### Cash Discount

**126. Purpose of Cash Discount.** In order to encourage the prompt settlement of accounts, merchants frequently offer to deduct a certain per cent of the bill if it is paid within a fixed number of days. The deduction for prompt payment is called **cash discount.**

**127. Terms.** The terms state the discounts offered, and the time when the bill is due and payable without any deduction for discount. The terms are usually indicated in the following manner:

2/10; 1/30; N/60.

The figure at the left of the line indicates the rate of discount offered; the figure at the right of the line indicates the number of days within which payment must be made in order to obtain the discount. Thus, the terms stated above mean:

2% discount from the face of the bill, if paid within 10 days;
1% discount from the face of the bill, if paid after 10 days but before 30 days;

No discount after thirty days and the bill is due in 60 days.
The terms are read as follows: "Two, ten; one, thirty; net, sixty."

**128. Definitions.** The amount of the purchase before subtracting the discount is called the **list price.**

The remainder after subtracting the discount is called the **net price.**
The **rate of discount** is stated as a per cent of the list price.

**129. Cash Discount Illustrated.** The terms of the invoice on page 145 were 2/10; N/30.

In order to obtain the discount, Scobey & Co. must pay the bill on or before June 20, 1922.

The bill is due and payment is expected **in full** on July 10, 1922.

No discount is allowed if payment is made between June 20 and July 10.

**130. Computing the Discount and the Net Price.**

*First Method;*

| | |
|---|---|
| $ 95.25 | List price |
| .02 | Rate of discount |
| $1.9050 | Discount |

| | |
|---|---|
| $ 95.25 | List price |
| 1.91 | Discount |
| $ 93.34 | Net price |

State a rule for finding discount and net price by this method.

(Note that five mills or more in the discount are considered a cent. Thus, $1.905 is considered $1.91.)

*Second Method;*

| | |
|---|---|
| 100 % | List price |
| 2 % | Rate of discount |
| 98 % | Per cent of bill to be paid |

| | |
|---|---|
| $95.25 | List price |
| .98 | |
| $93.345 | Net price |
| or $93.34 | |

State a rule for computing the net price by this method.

**131. The Advantages of Cash Discount.** Cash discounts may benefit both the purchaser and the seller. Merchants offer cash discounts because they encourage prompt payments, and thus decrease:

*a.* the loss from bad debts;
*b.* the cost of collecting accounts;
*c.* the amount of capital tied up in outstanding accounts.

It is usually good business for the purchaser to "take his discount" (pay the bill before it is due), even though the rate of discount may be small. An illustration will show this fact.

Suppose the terms of the sale are, Cash less 1%; net 30 days. If the purchaser pays cash, he receives 1% for the use of his money for one month. 1% a month is 12% a year; a high rate of interest.

The rates of cash discount are usually small, varying from 1% to 3%.

Small discounts are offered if the bill is due in a short time.

For example, 1/10; N/30.

Larger discounts are offered if the bill is due after a greater length of time.

For example, 3/30; N/4 months.

As a further means of insuring payment of invoices, merchants frequently charge interest on bills which are not paid when due.

### Written Work

Find the list price, the discount, and the net price of each of the following purchases, and answer the following questions about each invoice:

*a.* What is the last day on which payment can be made and the discount secured?

*b.* When is the invoice due?

**1.** Invoice dated April 4. Terms 2/10; N/30.

| | | |
|---|---|---|
| 7 cases Acme Peas | 14 doz. | $1.85 |
| 3 cases Osea Soap | | 5.45 |
| 7 bbl. Northern Salt | | 2.80 |
| 15 bbl. $\frac{1}{8}$ sacks A. D. Flour | | 8.85 |

**2.** Invoice dated October 26. Terms 1/15; N/30.

| | |
|---|---|
| 12 bbl. H. E. Granulated Sugar, 329, 335, 347, 351, 344, 347, 350, 331, 342, 355, 349, 333# | $8.20 |
| 5 sacks H. E. Granulated Sugar, 500# | 8.15 |

**3.** Invoice dated January 5. Terms 2/10; 1/20; N/30.

| | | |
|---|---|---|
| 16 bolts White Dress Linen, 10, 12, $11^1$, $12^2$, $10^3$, $11^3$, $10^3$, 12, $10^1$, $11^2$, 12, $11^1$, $10^2$, $11^3$, $12^1$, $10^2$ | yd. | $0.97\frac{1}{2}$ |
| 12 bolts Persian Lawn, $24^3$, $25^1$, $23^2$, 25, $24^2$, $24^3$, $25^2$, $24^1$, $23^3$, $24^3$, $25^2$, 25 | yd. | $.45\frac{3}{4}$ |
| 48 bolts French Nainsook, 12-yard pieces | yd. | $.49\frac{1}{2}$ |

| | | |
|---|---|---|
| 36 bolts Mercerized Lingerie Batiste, 24-yard pieces . | yd. | .43 |
| 24 bolts Imported Linen Lawn, 10-yard pieces . . . . | yd. | .96¾ |

NOTE. There will be two possible net prices for this invoice.

4. Rule a form similar to the following table.

Combine the dollars in the column at the left, with the cents in the row near the top, to form the list price. Deduct the discount at the rate shown above the cents, and enter the net price in the table. The two results entered in the table show the method. Thus,

| | | | |
|---|---|---|---|
| *a.* | $26.10 | the list price | |
| | .02 | the discount | |
| | $ 0.52 | the discount at 2 % | |
| | $26.10 | | |
| | .52 | | |
| | $25.58 | Net price | |

| | | | |
|---|---|---|---|
| *b.* | $641.67 | | |
| | .025 | | |
| | $ 16.04 | the discount at 2½ % | |
| | $641.67 | | |
| | 16.04 | | |
| | $625.63 | Net price | |

| | Less 2% | | Less 1½% | | Less 3% | | Less 2½% | |
|---|---|---|---|---|---|---|---|---|
| | | 10 | | 45 | | 75 | | 67 |
| $26 00 | $25 | 58 | | | | | | |
| 14 00 | | | | | | | | |
| 73 00 | | | | | | | | |
| 19 00 | | | | | | | | |
| 261 00 | | | | | | | | |
| 112 00 | | | | | | | | |
| 317 00 | | | | | | | | |
| 216 00 | | | | | | | | |
| 641 00 | | | | | | | $625 | 63 |

## TRADE DISCOUNT

**132. Purpose of Trade Discount.** In some lines of business merchants sell both at wholesale and retail. They advertise their goods at a certain price, but when they sell to dealers, they frequently deduct a part of this price. The amount deducted is called a **Trade Discount.**

Trade discounts are common in businesses which issue catalogues. When the catalogue is sent to a dealer, a discount sheet is inclosed.

## Specimen Discount Sheet

DISCOUNT SHEET

The following discounts are offered on articles listed in catalogue No. A 23.

| | |
|---|---|
| Pages 1 to 15 . . . . . | 20% |
| Pages 16 to 39 . . . . . | 28% |
| Pages 39 to 67 . . . . . | 40% |
| Pages 68 to 90 . . . . . | Net |
| Pages 91 to 136 . . . . . | 14⅔% |

Do not show this discount sheet to your customers.

NOTE. "Pages 68 to 90 Net" means, no trade discount is offered for goods listed on these pages.

Not Responsible for Goods Lost or Damaged in Transit. Claims for Allowance must be made upon Receipt of Goods. Address all Communications to Abercrombie & Co., Chicago

**ABERCROMBIE & COMPANY**

PUBLISHERS & BOOKSELLERS

245 WABASH AVENUE

CHICAGO, March 3, 1922

SOLD TO W. M. Rickert Co.,
Home,
Iowa

YOUR ORDER NO. 121
TERMS: 2/10: N/30
CONVEYANCE, U. S. Ex.

IN REFERRING TO THIS ORDER MENTION NO. 4336 ENTERED W. B. 3/3/22.

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | Copies H. & J. Second Year English | .85 | 10 | 20 | | |
| 21 | " Beeman's Algebra | .90 | 18 | 90 | | |
| | | | 29 | 10 | | |
| | Less 18% | | 5 | 24 | 3 | 86 |

### Class Discussion

1. Why do merchants offer trade discounts?

2. What is the catalogue price of the H. & J. Second Year English per copy?

3. How much does each copy of the English book actually cost the retail dealer? Ignore cash discount.

4. If Mr. Rickert sells this book for the catalogue price, 85 cents, how much profit does he make on each book?

5. Trade discounts are usually larger than cash discounts. Why do you think this is the case?

**133. Computing Trade Discounts.** Trade discounts are computed in the same manner as cash discounts.

**Problem.** What will goods cost a dealer if they are sold to him for \$46, less a trade discount of 25%?

SOLUTION.

| | |
|---|---|
| \$46.00 List price | \$46.00 List Price |
| .25 Rate of discount | 11.50 Trade discount |
| \$11.50 Trade discount | \$34.50 Price to dealer |

### Written Work

Rule and complete the following table. Enter the price to dealer after deducting the trade discount.

| Rate of Trade Discount | Catalogue Price | | | | | |
|---|---|---|---|---|---|---|
| | \$27 | 85 | \$126 | 39 | \$96 | 45 |
| $14\frac{2}{7}$% | | | | | | |
| 18% | | | | | | |
| 20% | | | | | | |
| 22% | | | | | | |
| 25% | | | | | | |
| 15% | | | | | | |
| 17% | | | | | | |
| 28% | | | | | | |
| $16\frac{2}{3}$% | | | | | | |
| 10% | | | | | | |

**134. Determining the Rate of Trade Discount.** In many lines of business the market price of the goods is continually fluctuating. This may be due to the changes in the prices of the raw material used in manufacturing, or to other causes. If new catalogues were published every time the prices changed, there would be an enormous expense in printing and mailing. Moreover, the time required for printing might be so great that new changes in prices would make the catalogue out of date before it reached the customers. To avoid this expense and delay the simple device is adopted of sending customers a new discount sheet.

For example, suppose an article made from steel (the market price of which varies at frequent intervals) is quoted in the catalogue at \$8.

When the market price of steel drops and the article can be sold for \$7, a discount of $12\frac{1}{2}\%$ can be offered.

If raw steel should drop in price so that the article could be sold for \$6, a discount of 25% could be offered.

**Problem.** What rate of discount will change a catalogue price of \$3 to a market price of \$2.50?

SOLUTION.

| | |
|---|---|
| \$3.00 | Catalogue price |
| 2.50 | Price at which article is to be sold |
| \$0.50 | Amount to be deducted as discount |

\$0.50 ÷ \$3.00 = $16\frac{2}{3}\%$, rate of discount to be offered

State a rule for finding the rate of discount by this method.

When the quotient obtained by the division involves a fractional per cent, the next lower whole number is sometimes taken as the per cent. Thus, a quotient of $15\frac{1}{7}\%$ may be regarded as 15%; a discount of 18.27% may be regarded as 18%.

**Problem.** An article is listed at \$24.50. The market conditions are such that it should sell for about \$21.30. What rate of discount should be offered?

SOLUTION.

| | |
|---|---|
| \$24.50 | List price |
| 21.30 | Market price |
| \$ 3.20 | Amount to be deducted by discount |

\$3.20 ÷ \$24.50 = 13.06%.

13% would therefore be offered.

This discount would make the net price \$ . ?

### Written Work

Rule and complete the following table, showing what rates of discount would be offered to reduce the list prices at the left to the market prices shown at the top of the table.

Express results to the nearest per cent which will not reduce the market prices below those stated in the table.

| List Prices | Market Prices | | | | | |
|---|---|---|---|---|---|---|
| | $8.50 | $7.75 | $8.00 | $7.60 | $6.00 | |
| $10.00 | | | | | | |
| 12.00 | | | | | | |
| 11.50 | | | | | | |
| 9.80 | | | | | | |
| 10.50 | | | | | | |

## Quantity Discounts

**135. Purpose of Quantity Discounts.** In some lines of business, particularly in manufacturing, it is customary to give a larger rate of discount on a large order than on a small one.

A mail order firm advertises:

1% discount on an invoice of $10; 2% on $20; 5% on $35.

What is the purpose of such quantity discounts?

A printer advertised the following rates:

$10 per thousand for the first thousand copies. On an order of more than 1000 copies, a discount of 15%. This discount is offered because the work of typesetting, making up the forms, and making ready the press must all be done, even if but one copy is to be printed. The cost of printing 2,000 copies, therefore, is not twice as much as the cost of printing 1,000 copies.

### Written Work

1. The following prices were quoted by a manufacturer of lockers: No. 6,094. Each $6.30.

Discounts:

Orders of from 50 to 100 — Less 8%.
Orders of from 101 to 300 — Less 10%.
Orders of from 301 to 500 — Less 12½%.

Find the total cost and the cost per locker of an order of 75 lockers; 140; 475.

**2.** A paper manufacturer quoted the following prices:

No. 020 note paper, cut to size 8 by 10, $.55 per ream.
An extra charge of 3 cents per ream will be made for wrapping in packages of 500 sheets. (500 sheets considered 1 ream.)
On orders of 100 reams or more a discount of 4% will be allowed.
Terms, 1/10; N/30.

Find the cost and state what discounts were allowed:

*a.* An order of 50 reams, unwrapped, payment made 20 days from date of sale.
*b.* An order of 150 reams, unwrapped, payment made 15 days from date of sale.
*c.* An order of 150 reams, wrapped, payment made 18 days from date of sale.
*d.* An order of 240 reams, wrapped, payment made 7 days from date of sale.
*e.* An order of 60 reams, wrapped, payment made 8 days from date of sale.

## Discount Series

**136. Definition.** When two or more discounts are allowed on an invoice, these discounts are known as a series.

The first discount applies to the list price; the second applies to the remainder after deducting the first discount; the third applies to the remainder after deducting the first two discounts; et cetera.

**137. Trade and Cash Discount.** The most common form of series consists of two discounts; the first a trade discount and the second a cash discount.

**Problem.** What is the smallest amount of money that will pay a bill of $236, subject to a trade discount of 20% and a cash discount of 2%?

SOLUTION.

| | | | |
|---|---|---|---|
| $236.00 | List price | $188.80 | Trade price |
| .20 | Rate of trade discount | .02 | Rate of cash discount |
| $ 47.20 | Trade discount | $3.7760 | Cash discount |
| $236.00 | List price | $188.80 | Trade price |
| 47.20 | Trade discount | 3.78 | Cash discount |
| $188.80 | Price to the trade | $185.02 | Net price |

The cash discount is computed on the trade price, *not* on the list price. Therefore, 20 % trade discount and 2 % cash discount are not the same as a single discount of 22 %.

### Written Work

Rule and complete the following table. From each list price deduct the trade discount indicated above the list price; and from the trade price thus found, deduct the cash discount at the rate shown at the left.

| CASH DISCOUNT | FACTS TO BE FOUND | TRADE DISCOUNT, 28 % | TRADE DISCOUNT, 25 % | TRADE DISCOUNT, 33⅓ % |
|---|---|---|---|---|
| 2 % | List Price<br>Trade Discount<br>Trade Price<br>Cash Discount<br>Net Price | $36 45 | $68 40 | $239 64 |
| 3 % | List Price<br>Trade Discount<br>Trade Price<br>Cash Discount<br>Net Price | 205 08 | 333 95 | 47 30 |
| 5 % | List Price<br>Trade Discount<br>Trade Price<br>Cash Discount<br>Net Price | 35 00 | 713 20 | 609 49 |

**138. Series of Trade Discounts.** In some lines of business it is customary to offer a series of trade discounts rather than a single rate.

**Problem.** What is the trade price of an invoice of $175.00 subject to discounts of 25%, 20%, and 10%?

SOLUTION.

| | | | |
|---|---|---|---|
| $175.00 | List price | $175.00 | List price |
| .25 | First discount rate | 43.75 | First discount |
| 43.75 | First discount | 131.25 | After first discount |
| $131.25 | After first discount | $131.25 | After first discount |
| .20 | Second discount rate | 26.25 | Second discount |
| 26.25 | Second discount | 105.00 | After second discount |
| $105.00 | After second discount | $105.00 | After second discount |
| .10 | Third discount rate | 10.50 | Third discount |
| 10.50 | Third discount | 94.50 | After third discount |

### Written Work

Find the net cost of the following bills:

1. $45.00 less 20%, 15%, and 10%.
2. $28.00 less 25%, 10%, and 5%.
3. $70.00 less 30%, 20%, and 2%.
4. $105.50 less 20%, 12½%, and 8%.
5. $39.80 less 15%, 10%, and 4%.
6. $2,806.75 less 28%, 14%, and 2%.
7. $467.70 less 25%, 20%, and 9%.
8. $369.05 less 40%, 20%, and 10%.
9. $294.46 less 33⅓%, 20%, and 12½%.
10. $723.34 less 37½%, 10%, and 20%.

**139. Computing a Single Rate Equivalent to a Series.** To compute a single rate equivalent to a series:

Subtract each rate from 100%;

Find the product of these remainders;

Subtract this product from 100%.

**Problem.** What single discount rate is equivalent to a trade discount of 20% and a cash discount of 2%?

SOLUTION.

100% − 20% = 80%
100% − 2% = 98%
80% × 98% = 78.4%
100% − 78.4% = 21.6% The single equivalent rate

**Problem.** What single rate is equivalent to 25 %, 20 %, and 10 %?

SOLUTION.

100 % − 25 % = 75 %
100 % − 20 % = 80 %
100 % − 10 % = 90 %
75 % × 80 % × 90 % = 54 %
100 % − 54 % = 46 % The single equivalent rate

### Written Work

The buyers in some large business houses prepare an elaborate table similar to the one which follows. When manufacturers quote prices subject to a discount series, the buyers can tell by a glance at the table what single discount the series equals.

Rule and complete the following table, using the method just explained to find single discounts equivalent to a discount series.

| TRADE DISCOUNT | CASH DISCOUNT, 1% | CASH DISCOUNT, 1½% | CASH DISCOUNT, 2% | CASH DISCOUNT, 2½% | CASH DISCOUNT, 3% |
|---|---|---|---|---|---|
| 18 % | | | | | |
| 15 | | | | | |
| 20 | | | | | |
| 25 | | | | | |
| 33⅓ | | | | | |

With the aid of the table, find the net price of the following:

*a.* An invoice of $215, less 18 % and 2 %.
*b.* An invoice of $464.20 less 18 % and 3 %.
*c.* An invoice of $12.40 less 20 % and 1½ %.
*d.* An invoice of $23.87 less 33⅓ % and 3 %.

Referring to the previous page; determine what single rate is equivalent to each series and compute the net amount of each bill using the single rates.

### Written Review

1. An implement dealer purchased the following invoice:

Dated April 17.

4 No. 364 Plows, $38.75, less 20 %.
7 Self Dump Hay Rakes, $20.50, less 18 %.
7 No. 264 Hay Stackers, $41.50, less 15 %.

Terms, 2% for Cash in 20 days. Net 90 days.
On May 2, he paid the bill with a check for $——.

2. Which of the following prices is better for the purchaser:

$45, less 25%, 18%, and 2%;
$60, less 28%, 25%, and 1%?

3. A merchant's discounts were 25% and 15%. A clerk sold an invoice of $72 and gave a single discount of 40%. How much did his error cost his employer?

4. A merchant lists a desk at $45 less 20%. A competitor sells a similar desk for $48 less 33⅓%. In order to meet his competitor's price, the first merchant decides to give an additional discount of —%.

5. Hewett paid an invoice in time to secure a discount of 3%. If the check sent was for $208.55, what was the list price of the invoice?

6. Graff Brothers sent a check to a wholesale house to pay an invoice. The check was for $801.90. What was the list price of the invoice if the discounts taken were 10% and 1%?

7. Compute the net price of an invoice of $100 subject to discounts as follows:

*a.* 40%
*b.* 25% 10%, and 5%
*c.* 25% 5%, and 10%
*d.* 10%, 25%, and 5%
*e.* 10%, 5%, and 25%
*f.* 5%, 10%, and 25%

What conclusions can you draw from this problem?

# CHAPTER XV

## RECORDING PURCHASES AND SALES

### Marking Goods

**140. The Purchases Book.** Merchants usually keep a record of purchases and sales. There are several different kinds of books used for this purpose. The following illustration shows a common form of the Purchases Book.

**PURCHASES BOOK**

| Date of Invoice | From Whom Purchased | Amount | Terms | Discount term Expires | Cash Discount | Due Date | When & How Paid |
|---|---|---|---|---|---|---|---|
| 1922 March 3 | Abercrombie & Co. | 23 86 | 2/10. N/30 | Mar 13 | 45 | Apr 2 | Mar 12 By Check $23.38 |
| | | | | | | | |
| | | | | | | | |

The entries in the purchases book are made from the invoice received at the time of the purchase. The model shows proper record of the purchase made from Abercrombie & Co., as shown by the invoice on page 156.

Notice that $23.86 is the amount of the invoice after deducting the trade discount of 18%. The cash discount is not deducted until the bill is paid.

The terms are taken from the invoice.

The discount term expires March 13. It is important to have this date recorded in the purchases book, as it is the last date on which payment may be made and the discount secured.

April 2 is the day when the bill is due and payment is expected. No discount is allowed when payment is made between March 13 and April 2.

**141. How to find the Date when an Invoice is Due.** If the terms are stated in days, count the actual number of days. Thus, 30 days from March 3 is April 2.

If the terms are stated in months, calendar months are counted. Thus, if the terms had been 2/10; N/1 month, the invoice would have been due on April 3.

No entry is made in the "When and How Paid" column until the invoice is paid.

It is not necessary to enter the items in the purchases book, because the invoice can be kept on file to supply this information.

**142. The Sales Book.** The following illustration shows a page from a commonly used form of sales book.

The entries were made from the invoice on page 156.

March 3, 1922.

| | | | | | |
|---|---|---|---|---|---|
| W. M. Rickert Co. | Home, Iowa | | | | |
| Terms, 2/10, N/30 | | | | | |
| 12 Copies Hr J. Second Year English | 85 | 10 | 20 | | |
| 21 " Beeman's Algebra | 70 | 14 | 70 | | |
| | | 24 | 90 | | |
| Less 18% | | 5 | 24 | 23 | 86 |

**143. Loose-leaf Sales Book.** Many merchants record their sales in a loose-leaf sales book. At the time the invoice is made, a carbon copy is also made. This requires very little extra labor, and the carbon copies, called "Charge Sheets," having holes punched at the side, can be bound together in a binder.

The loose leaf sales book has several advantages. Both the invoice and the charge sheet, which forms the sales book, can be made on the typewriter at the same time; fraud is prevented because the invoices are numbered, and a clerk cannot sell goods and make an invoice without also making a charge slip. He cannot keep the money and destroy the charge slip because one of the numbered sheets would be missing, and the fraud would be apparent.

The following illustration shows the charge sheet made as a carbon copy of the invoice shown on page 145.

TERMS. June 10, 1922

CHARGE

2% Cash in 10 days, Net 30 days.

Scobey & Company, Fayette, Iowa

| | | | | | | |
|---|---|---|---|---|---|---|
| | 2 Cases Tomato Catsup 4 doz. | 1.90 | 7 | 60 | | |
| | 6 Cases Elgin Canned Corn 12 doz | 1.80 | 21 | 60 | | |
| | 5 Cases Echo Peas 10 doz. | 1.85 | 18 | 50 | 47 | 70 |

## Written Review

Rule a Purchases Book and record the following purchases, making proper extensions.

1. From Eaton & Dunham, 213 Main Street, Indianapolis, Indiana.

Date, November 21, 1922.
Terms, 1/15; N/60.

| | | | |
|---|---|---|---|
| 6 doz. | Silk Four-in-hand Ties | each | $ .37½ |
| 3 doz. | White Lawn Ties | gross | 2.94 |
| 38 doz. | Assorted Style Amoryth Linen Collars | doz. | 1.92 |
| 18 doz. | Policeman's Suspenders | doz. | 4.37½ |
| 9 doz. | Khaki Overalls | doz. | 8.97 |
| 6 doz. | Boys' Flannelette Waists (ages 4 to 13 yr.) | doz. | 2.16¼ |
| 10 doz. | Men's Flannel Work Shirts (sizes 14 to 18) | doz. | 8.76½ |
| 2¼ doz. | Men's Negligee Shirts | doz. | 16.40 |

2. From J. B. Clark, Aurora, Illinois.

Date, October 27, 1922.
Terms 2/20; N/3 months.

| | | | |
|---|---|---|---|
| 7 doz. | Assorted Sizes Men's Silk Half Hose | doz. | $4.29 |
| 19½ doz. | Mercerized Lisle Hose | doz. | 4.78 |
| 3 doz. | Men's Black Overgaiters | doz. | 6.93 |
| 18 | Women's Taffeta Silk Waists (sizes 34 to 42) | each | 2.17 |
| 24 | Navy Blue Silk Chiffon Waists | each | 2.98 |
| 5 doz. | Embroidered Lace Coat Sets | doz. | 6.93¾ |
| 2 doz. | Amoskeag Gingham Aprons | doz. | 4.69½ |
| 2¼ doz. | Lace Vests | doz. | 4.25 |

**3.** From Bishop & McGee, Independence, Iowa.

Date, October 20, 1922.
Terms, 1/5; N/20.

| | | | |
|---|---|---|---|
| 10 doz. | Defiance Food Choppers | each | $2.85 |
| 9 doz. | Climax Food Choppers | each | .72 |
| 14 doz. | No. 7 size Skillets | each | .22½ |
| 30 sets | Enamel-lined Iron Kettles, each set containing | | |
| | 3 2 quarts at .21 | | |
| | 4 4 quarts at .29¼ | | |
| | 6 6 quarts at .34⅔ | | |
| | 4 8 quarts at .39¼ | | |
| | 2 10 quarts at .46⅔ | | |
| 2¾ doz. | H 921 Waffle Irons | doz. | $7.84⅜ |
| 6½ doz. | H 922 Waffle Irons | each | .93 |
| 25 | H 464 Soapstone Griddles (round) | each | .55 |
| 12 doz. | H 465 Soapstone Griddles (oval) | each | .65⅜ |

**4.** From A. D. McHaughton, Fairchild, Missouri.

Date, July 14, 1922.
Terms, 3/30; N/90.

| | | | |
|---|---|---|---|
| 5 doz. | H 731 Wire Waste Baskets | doz. | $. 89 |
| 15 doz. | H 732 Wire Letter Baskets | doz. | 1.23 |
| 28 | H 881 Spring Seats | each | .88 |
| 15 sets | H 393 Cobblers' Outfits | set | .96 |
| 19 doz. | H 242 Curry Combs | doz | .67 |
| 14 | F 272 Upholstered Rockers | each | 8.75 |
| 25 | F 279 Oak Rockers | each | 12.65 |
| 20 | F 212 Turkish Rockers | each | 12.75 |

Rule a Sales Book, and record the following sales, making proper extension.

**5.** To J. D. Preston, Monmouth, Illinois.

July 7, 1922.
Terms, 1/10; N/40.

| | | | | |
|---|---|---|---|---|
| 47 | No. L 601 Royal Worsted Wilton Rugs | 8 × 10 ft. | each | $29.25 |
| 52 | No. L 602 Royal Worsted Wilton Rugs | 6 × 7 ft. | each | 19.75 |
| 38 | No. L 603 Royal Worsted Wilton Rugs | 9 × 12 ft. | each | 32.35 |
| 65 | No. L 661 Worsted French Wiltons | 9 × 12 ft. | each | 44.60 |
| 50 | No. L 662 Plain Color Wiltons | 8 × 10 ft. | each | 31.75 |

6. To R. J. Noble, Dubuque, Iowa.

August 8, 1922.
Terms, 1/15; N/60.

| | | | |
|---|---|---|---|
| 25 doz. | Half-bleached Cotton Towels | 24 × 54 doz. | $ .98 |
| 16 doz. | Linen Monogram Towels | 22 × 39 doz. | 4.95 |
| 20 doz. | Bleached Turkish Bath Towels | 23 × 52 doz. | 2.37½ |
| 14 doz. | No. T 291 Cotton Face Cloths | 10 × 13 doz. | .42½ |

7. To Oscar Hamilton, Reynolds, North Dakota.

July 9, 1922.
Terms, 2/5; N/2 months.

| | | | |
|---|---|---|---|
| 15 doz. | Palmetto Fiber Scrub Brushes | doz. | $ .86 |
| 22 doz. | H 221 Kitchen Spoons, 10 inch | doz. | .23 |
| 19 doz. | H 241 Kitchen Spoons, 12 inch | doz. | .27 |
| 26 doz. | H 251 Kitchen Spoons, 14 inch | doz. | .33 |
| 38 doz. | H 333 Kitchen Forks, 13 inch | doz. | .19 |
| 36 doz. | H 334 Kitchen Forks, 15 inch | gross | 3.20 |
| 12 doz. | H 341 Perforated Steel Spoons | doz. | .84⅞ |
| 8 doz. | H 691 Kitchen Sets | doz. | 8.75 |
| 16 doz. | H 692 Kitchen Sets | doz. | 3.95 |

## Marking Goods

**144. Method of Marking Goods.** When stock is placed on the shelves in the salesroom, the cost of each article should be marked either on the goods, on the package which contains them, on tags attached to the goods, or on card lists placed near the goods.

**145. Advantage of Marking.** When the stock becomes low and the buyer wishes to purchase a new supply, he can compare the price paid for the goods on the shelves with quotations of prices made to him by the salesmen from the wholesale houses. It would also be a convenience to know the cost of an article if it proved to be a slow seller, and the manager determined to sell at a reduced price to unload the stock. One of the chief advantages of cost marking is in taking inventory of stock and finding its value at cost prices. In marking goods, the cost price is taken from the invoice.

**146. "Blind Price Lists."** It would be unwise to mark the cost in figures, as this would disclose the cost and the profit to purchasers.

It is, therefore, customary for merchants to adopt a set of symbols, called a cipher, or blind price list. Any symbols may be chosen, but they will be more easily written if letters are used, and these letters will be more easily remembered if they form a word or phrase. The word or phrase selected must not contain the same letter twice. Otherwise the same letter will represent two different figures.

The following will illustrate:

| | |
|---|---|
| a d m o n i s h e r | mah |
| 1 2 3 4 5 6 7 8 9 0 | 318 |
| F i t z a u b r e y | izy |
| 1 2 3 4 5 6 7 8 9 0 | 240 |

**147. The Repeater.** To further conceal the cost a "repeater" should be adopted. When the same figure is repeated, as in $1.55, the repeater sign is used for the repeated figure. "x" is frequently used as a repeater, but because it is so commonly used, some other repeater would perhaps be preferable. Words or phrases with eleven letters are frequently chosen as keys, one of the letters being used as a repeater.

Using "t" as the repeater, and "admonisher" as the key,

$3.88 would be written *mht.*

### Written Work

Using "blacksmith" as the key word, and "d" as the repeater indicate the following costs:

| | | | |
|---|---|---|---|
| **1.** $9.82. | **2.** $ .09. | **3.** $ .25. | **4.** $ 1.64. |
| **5.** $1.00. | **6.** $6.20. | **7.** $ .27. | **8.** $ 7.47. |
| **9.** $2.45. | **10.** $ .55. | **11.** $1.33. | **12.** $ .39. |
| **13.** $8.23. | **14.** $5.00. | **15.** $1.17. | **16.** $12.47. |
| **17.** $6.62. | **18.** $2.24. | **19.** $ .36. | **20.** $ 1.44. |

Many articles are bought by the dozen and sold by the piece. In marking goods bought in this way it is necessary to divide the cost by 12. This division will be facilitated if the decimal equivalents of the 12ths, from $\frac{1}{12}$ to $\frac{11}{12}$ inclusive, are memorized.

Make such a table of equivalents and memorize it. Thus,

$\frac{1}{12} = .08\frac{1}{3}$; $\frac{7}{12} = .58\frac{1}{3}$.

**Problem.** What is the cost of one hat at the rate of \$29 a dozen?

SOLUTION. $\frac{1}{12}$ of \$29 = \$$2\frac{5}{12}$ = \$2.41$\frac{2}{3}$.

**Oral Work**

What is the cost per article when the cost per dozen is:

| | | | | |
|---|---|---|---|---|
| **1.** \$27? | **2.** \$34? | **3.** \$64? | **4.** \$11? | **5.** \$3.40? |
| **6.** \$14.30? | **7.** \$7.50? | **8.** \$19.50? | **9.** \$46.60? | **10.** \$37.40? |

**148. Showing Cost and Selling Price.** When both the cost price and the selling price are shown, it is customary to write the cost price above and the selling price below a line, thus:

Cost of an article, \$6.25.
Selling price, \$8.00.
Key word, Fitzaubrey.
Price mark, $\frac{\text{uia}}{8.00}$.

Key words or phrases are sometimes used to mark the selling price as well as the cost, thus:

Cost key, " p u r c h a s e d i t "
1 2 3 4 5 6 7 8 9 0 "t" is the repeater.

Selling key, " s t u d y i n g h e r "
1 2 3 4 5 6 7 8 9 0 "r" is the repeater.

Cost, \$3.14.
Selling price, \$4.25.

Mark, $\frac{\text{rpc}}{\text{dty}}$.

**Written Work**

Using the above keys, show markings for the following:

| | COST | SELLING PRICE |
|---|---|---|
| **1.** | \$1.15 | \$1.50 |
| **2.** | \$ .65 | \$ .90 |
| **3.** | \$1.30 | \$1.75 |
| **4.** | \$2.18 | \$2.50 |
| **5.** | \$1.25 | \$2.60 |

What per cent of profit would be realized from selling goods marked as follows, using " purchased it " as the cost key and " studying her " as the selling key?

6. $\frac{\text{phi}}{1.75}$. 7. $\frac{\text{ah}}{\text{ge}}$. 8. $\frac{\text{dh}}{\text{sty}}$. 9. $\frac{\text{at}}{\text{ny}}$.

Mark the cost of each of the following articles, using " Now Pay Quick " as the purchase key.

10. $3.00 per dozen.
11. $185.00 per hundred.
12. $11.52 per gross.
13. $8.64 per case of 4 doz.

Mark both the cost and the selling price of the following. Devise a cost key of your own, and mark the selling price by using the key " importance."

14. Cost $2.60, marked to gain 22%.
15. Cost $1.75, marked to gain 18%.
16. Cost $25.00, marked to gain 20%.

The following is a list of key words and phrases:

| | | | |
|---|---|---|---|
| Buy for Cash | Our Last Key | Frank Smith | You Mark His |
| The Big Four | Republican | Big Factory | Now Be Sharp |
| Black Horse | Charleston | Don't Be Lazy | He Saw It Run |
| Buckingham | Cumberland | Market Sign | Hard Moneys |
| Authorizes | Dozen Black | Now Be Quick | What Prices |

**149. Adding the Buying Expenses to the Cost of the Goods.** The cost of the goods when they are finally placed on the merchant's shelves includes not only the wholesale price but also the freight and drayage, import duties paid, and any other expenses of purchasing and holding the goods for sale, such as rent of storage warehouses. These are known as buying expenses. In the statements which the merchant prepares to determine his profits these buying expenses should be added to the cost of the goods. It is a matter of choice whether the buying expenses shall be included in the cost marked on the goods.

If it is desired to include the buying expenses in the marked costs, the simplest method is to estimate the per cent of total buying expenses to total purchases. When records of total purchases and total buying

expenses have been kept for several years, the average per cent of buying expenses may be computed as shown in the following illustration.

To find an estimated rate of buying expenses to add to the cost of purchases in 1922 based on the figures of 1919, 1920, and 1921:

| | | | | |
|---|---|---|---|---|
| 1919, | Buying Expenses, | $ 785.75 | Purchases, | $16,240.00 |
| 1920, | Buying Expenses, | 835.50 | Purchases, | 19,360.00 |
| 1921, | Buying Expenses, | 923.90 | Purchases, | 21,365.00 |
| Total | Buying Expenses, | $2,545.15 | Purchases, | $56,965.00 |

$2,545.15 ÷ $56,965.00 = 4.46+ %, the per cent of buying expenses

In marking costs, 4.5 % of the cost of each article should be added as buying expenses.

NOTE. Probably 5 % might be used for convenience.

### Written Work

**1.** Compute the per cent of buying expenses to be added to the wholesale cost of goods purchased in 1922. Carry the result to the nearest whole per cent.

| YEAR | PURCHASES | BUYING EXPENSES |
|---|---|---|
| 1919 | $10,246.80 | $534.50 |
| 1920 | 12,726.95 | 592.90 |
| 1921 | 14,825.75 | 615.45 |

**2.** Find the marked cost of each of the following articles after adding the buying expenses. Use the per cent of buying expenses found in Problem 1.

*a.* Wholesale cost, $16.20.
*b.* Wholesale cost, 15.75.
*c.* Wholesale cost, 2.35.

**3.** At what price should each of these articles be sold to gain 23 % on the total cost including buying expenses?

**4.** The purchases made by a store in 1921 were $3,287.20; the buying expenses for the same year were $327.90. On the basis of these figures, what per cent should be added for buying expenses on purchases made in 1922? Approximate result to the nearest per cent.

**5.** What objection do you see to determining a per cent of buying expenses from the data of only one preceding year?

**6.** Find the marked cost of each of the following purchases, after adding the per cent of buying expenses. Use the per cent of buying expenses found in Problem 4.

*a.* Wholesale cost, $ 2.45.
*b.* Wholesale cost, 1.60.
*c.* Wholesale cost, 34.50.

**7.** At what selling price should each of the articles in Problem 6 be marked to gain $16\frac{2}{3}\%$ of the total cost?

# CHAPTER XVI

## PAYING FOR GOODS

**150. Making Change.** The payment of debts between persons in the same community is usually made with either cash or checks. When cash is the medium of payment, it is often necessary to " make change."

Speed and accuracy in making change are desirable. The following method is generally used by experienced tellers and cashiers. Beginning with the amount of the purchase, take from the cash drawer enough coins to bring the total to even dollars, using as few coins as possible, then take out dollars or larger denominations until the total cost of the purchase and change returned equals the payment made.

**Problem.** A five-dollar bill was given in payment for a purchase of \$.39. How should change be made?

SOLUTION. Take 1 penny,
1 dime,
1 fifty-cent piece,
4 dollars.

As a check on the accuracy of the change, say as you give the customer the money: "39 cents, 40, 50, \$1, \$5.00."

### Oral Work

Following the method described above, state what coins and bills should be given to make change for the following purchases, using the largest denominations possible:

| | PURCHASE | PAYMENT | | PURCHASE | PAYMENT |
|---|---|---|---|---|---|
| 1. | \$ .07 | \$ .50 | 2. | \$ .21 | \$ .50 |
| 3. | \$ .56 | \$ 1.00 | 4. | \$ .63 | \$ 2.00 |
| 5. | \$1.36 | \$ 5.00 | 6. | \$2.79 | \$ 5.00 |
| 7. | \$3.66 | \$ 5.00 | 8. | \$4.24 | \$10.00 |
| 9. | \$5.70 | \$10.00 | 10. | \$6.32 | \$20.00 |
| 11. | \$ .98 | \$ 5.00 | 12. | \$2.77 | \$10.00 |

| | Purchase | Payment | | Purchase | Payment |
|---|---|---|---|---|---|
| 13. | $ 3.46 | $20.00 | 14. | $11.87 | $15.00 |
| 15. | $14.42 | $20.00 | 16. | $ 7.37 | $ 8.00 |
| 17. | $11.13 | $20.00 | 18. | $ 1.13 | $20.00 |

**151. Payments by Check.** The payment of all bills by the actual transfer of money would be so inconvenient that the giving of checks has been substituted to a great extent and it is said that about 90 % of all bills are now paid by checks.

**152. Deposits and Withdrawals.** Business men keep the greater part of their cash funds on deposit with banks or trust companies. When money is deposited, a deposit slip similar to the illustration below is filled out by the depositor, showing his name, the date, the items, and the amount of the deposit.

DEPOSITED BY

C. L. Stevenson

IN THE

**STATE BANK OF OAK PARK**

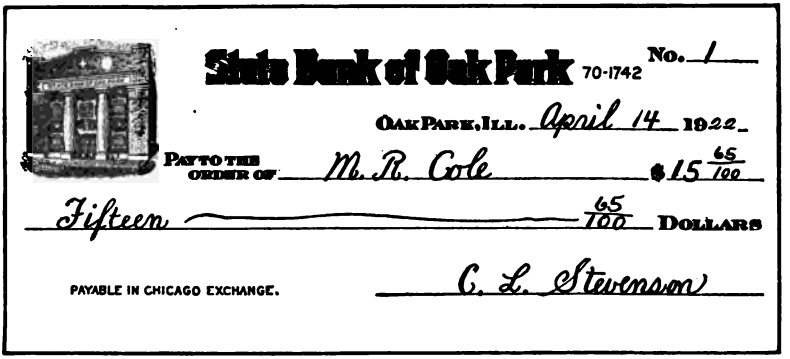
OAK PARK, ILL.

April 10 1922

| | DOLLARS | CENTS |
|---|---|---|
| GOLD | 40 | 00 |
| SILVER | 7 | 35 |
| BILLS | 14 | 00 |
| *Total* | 61 | 35 |

A Deposit Slip

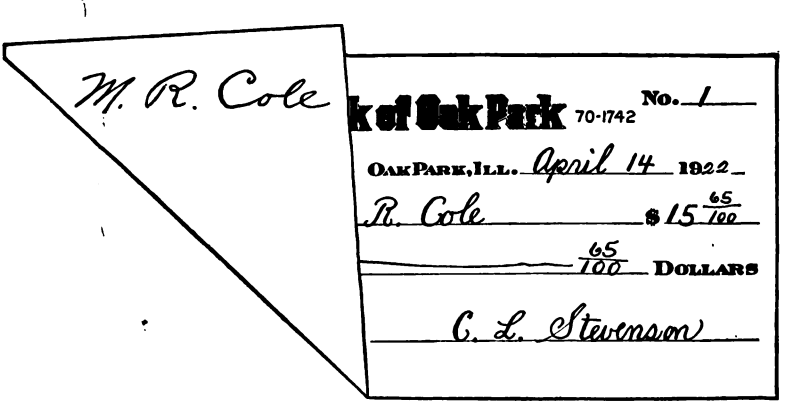
The depositor, wishing to pay a bill, draws a check ordering the bank to pay from the funds on deposit the sum of money stated on the check to the person named thereon. If the depositor wishes to draw cash from his account, he may make the check payable to " Self " or to " Cash."


State Bank of Oak Park 70-1742 No. 1
Oak Park, Ill. April 14 1922
Pay to the order of M. R. Cole $15 65/100
Fifteen 65/100 Dollars
Payable in Chicago Exchange.
C. L. Stevenson

A Check

C. L. Stevenson, the *drawer* of this check, is paying M. R. Cole, the *payee*, $15.65.

Depositors are credited by the bank with their deposits, and are charged with the checks drawn by them. Checks received from other people may be cashed at the bank, or they may be deposited the same as cash. All checks cashed or deposited must be indorsed; that is, they must be signed on the *back* by the person cashing or depositing them. Checks must also be indorsed if they are transferred to another person. Indorsements should be made on the back of the left end of the check.




A Check Indorsed

M. R. Cole's indorsement before cashing the check

## Indorsements

**153. Blank Indorsement.** This form of indorsement is illustrated on the back of the check on page 177. It makes the check payable to bearer.

**154. Indorsement in Full.** This form is illustrated below.

Pay to the order of
J. B. Banning
M. R. Cole

Checks indorsed in full must be indorsed by the new owner before being cashed.

**New Netherland Bank**
OF NEW YORK
DEPOSITED BY
V. E. Finch
NEW YORK. May 6, 1922

PLEASE LIST EACH CHECK SEPARATELY

| | DOLLARS | CENTS |
|---|---|---|
| BILLS | 16 | 00 |
| GOLD | 25 | 00 |
| SILVER | 12 | 45 |
| CHECKS Buffalo | 4 | 35 |
| " Newark NJ | 14 | 23 |
| " Boston | 213 | 25 |
| " | | |
| " | | |
| " | | |
| " | | |
| " | | |
| " | | |
| " | | |
| " | | |
| Total | 285 | 28 |

Deposit Slip Showing Cash and Check Deposit

If a check, note, or draft is not paid by the party primarily liable each indorser in turn becomes responsible for its payment. An indorser may limit his responsibility by writing the words "without recourse" above his signature. When this is done, the indorsement is said to be *qualified*.

### Written Work

**1.** You are opening an account by depositing to-day in some bank of your city, gold, $5; silver, $2.25; bills, $7; and the three following checks: 1st, drawn by Henry Bailey on the Merchants' National Bank of Fargo, North Dakota, amount $26.30; 2d, drawn by W. A. Owen on the Corn Exchange National Bank of Chicago, amount $8.35; 3d, drawn by Frank Mitchell on Carter's Bank, Eldora, Missouri, amount $12.40.

Rule a deposit slip and enter this deposit properly.

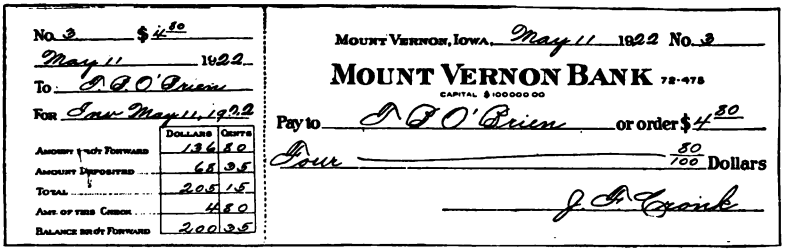
**2.** On a blank sheet of paper draw a check on the bank in which you have just made this deposit, paying F. G. Benton $23.25.

What is your balance in the bank after drawing this check?

**3.** Seven days ago you bought goods from F. G. Peterson, amounting to $28.50. The terms of the invoice were 2/10; N/30. To-day you pay the bill by check. Draw the check.

What is your balance in the bank?

**155. The Check Book.** Blank checks are furnished by the bank, bound in a check book. Checks are usually attached to stubs, on which are recorded, at the time of drawing the check, its number, the date, the name of the payee, the purpose for which it was given, and the amount. On the stub there is also kept a record of the balance remaining in the bank. As shown by the following illustration, a balance of $136.80 was brought over from May 10, and on May 11 a deposit of $68.35 was made and a check for $4.80 was drawn.



No. 3 $4.80
May 11 1922
To P. O'Brien
For Invoice May 11, 1922

| | Dollars | Cents |
|---|---|---|
| Amount brought Forward | 136 | 80 |
| Amount Deposited | 68 | 35 |
| Total | 205 | 15 |
| Amt. of this Check | 4 | 80 |
| Balance brought Forward | 200 | 35 |

Mount Vernon, Iowa, May 11 1922 No. 3

MOUNT VERNON BANK 78-478
CAPITAL $100000 00

Pay to P. O'Brien or order $4.80
Four 80/100 Dollars
J. A. Crank

**Written Work**

**1.** Using the following information, rule a check and stub similar to the preceding illustration, enter the necessary facts, and draw the check.

Name of bank, Fourth National, your city.

Balance from yesterday, $103.27. Deposit of $19.38.

Check No. 6, drawn by you to-day, payable to G. D. Fitzgerald and Co., in payment of the invoice purchased from them eight days ago. Amount of invoice, $73.28; terms, 2/10; N/60.

Write the stub.

What is the new balance?

**156. The Bank's Accounts with its Depositors.** All banks keep a depositor's ledger. Usually a page is devoted to the deposits and checks of each depositor.

*Williams & Harris*

| DATE | | | DEPOSITS | | CHECKS | | BALANCE | |
|---|---|---|---|---|---|---|---|---|
| 1922 | | | | | | | | |
| Jan. | 6 | | 143 | 65 | | | 143 | 65 |
| | 7 | | | | 26 | 25 | 117 | 40 |
| | 12 | | 68 | 60 | 13 | 25 | 172 | 75 |
| | 16 | | | | 6 | 80 | | |
| | | | | | 12 | 40 | | |
| | | | | | 3 | 75 | 149 | 80 |
| | 21 | | 6 | 00 | 43 | 74 | | |
| | | | | | 102 | 73 | 9 | 33 |
| | | | | | | | | |

AN ACCOUNT IN A DEPOSITORS' LEDGER

**Written Work**

Rule a page of a depositors' ledger, similar to the preceding illustration, and enter the following deposits and checks made by yourself, computing the daily balances. In computing balances, use the method of adding and subtracting explained in Section 12.

March 2, Deposit $127.80. March 3, Check $26.14. March 4, Check $23.19. March 5, Deposit $114.32; Check $12.19. March 6, Checks $13.14, $12.17. March 10, Deposits $32.76, $22.14; Check $17.14. March 11, Checks $26.19, $127.14. March 13, Deposits $12.19, $76.27; Checks $32.15, $27.19, $13.81. March 16, Deposit $27.94. March 18, Checks $23.42, $27.93.

**157. Reconciling the Bank Account.** It is the custom of depositors to reconcile the bank account monthly, to determine whether the balance shown by the stub of the check book agrees with the balance shown by the bank's records, and if the balances do not agree to determine the cause of the discrepancy. At the close of each month the bank gives the depositor a statement. This statement is a copy of the bank's account with the depositor, showing the balance at the beginning

of the month, the deposits and other credits, the checks cashed and any other debits, and the balance at the end of the month. The bank also returns the depositor's cancelled checks.

The depositor should now arrange the returned checks in numerical order and place a check mark on each stub corresponding to a returned check. Any stubs which are not marked represent checks which have been drawn but which have not been paid by the bank prior to the close of the month. A list should be made of these checks, showing the number and amount of each.

The deposits shown by the check book stubs should be compared with the credits shown on the bank statement. In many cases the deposits made on the last day of the month will not be put through the bank's books until the first day of the following month because of the bookkeeping involved. Hence the depositor's check book stubs may show a deposit which is not shown on the bank's statement.

If there are any other items which are not included in both the bank's and the depositor's records, these also should be noted. For instance, the bank may have allowed interest on the deposit while the depositor has not entered this on the check book stub; or the bank may have made a charge because the depositor's balance was less than a stipulated sum during the month.

After all of these items have been noted, a reconciliation statement should be made as follows:

RECONCILIATION STATEMENT

September 30, 1922

| | | |
|---|---|---|
| Balance per Check Book | | $457.68 |
| Add Interest Allowed | | .48 |
| Adjusted Balance | | 458.16 |
| Balance per Bank Statement | | 491.35 |
| Deduct Checks Outstanding: | | |
| No. 2109 | $36.21 | |
| No. 2112 | 19.45 | |
| No. 2114 | 7.46 | 63.12 |
| | | 428.23 |
| Add Deposit, September 30, 1922 | | 29.93 |
| Adjusted Balance | | 458.16 |

If the adjusted balances fail to agree the accounts do not reconcile, and the depositor should determine whether he has omitted some item from his records or made some other error, or whether an error has been made by the bank.

**Written Work**

On June 30, 1922, Walter Dayton's check book showed a balance of $178.65. The bank's statement showed a balance at the same date of $176.30. On comparing the cancelled checks with the stubs, Dayton found that the following checks were outstanding: No. 1315, $12.60; No. 1317, $7.80; No. 1320, $3.60. On June 30 he deposited $25.35 but this deposit did not appear on the bank's statement. The bank made a charge of $1.00 because the balance during the month had been less than $300.00. Prepare a reconciliation of the bank account.

**158. The Clearing House.** Any bank will cash the checks drawn on other banks, provided the person presenting them is known at the bank or is properly identified. For example, if you are given a check on the First National Bank, you can cash it or deposit it at the Second National Bank. The Second National then collects the amount of the check from the First National.

In the large cities, the checks are collected through a Clearing House. The representatives of the various banks meet at the Clearing House, bringing the checks which they have paid for other banks. Each bank is credited with the amount of the checks which it has paid for other banks, and debited with the amount of the checks which other banks have paid for it. The Clearing House collects from those having debit balances and pays those having credit balances.

Let us consider three of the banks in a city; the First National, the First State, and the City National. The representatives will meet, each bringing the checks paid by his bank for the other banks the preceding day, and a sheet showing the amount paid for each of the other banks. The statements brought by the representatives of the banks are shown below:

FIRST NATIONAL

| PAID FOR | AMOUNT |
|---|---|
| First State | $16,265.00 |
| City National | 12,146.80 |
| Total | $28,411.80 |

FIRST STATE

| PAID FOR | AMOUNT |
|---|---|
| First National | $ 8,368.28 |
| City National | 9,208.48 |
| Total | $17,576.76 |

CITY NATIONAL

| PAID FOR | AMOUNT |
|---|---|
| First National | $12,875.00 |
| First State | 4,738.80 |
| Total | $17,613.80 |

At the Clearing House a sheet similar to the following is prepared. In the debit column is placed the total value of checks paid *for* each bank; in the credit column is placed the total value of checks paid *by* each bank. The balance columns show the balances payable by or with each bank.

MANAGER'S SHEET

| | DEBITS | CREDITS | DEBIT BALANCE | CREDIT BALANCE |
|---|---|---|---|---|
| First National | $21,243.28 | $28,411.80 | | $7,168.52 |
| First State | 21,003.80 | 17,576.76 | $3,427.04 | |
| City National | 21,355.28 | 17,613.80 | 3,741.48 | |
| Totals | $63,602.36 | $63,602.36 | $7,168.52 | $7,168.52 |

The First State and the City National each turns over to the Clearing House the amount of its debit balance; the Clearing House pays this amount to the First National. In this way payment of over sixty-three thousand dollars is made by the actual transfer of only a little more than seven thousand dollars.

### Written Work

Prepare a Manager's Sheet, and find the debit or credit balance of each bank in the following Clearing House Association. The amount

of the checks paid by each bank is shown by the debit sheets which follow.

HARPER'S STATE BANK

| PAID FOR | AMOUNT |
|---|---|
| Harper's State | |
| First National | $23,756.27 |
| Traders' | 14,259.95 |
| Merchants' National | 25,726.87 |
| Farmers' & Mechanics' | 9,243.65 |
| First State | 12,060.[illegible] |

FIRST NATIONAL BANK

| PAID FOR | AMO[illegible] |
|---|---|
| Harper's State | $13,2[illegible] |
| First National | |
| Traders' | 8,836 |
| Merchants' National | 14,85[illegible] |
| Farmers' & Mechanics' | 8,8[illegible] |
| First State | 8,3[illegible] |

[illegible]aring
[illegible]anks.
[illegible]s paid
[illegible]hich
[illegible]those

TRADERS' BANK

| PAID FOR | AMOUNT |
|---|---|
| Harper's State | $ 5,273.[illegible] |
| First National | 12, 82 |
| Traders' | |
| Merchants' National | 7, 52.55 |
| Farmers' & Mechanics' | 11,25[illegible] |
| First State | 10,4[illegible]25 |

MERCHANTS' NATIONAL

| PAID FOR | [illegible]UNT |
|---|---|
| Harper's State | $ 3[illegible]3.64 |
| First National | 6, 30 |
| Traders' | 10,[illegible]40 |
| Merchants' National | |
| Farmers' & Mechanics' | 7,23 |
| First State | 9,2[illegible] |

FARMERS' & MECHANICS

| PAID FOR | AMOUNT |
|---|---|
| Harpers' State | $ 2,346.75 |
| First National | 5,236.73 |
| Traders' | 4,232.25 |
| Merchants' National | 7,727.86 |
| Farmers' & Mechanics' First State | 2,005.39 |

FIRST STATE

| PAID FOR | AMOUNT |
|---|---|
| Harper's State | $ 8,946.27 |
| First National | 1,856.23 |
| aders' | 11,472.75 |
| Aerchants' National | 923.88 |
| rmers' & Mechanics' First State | 2,423.73 |

h banks will pay to the Clearing House, and how much will ?

banks will receive from the Clearing House, and how much will h receive?

**159. Exchange on Checks.** Checks are frequently used to send money to people in cities other than the one in which the drawer of the check resides. For example, if you have money deposited in the Home National Bank of your city, you may draw a check against this deposit and send it to Mr. Perkins of Omaha. Mr. Perkins will take it to his bank in Omaha, which will pay the check, even though it has no knowledge of you or your deposit in the Home National Bank. It knows that Mr. Perkins is known at his bank, and that his endorsement is an agreement to refund the money if the check proves worthless is sufficient protection for his bank.

When banks pay out-of-town checks, they incur a certain amount of expense in collecting them. To cover this expense they often require persons for whom they cash checks to pay them "exchange." When exchange is charged, a common rate is $\frac{1}{10}$ of 1 per cent of the face of the check, with a minimum charge of from 10 cents to 25 cents.

**Problem.** Mr. Black, who lives in Bloomington, Illinois, received two checks in his mail. One was from Wm. Harris, of Lincoln, Ne-

braska, for $365.85; the other from H. B. Felter, of Cincinnati, Ohio, for $28.25. The bank at which Mr. Black cashed these checks charges for exchange $\frac{1}{10}$% of the face of each check, or a minimum of 10 cents. What was the exchange on the checks?

SOLUTION. $\frac{1}{10}$% of $365.85 = $.37
$\frac{1}{10}$% of $28.25 = $.028
Since $.028 is below the minimum, 10 cents exchange is charged.
The total exchange is $.47.

**Written Work**

Prepare deposit tickets for the following:

1. Deposited by Wm. Frend, of Peoria, Illinois, in the Peoria State Bank, on August 8, 1922:

Gold, $25; silver, $37.50; currency, $17; and checks as follows:
Check drawn by D. P. Snyder of Amboy, Illinois, $65.75.
Check drawn by Hammond & Davis of Lima, Iowa, $236.57.
Check drawn by B. J. Harris of Springfield, Illinois, $235.
The bank charges $\frac{1}{10}$% exchange. Minimum, 15 cents.
Find the total of the items and deduct the charge for collection.

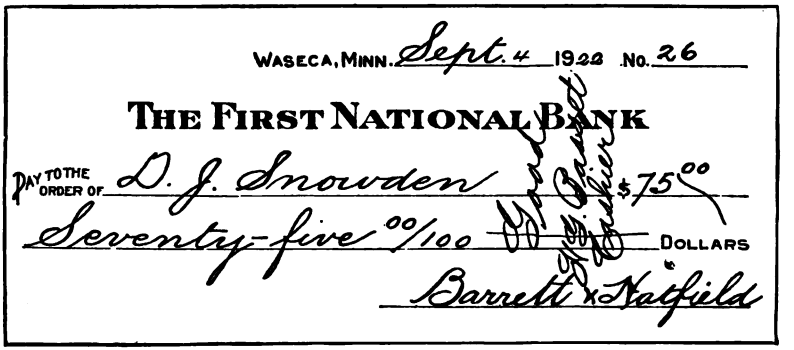
2. Deposited by G. D. Bernham of Hawkeye, Iowa, in the Hawkeye State Bank, on August 21, 1922:

Check drawn by S. Y. Benton of Clarinda, Iowa, $75.88.
Check drawn by D. W. Galbreath of Morris, Illinois, $57.93.
Check drawn by F. J. Ressler of Pasadena, California, $346.49.
Check drawn by D. T. Bailey of Buffalo, New York, $48.95.
The bank charges exchange on the California and New York checks; rate, $\frac{1}{10}$%, minimum 25 cents each.

NOTE. It is the custom of some banks to cash checks on banks in near-by states without charging exchange.

**160. Certified Checks.** If you wish to assure the person to whom you send a check that you have sufficient money in the bank to pay the check, you may have the check certified. Draw the check in the usual way and take it to your bank. The cashier will write or stamp "Accepted" or "Certified" or "Good" across its face and will sign his name. He will then set aside from your deposit the amount of the check, holding it in the bank's funds until the check is presented for

payment. Certifying the check reduces the receiver's risk of cashing a worthless check, but it does not relieve him from a possible exchange fee.

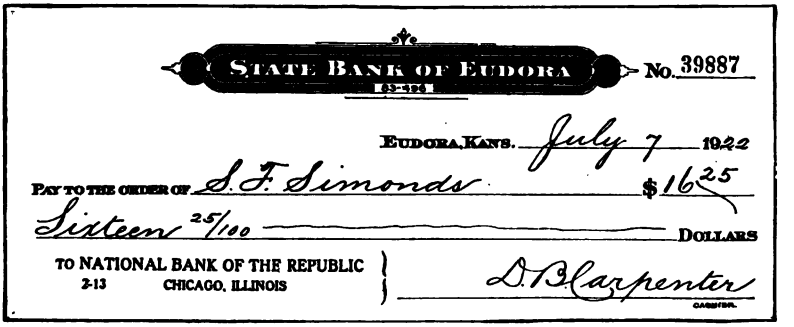

WASECA, MINN. Sept. 4 1922 No. 26
THE FIRST NATIONAL BANK
PAY TO THE ORDER OF D. J. Snowden $75.00
Seventy-five 00/100 DOLLARS
Barrett & Natfield

A CERTIFIED CHECK

**161. Bank Drafts.** Banks keep funds on deposit in the banks of the larger cities, and draw checks on these city banks just as you may draw a check on the bank in which you have deposited money. An order drawn by one bank against its deposit in another bank is called a "Bank Draft."

**162. Advantages of Bank Drafts.** A bank draft is preferable to a check when sending money from one town to another, for two reasons:

*a.* The draft is drawn by a bank, while the check is drawn by an individual. There is therefore a greater certainty that the bank draft is genuine, and will be paid when presented.


STATE BANK OF EUDORA
No. 39887
EUDORA, KANS. July 7 1922
PAY TO THE ORDER OF S. F. Simonds $16.25
Sixteen 25/100 DOLLARS
TO NATIONAL BANK OF THE REPUBLIC
2-13 CHICAGO, ILLINOIS
D. B. Carpenter
CASHIER

A BANK DRAFT

*b.* Bank drafts are drawn on banks in large cities because it is only in the large cities or business centers that other banks have any need for deposits. There is less expense incurred in collecting a bank draft drawn on a bank in a large city than in collecting a check drawn on a bank in a small town. Therefore banks seldom charge exchange when cashing bank drafts.

**Explanation.** D. B. Carpenter is the cashier of the State Bank of Eudora. This bank has funds on deposit in the National Bank of the Republic, Chicago. When the National Bank of the Republic pays this draft, it will deduct the amount from the balance of the Eudora Bank.

S. F. Simonds lived in Eudora and wished to send $16.25 to F. E. Craig of Bloomington, Illinois. When he purchased the draft from his bank, he had it made payable to himself. He might have had it made payable to F. E. Craig, but if he had done so, his (Simonds's) name would not have appeared on the draft, and if it had become separated from the letter accompanying it, Craig might have had difficulty in recalling from whom he received it.

After receiving the draft, Simonds indorsed it as follows:

Pay to the order of
F. E. Craig.
S. F. Simonds,

and sent it to Mr. Craig.

Mr. Craig indorsed it and cashed it at the Corn Belt National Bank of Bloomington.

We will now follow the steps by which the Corn Belt National collected the draft. This bank deposits funds with the Continental National Bank of Chicago, which is called its "correspondent," and it therefore sent this draft, with others paid the same day, to the Continental National Bank as a deposit.

The Continental National sent the draft to the Clearing House, which collected it from the National Bank of the Republic.

After paying the draft, the National Bank of the Republic charged the amount to the State Bank of Eudora.

The preceding statement may be summarized as follows:

State Bank of Eudora sells the draft to

↓

S. F. Simonds, who paid for it either with cash or a check; and sent it to

↓

F. E. Craig, who gave Simonds credit for $16.25; and deposited the draft at

↓

Bloomington Bank, which gave Craig $16.25; and sent the draft to

↓

Continental National, which gave the Bloomington Bank credit for $16.25; and collected the draft through the Clearing House from

↓

National Bank of the Republic, which paid the Continental National $16.25, and charged Eudora Bank $16.25.

INDORSEMENTS ON THE MODEL BANK DRAFT

Pay to the order of
F. E. Craig.
S. F. Simonds

Pay to the order of the
Corn Belt National Bank
F. E. Craig

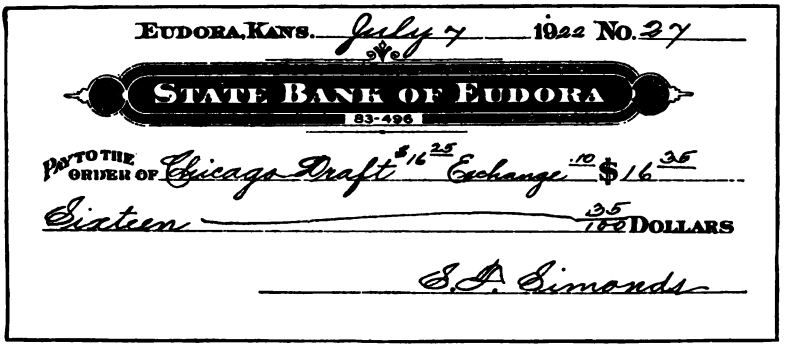
Pay to the order of
*Continental National Bank.*
All prior indorsements guaranteed
CORN BELT NATIONAL BANK,
BLOOMINGTON, ILL.,
F. A. FREND, Cashier.

Pay to the order of
*Any Bank, Banker, or Trust Co.*
Prior indorsements guaranteed.
July 12, 1922.
CONTINENTAL NATIONAL BANK,
M. B. JONES, Cashier.

These indorsements indicate the successive owners of the draft. The last two indorsements are made with rubber stamps. The last one is made payable to "any Bank, Banker, or Trust Company," for convenience in collecting through the Clearing House.

**163. The Cost of Bank Drafts.** When a bank sells a draft, it gives the purchaser the benefit of its deposit relations with a large city bank. For this convenience it may make a charge called "exchange." It will be noted that exchange on a check is paid by the person receiving and cashing it, while the exchange on a draft is paid by the person purchasing and sending it. When exchange is charged, $\frac{1}{10}\%$ with a minimum of 10 cents is a customary fee.

The following is a copy of the check given by S. F. Simonds to the State Bank of Eudora to pay for the draft purchased. There was a charge of 10 cents for exchange.


EUDORA, KANS. July 7 1922 No. 27

STATE BANK OF EUDORA

83-496

PAY TO THE ORDER OF Chicago Draft $16.25 Exchange .10 $16.35

Sixteen 35/100 DOLLARS

S. F. Simonds

**Written Work**

**1.** You owe H. J. Palmer, of Toledo, Ohio, $27.35. You buy a draft, payable to yourself for this amount, from the Merchants' State Bank of your city, of which A. R. Burton is the cashier. The draft is drawn on the Home National Bank of New York City. Exchange, 10 cents. Draft is made payable to you.

Write a draft similar to the one which the bank would give you, and the check which you would give the bank in payment.

Indorse the draft. How soon can Mr. Palmer get his money?

**2.** Mr. Palmer cashed the draft at the High Street National Bank of Toledo, B. F. Ohren, Cashier. The High Street National sent it to the Bankers' National Bank of New York, G. B. Martin, Cashier.

The Bankers' National collected it through the Clearing House.

Show the indorsements on the draft.

**3.** Find the cost of a draft sent to pay an invoice of $246.50, less a cash discount of 2%. Exchange, $\frac{1}{10}\%$ of the face of the draft; minimum, 15 cents.

4. On October 8, you purchased from Benedict & Meredith, Pittsburgh, Pennsylvania, an invoice amounting to $794.86. Terms, 2/15; N/60. On October 16, you paid the invoice by a draft purchased at the Claim Street State Bank of your city, J. D. Haines, Cashier. The draft was drawn on the First National Bank of New York. Exchange, $\frac{1}{10}\%$.

Write the draft which you received, the indorsement, and the check you gave to pay for the draft.

**164. Postal Money Orders.** A postal money order is an order drawn by one postmaster on the postmaster at some other office, calling for the payment of a stated sum of money to the person named on the order. Postal money orders are commonly used to make payments by mail. They are issued in any amount from $.01 to $100.00. As a means of protection, no order is issued for more than $100.00. If it is desired to send more than this amount, additional orders may be purchased.

**Fees** for money orders payable in the United States (which includes Hawaii and Porto Rico) and its possessions, comprising the Canal Zone, Guam, the Philippines, as well as in Bermuda, British Guiana, British Honduras, Canada, Cuba, Newfoundland, at the United States Postal Agency in Shanghai (China), in the Bahama Islands, and in certain other islands in the West Indies, are as follows:

| | | | | | |
|---|---|---|---|---|---|
| For orders | from | $ 0.01 | to | $ 2.50 | 3 cents |
| | from | 2.51 | to | 5.00 | 5 cents |
| | from | 5.01 | to | 10.00 | 8 cents |
| | from | 10.01 | to | 20.00 | 10 cents |
| | from | 20.01 | to | 30.00 | 12 cents |
| | from | 30.01 | to | 40.00 | 15 cents |
| | from | 40.01 | to | 50.00 | 18 cents |
| | from | 50.01 | to | 60.00 | 20 cents |
| | from | 60.01 | to | 75.00 | 25 cents |
| | from | 75.01 | to | 100.00 | 30 cents |

**Problems.** 1. What is the cost of an order for $27.35?

SOLUTION. $27.35 lies between $20.01 and $30.00, and the rate is 12 cents. Total cost of order, $27.35 + .12 = $27.47.

2. What is the cost of sending $267.95 by postal money orders?

SOLUTION. Two orders for $100.00 each, and one order for $67.95, will be purchased.

Fees of two $100.00 orders, 60 cents.
Fee of $67.95 order, 25 cents.
Total fees, 85 cents.
Total cost of orders, $267.95 + .85 = $268.80.

**165. Bank Drafts and Postal Money Orders Contrasted.** Note the following differences between a bank draft and a postal money order:

A bank draft may be indorsed as many times as desired.
A postal money order may be indorsed only once.
A bank draft may be cashed at *any* bank.
A postal money order must be presented to the post office on which it is drawn, or to a bank which can cash it at that post office.

**166. Express Money Orders.** Express money orders are similar in many respects to postal money orders. They can be purchased from the agent of the express company, and are payable from the funds of the express company on deposit in various banks specified in the order.

No order is issued for an amount larger than $50. If it is desired to send a larger amount, additional orders must be purchased.

**Express Money Order Rates.** The rates for express money orders are:

| | | | | | |
|---|---|---|---|---|---|
| For orders | from | $ .01 | to | $ 2.50 | 5 cents |
| | from | 2.51 | to | 5.00 | 7 cents |
| | from | 5.01 | to | 10.00 | 10 cents |
| | from | 10.01 | to | 25.00 | 12 cents |
| | from | 25.01 | to | 50.00 | 15 cents |
| | from | 50.01 | to | 75.00 | 18 cents |
| | from | 75.01 | to | 100.00 | 20 cents |

**Problem.** What will be the fee for the transfer of $85.60?

SOLUTION. The fee will be 20 cents, one order being issued for $50.00 and another for $35.60.

**Problem.** What will be the fee for the transfer of $260.75?

SOLUTION. $200.00 at the rate of 20 cents per hundred = 40 cents
60.75 at the rate of . . . . . 18 cents
Total . . . . . 58 cents

**167. Telegraph Money Transfers.** Money may be transferred by telegraph when there is an urgent necessity for immediate payment. The rates for this service between points in the United States are determined as follows:

To the tolls for a fifteen-word message between the office of deposit and the office of payment add the following charges:

| | |
|---|---|
| For $25.00 or less . . . . . | 25 cents |
| 25.01 up to $ 50 . . . . . | 35 cents |
| 50.01 up to 75 . . . . . | 60 cents |
| 75.01 up to 100 . . . . . | 85 cents |

For amounts above $100 add (to the $100 rate) 25 cents per hundred (or any part of $100) up to $3,000.

For amounts above $3,000 add (to the $3,000 rate) 20 cents per hundred (or any part of $100).

**Problems.** 1. What is the charge for sending $25 or less to a point where the fifteen-word rate is 65 cents?

SOLUTION.

| | |
|---|---|
| $.25 | Minimum charge |
| .65 | Tolls on 15-word message |
| $.90 | Total charge |

2. What is the total charge for transfer of $105 if the tolls on a fifteen-word message are 65 cents?

SOLUTION.

| | |
|---|---|
| $.85 | Charge for first $100 |
| .25 | Charge for fraction of second $100 |
| .65 | Tolls on 15-word message |
| $1.75 | Total rate |

### Written Work

1. Assuming that you owe the following bills:

F. G. Young, $ 39.40;
P. S. Sanborn, $112.75;
H. L. Colwell, $416.25, less 3%;

determine the fees for the purchase of either postal or express money orders for transferring money to make payments. Complete the following blank form.

| Name | Net Amount of Bills | Fees for Money Orders | Total Cost | Denominations of Orders Received from Ex. Co. | Denominations of Orders Received from P. O. |
|---|---|---|---|---|---|
| | | | | | |

2. What is the charge for a telegraph money transfer of $23.15 to a point where the fifteen-word message rate is 85 cents?

3. What is the charge for a telegraph money transfer of $52.85 to a point where the fifteen-word message rate is $1.30?

4. What is the total cost of a telegraph money transfer for $215 to a point where the fifteen-word rate is 65 cents?

# CHAPTER XVII

## COLLECTING BILLS

**168. Statements.** At periodical intervals, usually on the first day of each month, merchants send statements to their customers. This

**STATEMENT**

ACCOUNT NO. 5422

**WEBSTER & McCLELLAN**

16 JEFFERSON STREET

F. B. Turner,
La Grange, Ill

CHICAGO, June 1, 1922

All bills are due the first of the month after purchase.
This statement is intended to show you the condition of your account on our books. If this statement does not correspond with your accounts please notify us. If correct, please remit.

| | | | |
|---|---|---|---|
| | To balance, as per former statement | 17.65 | |
| May 3 | To Mdse, as per invoice | 23.78 | |
| May 16 | | 19.65 | |
| May 23 | | 27.84 | 88.92 |
| | Credits | | |
| May 20 | By Mdse returned | 2.85 | |
| May 26 | By Cash | 75.00 | 77.85 |
| | Balance | | 11.07 |

statement shows the date and amount of each purchase, the date and amount of each payment, and the balance due. The items sold are not enumerated in the above statement because the invoices may be consulted to obtain this information.

**HENDERSON & BISHOP**
143 WEST MONROE AVENUE

R. F. Bailey,
West Union, Iowa.

CHICAGO July 1, 1922

Interest charged on overdue accounts.
All claims for correction must be made on receipt of statement.

| DATE | ITEM | DEBIT | CREDIT | BALANCE |
|---|---|---|---|---|
| June 1 | Balance, May 31 | | | 75.60 |
| June 2 | Cash | | 75.00 | .60 |
| June 6 | Mdse | 34.55 | | 35.15 |
| June 12 | Mdse | 17.26 | | 52.41 |
| June 15 | Returned goods | | 4.35 | 48.06 |
| June 24 | Mdse | 5.96 | | 54.02 |

ANOTHER FORM OF STATEMENT

## Written Work

1. Rule a statement similar to the first illustration, and enter the following: Your transactions with S. J. Smith, Fayette, Missouri.

Balance, August 1, $127.86.

Purchases, August 3, $72.85; August 11, $23.89; August 14, $75.23; August 20, $14.76; August 28, $38.97.

Payments, August 2, Cash $126.00; August 13, Returned merchandise, $15.00.

Statement rendered September 1.

2. Rule a statement similar to the second illustration, and enter the following transactions with Beardsley & Russel, Montgomery, Alabama.

Balance, October 1, 1921, $32.11.

Purchases, October 5, $45.89; October 9, $23.87; October 12, $243.87; October 15, $65.00; October 21, $52.57; October 26, $40.50; October 29, $50.25.

**169. Commercial Drafts.** A commercial draft or bill of exchange is an unconditional order in writing signed by the drawer, ordering the person to whom it is addressed (called the **drawee**) to pay on demand or at a fixed or determinable future time a certain sum in money to the order of a designated person (called the **payee**).

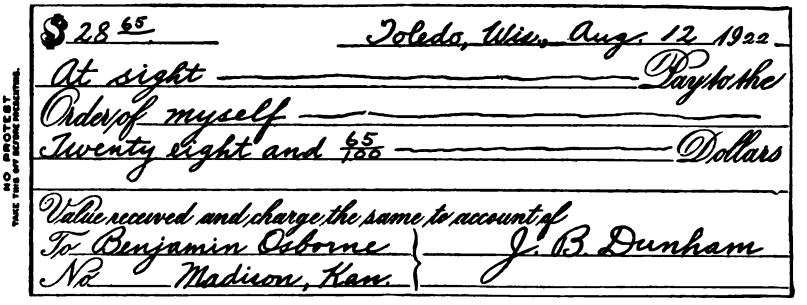
If the draft orders the drawee to pay money to the drawer, the paper is called a **two-party draft.** In such cases the drawer is also the payee. If the payee is a third party, the paper is called a **three-party draft.**

If the draft is payable at sight (that is, on demand), it is called a **sight draft.** If it is payable at a fixed or determinable future time, it is called a **time draft.** A time draft may be drawn payable a certain number of days or months after date; that is, after the date on which it is drawn. It may be drawn payable a certain number of days or months after sight.

**Acceptances.** An acceptance is a time draft which the drawee has agreed to pay, indicating this agreement by writing across its face "Accepted" together with his signature. If the draft is payable "—— days after sight" or "at —— days' sight," the drawee (who now becomes the acceptor) should also write the date on which he accepts. This is because "after sight" means after acceptance, and it is necessary to know the date of acceptance in order to know the date of maturity. If the draft is payable "after date" the drawee may write the date of acceptance if he desires, but it is not necessary to do so because the draft becomes due a stated time after its date regardless of the date of acceptance. An acceptance which represents a debt which arose out of the purchase of goods is called a **trade acceptance.**

**170. Drafts for the Collection of Accounts.** A sight draft or a time draft may be used as a means of collecting a book account. Wholesale houses often make terms similar to the following: "Sight draft in ten days, less 2%." At the expiration of ten days from the date of sale, the selling merchant sends a sight draft for the net amount of the bill to the purchasing merchant's bank. The bank presents it to the purchaser for collection.

To illustrate: J. B. Dunham of Toledo, Wisconsin, has sold goods to Benjamin Osborne of Madison, Kansas. In accordance with the terms of the sale, Dunham draws the following draft:


$28 65/100 Toledo, Wis. Aug. 12 1922
At sight Pay to the
Order of myself
Twenty Eight and 65/100 Dollars
NO PROTEST TAKE THIS OFF BEFORE PRESENTING
Value received and charge the same to account of
To Benjamin Osborne
No Madison, Kan.
J. B. Dunham

Mr. Dunham then indorses the draft as follows:

Pay to the order of
The First State Bank of Toledo
for collection only.
J. B. Dunham,

and gives it to his bank to collect.

The bank, in turn, indorses it as follows:

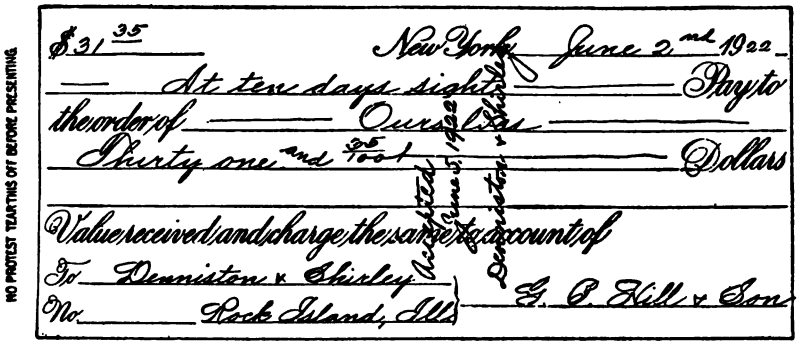
Pay to the order of
The Farmers National Bank of Madison
for collection only.
First State Bank,
Toledo, Wisconsin,
E. J. Baxter, Cashier,

and forwards it to the bank at Madison to collect. The Madison bank presents it to Mr. Osborne for payment.

If Osborne pays the draft, the bank stamps a receipt thereon, and gives the draft to Osborne. The bank keeps a certain amount as a collection fee, and returns the balance to the bank at Toledo, where Mr. Dunham receives the proceeds in cash or as a deposit credit.

If Osborne does not pay the draft, it is returned to the bank at Toledo with a statement of the reason for non-payment.

At the time of selling the goods the wholesale house may require the purchaser to accept a time draft. The wholesale house can then use the trade acceptance as security to borrow money at the bank, or it can sell the acceptance to the bank. At maturity the acceptance is presented to the acceptor for payment. The following is an illustration of such an acceptance:


NO PROTEST TEARTHIS OFF BEFORE PRESENTING
$31.35
New York June 2nd 1922
At ten days sight Pay to
the order of Ourselves
Thirty one and 35/100
Dollars
Value received and charge the same to account of
To Denniston & Shirley
No. Rock Island, Ills.
G. P. Hill & Son
Accepted June 5, 1922
Denniston & Shirley

AN ACCEPTED TIME DRAFT

This draft is due thirty days after June 6, or July 6.

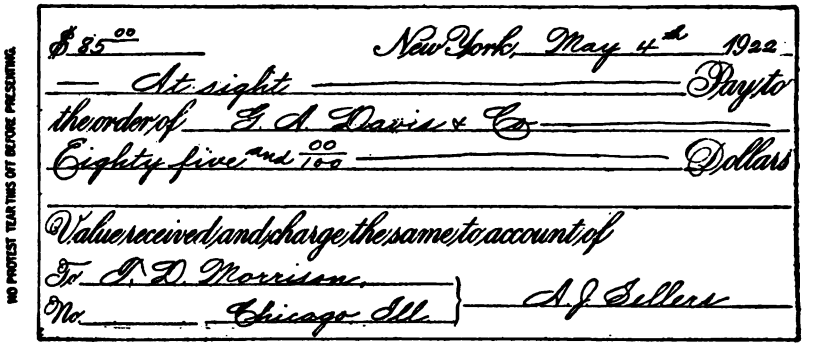
If a creditor has difficulty in collecting an account from a debtor he may draw a draft on him even though the terms at the time of sale made no mention of a draft. The debtor is not obliged to accept it or pay it, but since it will be presented to him by a bank in his own town he may do so in order to keep his reputation good with the bank.

**171. Drafts with C. O. D. Shipments by Freight.** When a merchant sells goods to be shipped by freight, he receives a bill of lading from the railroad company. This bill of lading is the railroad's receipt for the goods. To make a C. O. D. shipment the seller obtains an Order Bill of Lading from the railroad. The purchaser cannot get the goods from the railroad without surrendering the bill of lading. Therefore, a selling merchant, instead of sending the bill of lading direct to the purchaser, attaches it to a sight draft and sends the draft and the bill of lading to a bank in the purchaser's town.

In order to get the goods the purchaser must have the bill of lading; in order to get the bill of lading he must pay the draft at the bank.

**172. Drafts to Avoid the Transfer of Funds.** Three-party drafts are used for this purpose, but their use is not as customary as in the past. The following illustration will show the procedure.

T. D. Morrison of Chicago owes A. J. Sellers of New York $85.00. Mr. Sellers owes G. A. Davis & Co. of Chicago $150.00. Mr. Sellers draws the following draft, asking Mr. Morrison to pay G. A. Davis & Co. $85.00.


$85.00
New York, May 4th 1922
At sight
Pay to
the order of G. A. Davis & Co
Eighty five and 00/100
Dollars
Value received and charge the same to account of
To A. D. Morrison,
No. Chicago, Ill.
A. J. Sellers
NO PROTEST TEAR THIS OFF BEFORE PRESENTING

THREE-PARTY SIGHT DRAFT

### Oral Work

What two parties make a payment when this draft is used?

Explain how one transfer of money is saved.

How much will Sellers owe G. A. Davis & Co. after they have received this draft?

Three-party drafts may also be made payable after a stated time by using the terms explained on page 197.

### Written Work

**1.** On July 13, you purchased from A. B. Butler of Minneapolis, Minnesota, an invoice of goods amounting to $123.60; terms, sight draft in 10 days, less 3%. On July 20, Mr. Butler drew a sight draft on you for the net amount of the bill. He indorsed the draft, and gave it to the Fourth Street Bank of Minneapolis for collection. The Minneapolis bank indorsed it and sent it to the First National Bank of your city. The draft was presented to you by the bank on the 23d of July.

Write the draft with indorsements.

If the First National Bank charged 15 cents for collection, how much did Mr. Butler receive in payment of the draft?

How much did you pay to the bank?

**2.** You purchased an invoice of goods from Winthrop and Monroe of Toledo, Ohio. Amount of invoice, $365.00. It was your first transaction with them and consequently you were unknown to them. As you wanted the goods immediately and did not wish to delay while inquiries were made regarding your credit, you instructed them to send the goods

by freight and send a sight draft, with bill of lading attached, to the First National Bank of your city. The sight draft, which was dated yesterday, arrived this morning. It was made payable to Winthrop and Monroe and was indorsed to the order of the bank.

Write the draft and show the indorsement made by Winthrop and Monroe.

Write a check in favor of the bank to pay the draft.

What will you receive in exchange for the check?

How will Winthrop and Monroe get their money?

**3.** On September 3, you purchased an invoice of goods from S. L. Norton of Springfield, Illinois. Terms, thirty-day draft from date of sale, less 1%. Amount of invoice, $75.00. On September 5, you received the draft in the mail; it was dated September 3, and was payable 30 days after date. You immediately accepted the draft and returned it to Mr. Norton.

Write the draft, and show your acceptance.

When will you be expected to pay the draft?

**4.** You owed S. D. Briggs of Winona, Minnesota, $75.00; Mr. Briggs owed F. R. Tuttle of your city, $136.00. On June 19, Mr. Briggs drew a sight draft on you for the amount of your debt, and sent it to Mr. Tuttle as part payment of his debt. Mr. Tuttle presented it to you. You gave Mr. Tuttle cash in payment of the draft, and received the draft as a receipt.

Write the draft.

How much did Briggs owe Tuttle after the draft was paid?

## CHAPTER XVIII

### ACCOUNTS

AN account is an orderly record of the transactions pertaining to any one person or thing.

**173. The Cash Account.** In keeping a record of cash,

All cash receipts are entered on the left, or debit side;

All cash payments are entered on the right, or credit side;

The difference between the total receipts and total payments is the balance of cash on hand.

The following transactions are recorded in the model cash account in the illustration.

April 6, 1922, C. D. Smith invested $5,000 in business; April 7, he paid $1,640 for goods; April 10, he paid his store rent, $75; April 11, he received $275 for merchandise; April 13, he received $36 from John Appleton; April 14, he paid F. G. Barton $128; April 18, William Hobart paid him $60 on account; April 28, he paid D. F. Hilton $30 on account.

Cash

| 1922 | | | | 1922 | | | |
|---|---|---|---|---|---|---|---|
| April | 6 | C. D. Smith | 5000— | April | 7 | Merchandise | 1640— |
| | 10 | Mdse. Sale | 275— | | 10 | Store Rent | 75— |
| | 13 | John Appleton | 36— | | 14 | F. G. Barton | 128— |
| | 16 | William Hobart | 60— | | 28 | D. F. Hilton | 30— |
| | | | | | 30 | Balance | 3498— |
| | | | 5371— | | | | 5371— |
| May | 1 | Balance | 3498— | | | | |

**174. Personal Accounts.** Transactions with persons are recorded in personal accounts. Persons to whom goods are sold are debited for all goods sold to them on account; they are credited for payments made by them on account.

Persons from whom goods are bought, are credited for goods purchased from them on account; they are debited for payments made to them on account.

The two illustrations which follow will show the two common rulings for personal accounts.

May 2, 1922, purchased from Harold Booth, merchandise, $260. May 8, purchased merchandise from him, $37.90; May 10, paid him for the invoice of May 1, less 1% discount, Cash $257.40, Discount $2.60; May 17, paid him for the invoice of May 8, less 1% discount, Cash $36.52, Discount $.38; May 20, purchased merchandise, $271.25.

Harold Booth

| 1922 | | | | | 1922 | | | |
|---|---|---|---|---|---|---|---|---|
| May | 10 | Dis. 2.60 Cash 257.40 | | 260 — | May | 1 | Mdse | 260 — |
| | 17 | Dis. .38 Cash 36.52 | | 37 90 | | 8 | Mdse | 37 90 |
| | | | | | | 20 | Mdse | 271 25 |

June 1, 1922, sold to B. A. Newcomber, goods amounting to $345.70; June 5, sold to Newcomber merchandise amounting to $38.45; June 12, received payment for invoice of June 1, less 2%, Cash $338.79, Discount $6.91; June 13, sold Newcomber an invoice amounting to $83.92; June 14, received payment for the invoice of the 5th, less 1%, Cash $38.07, Discount $.38.

B. A. Newcomber

| Date | | Sale | Cash | Discount | Balance |
|---|---|---|---|---|---|
| 1922 June | 1 | 345 70 | | | 345 70 |
| " | 5 | 38 45 | | | 384 15 |
| " | 12 | | 338 79 | 6 91 | 38 45 |
| " | 13 | 83 92 | | | 122 97 |
| " | 14 | | 38 07 | 38 | 83 92 |

**175. Accounts Receivable and Accounts Payable.** If a person's account is larger on the debit side, it shows a balance owed *by* that person; it is therefore an account receivable, and the balance is an asset.

If a person's account is larger on the credit side, it shows a balance owed *to* that person; it is therefore an account payable, and the balance is a liability.

**Written Work**

**1.** Rule a cash account and enter the following transactions; find the balance, and rule the account.

August 2, you invest \$5,000 in a grocery business; August 3, purchase merchandise, paying cash for the same, \$1,345.75; August 6, receive cash for merchandise sold, \$127.50; August 10, receive cash for goods sold, \$50.25; August 12, pay for advertising, \$5.60; August 17, receive cash for merchandise sold, \$23.74; August 18, pay cash for an invoice of merchandise, \$56.35; August 21, receive cash from Henry Belmont on account, \$45.80; August 23, pay cash to Oscar Haines on account, \$54.85; August 31, pay store rent, \$30, and clerk hire, \$35.

**2.** Rule a personal account similar to the illustration of Harold Booth's account, page 203, and enter the following transactions:

September 3, you purchase from R. G. Henderson, on account, 2/10; N/60, an invoice of goods amounting to \$237.40; September 9, purchase an invoice of goods from Henderson, \$58.35, terms 1/15; N/2 months; September 11, purchase an invoice of merchandise from Henderson, \$123.60, terms 2/5; N/30; September 12, pay the invoice of the 3d, less the discount; September 14, purchase goods amounting to \$25; September 16, pay the invoice of the 11th, less the discount. What is the balance of Mr. Henderson's account? Is this an account receivable or an account payable? Is the balance an asset or a liability?

**3.** Rule an account similar to the illustration of B. A. Newcomber's account, page 203, and enter the following transactions:

July 2, sold R. S. Clark, terms 1/5; N/30, merchandise amounting to \$57.82; July 5, sold Clark goods to the value of \$23.95, terms 2/10; N/60; July 7, received payment for the invoice of the 2d, less the discount; July 9, sold Clark an invoice of \$75, less a trade discount of 10%, terms 1/10; N/30; July 11, sold him a bill of goods amounting to \$40; July 15, received cash for the invoice of the 5th, less the discount.

Is this account an account receivable or an account payable?

Is the balance an asset or a liability?

# CHAPTER XIX

## TAKING INVENTORY

At certain regular intervals an inventory is taken to determine the value of the stock on hand. Two clerks usually work together in taking the inventory. One counts the number of items of each kind, and reads aloud the cost price marked on the goods. The second clerk records these facts. The inventory is then sent to the office, where the value of each item is extended and the total value of the stock on hand is determined.

Inventories are entered in various forms, depending upon the details of information desired.

**176. Periodic Inventories.** A simple form like the following may be used:

| QUANTITY | NAME OF ITEM | COST | | COST EXTENSION | |
|---|---|---|---|---|---|
| 19 bbl. | Herald Flour | 5 | 60 | 106 | 40 |
| 345 lb | Java Coffee | | 26 | 89 | 70 |
| 28 lbs | Ceylon Tea | | 43 | 12 | 26 |

### Written Work

Rule an inventory similar to the preceding form, enter the following items, find the cost of each item, and the total cost.

This is one of several inventory sheets used in taking stock in a grocery.

| | | | | |
|---|---|---|---|---|
| 87 | lb. | Mexican Java Coffee | lb. | $ .23½ |
| 136 | lb. | Ceylon Tea | lb. | .42 |
| 167 | pkg. | Half-pounds Ceylon Tea | lb. | .47 |
| 2 | cases | Boneless Herring (4 doz.) | doz. | 1.10 |
| 83 | lb. | Salt Mackerel | lb. | .22 |
| $6\frac{5}{12}$ | doz. | Cans Corn | doz. | .85 |

| | | | | | |
|---|---|---|---|---|---|
| $4\frac{11}{12}$ | doz. | Cans Tomatoes | doz. | .95 |
| $8\frac{3}{4}$ | doz. | Cans Peas | doz. | 1.25 |
| $9\frac{1}{6}$ | doz. | Cans Beans | doz. | 1.35 |
| $3\frac{5}{6}$ | doz. | Bottles Olives | doz. | 1.00 |
| $4\frac{1}{3}$ | doz. | Bottles Olives (18 oz.) | doz. | 3.20 |
| $\frac{7}{12}$ | doz. | Bottles Stuffed Olives (10 oz.) | doz. | $.82\frac{1}{2}$ |
| 345 | lb. | Evaporated Apricots | lb. | $.08\frac{3}{4}$ |
| 163 | lb. | Evaporated Apples | lb. | $.07\frac{1}{4}$ |
| 109 | lb. | Dried Prunes | lb. | $.05\frac{1}{2}$ |
| 230 | lb. | Dried Peaches | lb. | $.07\frac{3}{4}$ |
| 67 | lb. | Seedless Raisins | lb. | $.09\frac{1}{2}$ |
| 1 | case | Currants (36 lb.) | lb. | $.08\frac{1}{2}$ |

A more elaborate form of inventory similar to the following may be used when desired.

| Add | Deduct | Lot No. | Size | Name and Quantity | Cost Price | Cost Extension | % Dep. | Loss Assumed |
|---|---|---|---|---|---|---|---|---|
| | | A 42 | 6 | 3 Pairs Mens Gun Metal Oxfords | 2 80 | 8 40 | | |
| | 1 | A 42 | 6½ | 4 Pairs do | 2 80 | 8 40 | | |
| 4 | | A 42 | 7 | 5 Pairs do | 2 80 | 25 20 | | |
| | | A 42 | 7½ | 6 Pairs do | 2 80 | 16 80 | | |
| | | A 42 | 8 | 2 Pairs do | 2 80 | 5 60 | | |
| | | C 9 | 7 7½ | 3 Pairs Mens Blucher Oxfords | 3 10 | 9 30 | 10% | .93 |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

**177. Explanation of Form.** The Lot Number is a number given an article by either the manufacturer or the merchant. Since different styles of goods are given different lot numbers, this number may be entered in the inventory in place of a description.

The "size" column is used to show the different sizes of an article in stock. As illustrated in the model inventory, each size of an article may appear on a separate line, or all sizes may be entered on the same line. When the latter method is followed, the quantity of a certain

size is indicated by writing the size below a short line, and the number of articles of that size above the line. Thus, $\frac{3}{7}$, $\frac{5}{8}$, means 3 No. 7's, 5 No. 8's.

When an error is made in the count, this error may be corrected by using the "Add" or "Deduct" columns. These columns are also used when goods are purchased or sold while the inventory is being taken. By adding goods purchased, and deducting goods sold during the taking of inventory, the value of stock at the end of the inventory may be determined.

To find the cost extension, multiply the cost by the number of items in the "Quantity" column plus the number of items in the "Add" column or minus those in the "Deduct" column.

If goods become shopworn or out of style, if the market price has decreased, or if, for any other reason, the value has become less than the cost marked on the goods and entered in the inventory, it is necessary to make some allowance for this depreciation in value.

Perhaps an article is worth only half its cost price. In that case a 50% depreciation should be entered in the "% Depreciation" column. Then 50% of the cost extension for that item is entered in the "Loss Assumed" column, and the total of this column is subtracted from the total of the "Cost Extension" column to determine the actual value of the stock on hand.

### Oral Work

1. What does 25% depreciation mean? 10% depreciation? 20% depreciation?

2. If stock is damaged so that it is worth only $\frac{3}{4}$ of its original cost, what per cent depreciation should be entered?

3. Goods are worth $\frac{2}{5}$ of their original cost. What is the per cent depreciation?

4. Goods are worth $\frac{4}{5}$ of their original cost. What is the per cent depreciation?

5. Goods are worth $\frac{7}{8}$ of their original cost. What is the per cent depreciation?

### Written Work

Rule an inventory form similar to the one on page 206 and enter the following items which form a portion of the inventory of a shoe stock. Enter one size to a line, find the value of each item listed, and

the total value of the stock shown on the page after making corrections for stock returned, for sales, and for depreciation.

Lot No. 219 B; $\frac{2}{6}, \frac{1}{6\frac{1}{2}}, \frac{3}{7}, \frac{4}{7\frac{1}{2}}, \frac{2}{8}, \frac{2}{8\frac{1}{2}}$. Men's Tan Oxfords, $3.15 per pair.

Lot No. 712; $\frac{2}{6}, \frac{3}{6\frac{1}{2}}, \frac{4}{7}, \frac{4}{7\frac{1}{2}}, \frac{3}{8}, \frac{4}{8\frac{1}{2}}, \frac{2}{9}$. Men's Vici Kid Bluchers, $2.90 per pair.

Lot No. 322 A; $\frac{1}{6}, \frac{1}{6\frac{1}{2}}, \frac{2}{7}$. Romeos at $1.90 per pair.

Lot No. 618 D1; $\frac{2}{7}, \frac{3}{7\frac{1}{2}}, \frac{3}{8}, \frac{2}{8\frac{1}{2}}, \frac{2}{9}, \frac{1}{9\frac{1}{2}}$. Men's Russian Calf Boots, $3.20 per pair.

One pair of Lot No. 712, Size 7, taken home by a customer before taking inventory, was returned after Lot No. 712 was listed.

During stock taking, the following sales were made:

1 pair Lot No. 712, size $7\frac{1}{2}$.
1 pair Lot No. 618 D1, size $7\frac{1}{2}$.
1 pair Lot No. 618 D1, size $9\frac{1}{2}$.

Lot No. 322 A can now be purchased for $1.75.

**178. Perpetual Inventory.** In some lines of business a perpetual inventory is kept. It is also called a Stock Record. When such a record is kept, a book is required for the purpose, one page being devoted to each item carried in stock. Both purchases and sales are recorded at cost price. The following model illustrates a form of perpetual inventory.

**179. Value of Perpetual Inventory.** Considerable labor is required to maintain a perpetual inventory, but it is valuable for several reasons.

When stock is kept in warehouses or in storerooms at some distance from the salesrooms, the perpetual inventory is a great convenience since it shows without delay the amount of stock on hand.

A " danger point " is fixed for each item. When the balance on hand has decreased to this danger point, a new supply is purchased.

An inventory may be taken at periodical intervals, and the quantities shown compared with the stock record. If the inventory and the stock record do not agree, an investigation may be made to discover the cause of the discrepancy.

No. A 603 Brace Castings

Danger Point 15

| Date | | Purchases | | | Sales | | | Balance | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1922 | | No. | Cost | Value | No. | Value | | No. | Value | |
| July | 8 | 60 | 18 | 10 80 | | | | 60 | 10 | 80 |
| " | 17 | | | | 14 | 2 | 52 | 46 | 8 | 28 |
| " | 22 | | | | 26 | 4 | 68 | 20 | 3 | 60 |
| " | 31 | 80 | 18 | 14 40 | | | | | 18 | 00 |
| Aug. | 3 | | | | 25 | 4 | 50 | | 13 | 50 |

**Written Work**

Rule a page of a stock record similar to the model on this page, and enter the following facts, making a perpetual inventory of the stock of Solvay Coke.

PURCHASES

| | |
|---|---|
| October 22, 54,200# | at $4.45 per ton |
| November 18, 57,600# | at $4.45 per ton |
| December 7, 52,800# | at $4.45 per ton |

SALES

October 29, 2 tons; October 30, 5 tons; October 31, 7,500#; November 2, 9,500#; November 5, 7 tons; November 17, 2 tons; November 19, 12 tons; November 20, 7,000#; November 23, 4 tons; November 24, 9,000#; November 30, 2 tons; December 9, 13,000#; December 11, 5 tons.

## CHAPTER XX

## GROSS PROFIT ON SALES

**180. Definition.** Gross Profit on Sales is the excess of the selling price of goods over their cost.

**181. Cost of Goods Sold.** The cost of goods sold is computed as follows: To the inventory at the beginning of the period add the purchases and the buying expenses such as freight and drayage; from this total deduct the inventory at the close of the period.

**182. The Per Cent of Gross Profit** may be computed by dividing the gross profit by the cost of the goods sold or by the selling price. Textbooks have usually stated that cost should be used as the basis, but business men almost invariably use the selling price as the basis. Therefore, when the expression " per cent of gross profit " is used in this text it will be understood to refer to the rate obtained by dividing the gross profit by the amount of the sales.

### Oral Work

1. A hat which cost \$2.00 is sold for \$3.00. What is the profit? What is the per cent of profit?

2. A set of books cost a dealer \$20.00. In order to sell them the dealer lost \$2.00. What was the per cent of loss?

3. Through an error in an advertisement a merchant was obliged to sell some parasols at $16\frac{2}{3}\%$ less than their cost. He lost 30 cents on each parasol. What did they cost, and what was the selling price? At what price should they have been sold to gain 10% of the cost?

### Written Work

1. Mr. Fisher bought a carriage for \$55 and sold it for \$75. What was the gross profit? What was the per cent of gross profit?

2. During a certain year a merchant purchased goods which cost him \$14,000. At the close of the year this stock inventoried \$3,000. What was the cost of the goods sold?

His sales for the year were $14,080. What was his gross profit? What was the per cent of gross profit?

**3.** The inventory of a dry goods merchant's stock at the beginning of the year was $3,560. The purchases during the year were $28,265. At the close of the year the inventory showed a stock of $3,940 unsold. What was the cost of the goods sold?

His sales for the year were $31,673. What was the gross profit? What was the per cent of gross profit?

**4.** Mr. Tracey bought the lumber business formerly conducted by Mr. Boyce. The stock on hand inventoried $5,280. Mr. Tracey's purchases during the year were $16,385. The freight was $236, the cost of labor in placing lumber in the yards and all other buying expenses $385. At the end of the year the stock inventoried $6,125. What was the cost of the goods sold during the year?

If the sales during the year were $19,245, what was the gross profit and per cent of gross profit?

**5.** Complete the following table:

| Inventory, Jan. 1, 19— | Purchases | Buying Expenses | Total | Inventory, Dec. 31, 19— | Cost of Goods Sold | Sales | Profit or Loss | Per Cent Profit or Loss |
|---|---|---|---|---|---|---|---|---|
| 3,125 75 | 16,294 90 | 3,70 25 | | 4,685 60 | | 18,495 40 | | |
| 5,428 80 | 23,926 56 | 627 90 | | 5,626 35 | | 33,756 20 | | |
| 1,290 00 | 6,754 00 | 271 25 | | 2,140 00 | | 8,325 75 | | |
| 4,728 50 | 12,398 80 | 1,417 25 | | 1,265 70 | | 13,215 75 | | |
| 3,680 00 | 15,927 00 | 485 00 | | 2,726 00 | | 14,850 00 | | |

Enter profit and per cent of profit in black ink.
Enter loss and per cent of loss in red ink.

### Written Review

**1.** Compute the cost of the following invoice subject to a 28% trade discount:

| | | |
|---|---|---|
| 8 Tables | @ | $27.50 |
| 12 Dressers | @ | 54.00 |
| 12 Beds | @ | 65.00 |
| 36 Chairs | @ | 14.80 |

**2.** If 4% is added to cost to cover buying expenses, what is the marked cost of each of the articles purchased?

**3.** How would the cost of each article be marked using the key word Charleston?

**4.** The invoice is subject to a cash discount of 2% if paid within ten days. How much is the cash discount, and what is the net amount of the invoice?

**5.** What is the net amount of an invoice of $745.60, subject to discounts of 20%, 10%, and 2%?

**6.** What single discount is equivalent to the series 40%, 20% and 1%?

**7.** A's check book shows a balance of $271.60. The bank's statement at the same date shows a balance of $316.95. There are outstanding checks as follows:

| | |
|---|---|
| No. 161 | $14.75 |
| 170 | 23.80 |
| 172 | 6.95 |

The bank has charged 15 cents for collecting a draft and A has not deducted the charge on his check book stubs. Prepare a reconciliation of the bank account.

**8.** What is the fee for sending $75.60 by postal money order?

**9.** What is the value of £75 4*s*, 6*d* at 4.655?

**10.** How many francs can be obtained for $250.00 if exchange is quoted at 6.14?

**11.** A merchant's inventory January 1, 1922, was $6,340.65; his purchases for the year were $75,210.93; and his inventory December 31, 1922, was $7,246.80. His sales for the year were $108,215.60. What was the cost of sales? the gross profit on sales? the per cent of gross profit?

# CHAPTER XXI

## INTEREST

**183. Definitions.** **Interest** is the income earned by loaning money or extending credit, or the expense incurred by borrowing money or obtaining credit.

**Accrued interest** may be either receivable or payable. Accrued interest receivable is unpaid interest due from a debtor; accrued interest payable is unpaid interest due to a creditor.

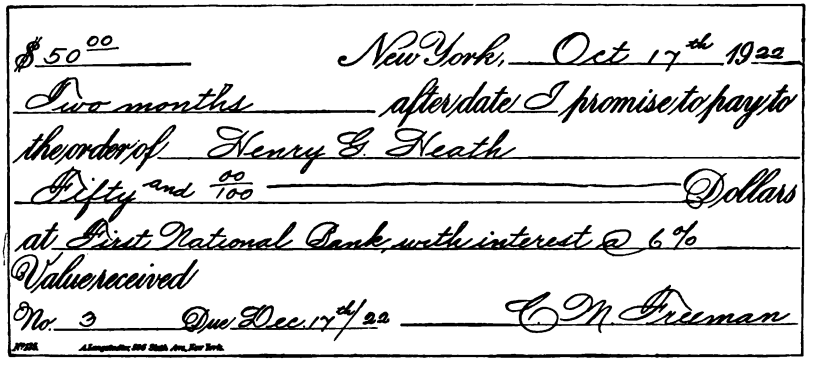
The sum borrowed is called the **principal.**

The per cent of the principal to be paid each year for the use thereof is called the **rate.**

The total of the principal and the unpaid or accrued interest is called the **amount.**

Assume that James Smith borrowed $1,000 from Henry Davis at 6% for one year. He paid a half year's interest at the middle of the year, but has not paid the interest at the end of the year. The principal is $1,000; the rate is 6%; the interest for the year is $60, and this interest is an earning for Davis who loaned the money and an expense for Smith who borrowed it. At the end of the year there is $30 unpaid or accrued interest; from Smith's standpoint this is accrued interest payable; from Davis's standpoint it is accrued interest receivable.

A **promissory** note is an unconditional promise in writing signed by the **maker** engaging to pay on demand or at a fixed or determinable future time a certain sum of money to the **payee** or his order.


$50^{00}$ New York, Oct 17th 1922

Two months after date I promise to pay to

the order of Henry G. Heath

Fifty and $^{00}/_{100}$ Dollars

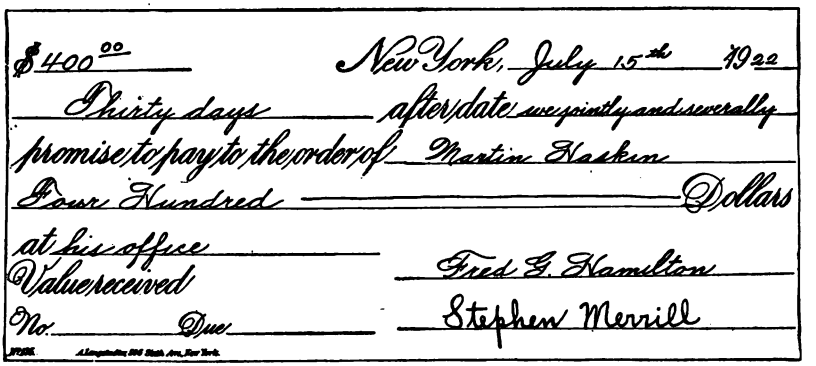
at First National Bank, with interest @ 6%

Value received

No. 3 Due Dec 17th/22 C M Freeman

PROMISSORY NOTE

If the borrower is a person of small means, or if, for any reason, it may be unsafe to loan money to him, he may be requested to have some person of recognized financial standing sign the note with him. Such notes are either **joint notes,** or **joint and several notes,** depending upon the wording.



$400.00 New York, July 15th 1922

Thirty days after date we jointly and severally promise to pay to the order of Martin Haskin

Four Hundred ——— Dollars

at his office

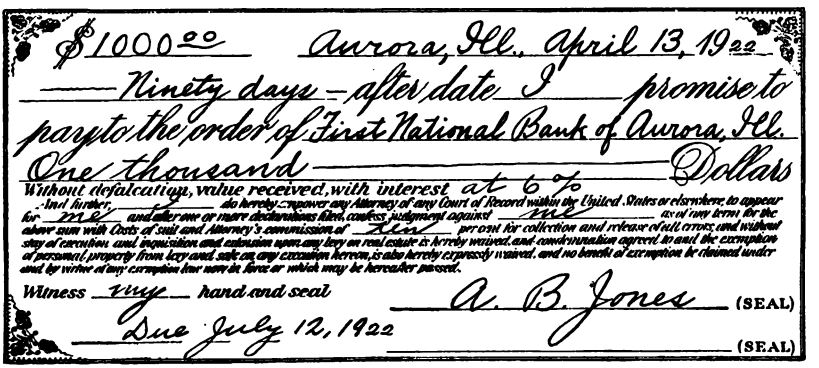
Value received

No. Due

Fred G. Hamilton

Stephen Merrill

JOINT AND SEVERAL NOTE



$1000.00 Aurora, Ill., April 13, 1922

Ninety days after date I promise to pay to the order of First National Bank of Aurora, Ill.

One thousand ——— Dollars

Without defalcation, value received, with interest at 6%

And further ——— do hereby empower any Attorney of any Court of Record within the United States or elsewhere, to appear for me and enter one or more declarations filed, confess judgment against me as of any term for the above sum with Costs of suit and Attorney's commission of Ten per cent for collection and release of all errors, and without stay of execution, and inquisition and extension upon any levy on real estate is hereby waived, and condemnation agreed to, and the exemption of personal property from levy and sale on any execution hereon, is also hereby expressly waived, and no benefit of exemption be claimed under and by virtue of any exemption law now in force or which may be hereafter passed.

Witness my hand and seal A. B. Jones (SEAL)

Due July 12, 1922 ——— (SEAL)

JUDGMENT NOTE

**184. To find interest at 6% for 6, 60, 600, or 6,000 days.**

| | |
|---|---|
| The interest on $1.00 for 1 year at 6% is | $.06 |
| The interest on $1.00 for 2 months (60 da.) is | .01 |
| The interest on $1.00 for 6 days ($\frac{1}{10}$ of 60 da.) is | .001 |

Thus, to find the interest on $1.00 for 60 days at 6%, move the decimal point two places to the left; to find the interest for 6 days, move

the point three places to the left. Since this is true if the principal is $1.00, it will be true of any principal, and we have the following rule:

Given any principal, to find the interest at 6%.

*For 6 days, move the decimal point three places to the left; for 60 days, two places to the left; and for 600 days, one place to the left. For 6,000 days the interest will be the same as the principal.*

**Oral Work**

State the interest on each of the following principals at 6%, for 6, 60, 600, and 6,000 days:

| | | | |
|---|---|---|---|
| 1. $1,000. | 2. $350. | 3. $125.60. | 4. $3,620. |
| 5. $12.20. | 6. $325.50. | 7. $6. | 8. $4,285 |

Find the interest at 6% on:

8. $245 for 6 days.
9. $27 for 6 days.
10. $37.50 for 60 days.
11. $12.50 for 6 days.

**185. To find interest at 6% for fractions or multiples of 6, 60, 600, or 6,000 days.**

The time is not always 6, 60, or 600 days, yet the above method can be easily used when the time is an easy fraction or an exact multiple of 6, 60, 600, or 6,000 days.

**Problems.** 1. Find the interest on $500 for 12 days at 6%.

SOLUTION. $.50 = the interest for 6 days.
$2 \times \$.50 = \$1.00$, the interest for 12 days.

2. Find the interest on $700 for 40 days at 6%.

SOLUTION. $7 = the interest for 60 days.
$\frac{2}{3}$ of $7 = $4.67, the interest for 40 days.

3. Find the interest on $300 for 3 months at 6%.

SOLUTION. $3 = the interest for 2 months (60 days).
$\$3 \div 2 = \$1.50$, the interest for 1 month.
$\$1.50 \times 3 = \$4.50$, the interest for 3 months.

### Written Work

Rule a form similar to the following model; compute the interest at 6% on the various principals for the different times.

| Principals | 24 Da. | 30 Da. (1 Mo.) | 40 Da. | 80 Da. | 90 Da. (3 Mo.) | 18 Da. | 50 Da. | 70 Da. | 4 Mo. | 6 Mo. |
|---|---|---|---|---|---|---|---|---|---|---|
| $ 125 60 | | | | | | | | | | |
| 28 95 | | | | | | | | | | |
| 378 45 | | | | | | | | | | |
| 500 00 | | | | | | | | | | |
| 960 00 | | | | | | | | | | |
| 96 00 | | | | | | | | | | |
| 9 60 | | | | | | | | | | |
| 475 00 | | | | | | | | | | |
| 4 75 | | | | | | | | | | |
| 30 00 | | | | | | | | | | |
| 146 00 | | | | | | | | | | |
| 419 50 | | | | | | | | | | |
| 395 00 | | | | | | | | | | |
| 25,000 00 | | | | | | | | | | |

When the time is not an easy fraction or an exact multiple of 6, 60, or 600 days, an adaptation of the method shown in the last exercise may be used.

**Problems.** 1. Find the interest on $500 for 99 days at 6%.

Solution.

$5.00 = the interest for 60 days.
2.50 = the interest for 30 days.
.50 = the interest for 6 days.
.25 = the interest for 3 days.
$8.25 = the interest for 99 days.

2. Find the interest on $1,000 for 82 days at 6%.

Solution.

$10.00 = the interest for 60 days.
$3.33\frac{1}{3}$ = the interest for 20 days.
$.33\frac{1}{3}$ = the interest for 2 days.
$13.67 = the interest for 82 days.

### Oral Work

How would you find the interest on any principal for the following number of days at 6%:

3, 4, 5, 7, 8, 9, 36, 66, 69?

### Written Work

Enter the interest at 6% in the proper place on a form ruled like the following model:

| | $845.00 | $632.00 | $1,296.00 | $5,483.00 | $74.60 | $143.40 | $746.95 | $25.00 | $130.00 |
|---|---|---|---|---|---|---|---|---|---|
| 10 da. | | | | | | | | | |
| 66 da. | | | | | | | | | |
| 63 da. | | | | | | | | | |
| 68 da. | | | | | | | | | |
| 75 da. | | | | | | | | | |
| 80 da. | | | | | | | | | |
| 85 da. | | | | | | | | | |
| 96 da. | | | | | | | | | |
| 98 da. | | | | | | | | | |
| 45 da. | | | | | | | | | |
| 82 da. | | | | | | | | | |
| 28 da. | | | | | | | | | |

**186. To find interest at 6% for any number of days.**

Sometimes the method of adding the interest for different fractions or multiples of 6 and 60 days cannot conveniently be used.

In such cases

*Point off three places in the principal; this will give the interest for six days. Multiply by the number of days; this will give the interest for six times the number of days. Divide by six; this will give the interest for the required number of days.*

**Problem.** Find the interest on $500 for 19 days at 6%.

SOLUTION. $.50 = the interest for 6 days.
$.50 × 19 = $9.50, the interest for 19 × 6 days
$9.50 ÷ 6 = $1.58, the interest for 19 days.

### Written Work

Enter the interest at 6% on the various principals for the different times in the proper place on a form ruled like the following model:

| | $435.00 | $127.90 | $213.65 | $472.90 | $123.50 | $12.35 |
|---|---|---|---|---|---|---|
| 23 da. | | | | | | |
| 29 da. | | | | | | |
| 37 da. | | | | | | |
| 19 da. | | | | | | |
| 25 da. | | | | | | |
| 73 da. | | | | | | |
| 47 da. | | | | | | |
| 81 da. | | | | | | |
| 114 da. | | | | | | |
| 95 da. | | | | | | |
| 61 da. | | | | | | |
| 77 da. | | | | | | |
| 183 da. | | | | | | |
| 211 da. | | | | | | |

**187. Interchanging Principal and Time.** There is a short method which may sometimes be used when the days are not exact multiples of 6, 60, 600, or 6,000.

The interest is the same in each of the following cases:

$600.00 for 29 days at 6%, and
$ 29.00 for 600 days at 6%.

It is much easier to compute the interest in the latter case, because all that is necessary is to point off one place in the principal, giving the amount of interest immediately, $2.90.

Therefore, when the days are not exact multiples or easy fractions of 6, 60, 600, or 6,000, and when the principal *is* an exact multiple or easy fraction of one of these numbers, interchange the number of dollars and days, and proceed as before.

**Problems.** 1. Find the interest on $360.00 for 83 days at 6%.

SOLUTION. Interchanging, we have $ 83.00 for 360 days at 6%.
Pointing off two places, $.83 interest for 60 days.
Multiply by 6 for 360 days, and the result is $4.98.

2. Find the interest on $200.00 for 47 days at 6%.

SOLUTION. Interchanging, we have $47.00 for 200 days at 6%.
Point off one place for 600 days, $4.70.
Divide by 3 for 200 days, $1.57.

### Oral Work

How would you find the interest on the following at 6%?

1. $700.00 for 71 days.
2. $150.00 for 59 days.
3. $420.00 for 113 days.
4. $30.00 for 57 days.
5. $66.00 for 171 days.
6. $1200 for 37 days.

### Written Work

Compute and enter the interest at 6% on a form ruled like the following model:

| | 117 Da. | 34 Da. | 161 Da. | 50 Da. | 74 Da. | 3 Mo. 7 Da. |
|---|---|---|---|---|---|---|
| $300 00 | | | | | | |
| 1,200 00 | | | | | | |
| 240 00 | | | | | | |
| 4,200 00 | | | | | | |
| 150 00 | | | | | | |
| 72 00 | | | | | | |
| 450 00 | | | | | | |
| 54 00 | | | | | | |
| 6,000 00 | | | | | | |
| 5,000 00 | | | | | | |

**188. To find the interest when the rate is other than 6%.**

*First find the interest at 6%, then take $\frac{1}{6}$ of the result; this will give the interest at 1%. Multiply this quotient by the given rate.*

The following short methods are of value in computing interest:
If the rate is 7%, add $\frac{1}{6}$ of the interest at 6%.
If the rate is 5%, subtract $\frac{1}{6}$ of the interest at 6%.
If the rate is $7\frac{1}{2}$%, add $\frac{1}{4}$ of the interest at 6%.
If the rate is $4\frac{1}{2}$%, subtract $\frac{1}{4}$ of the interest at 6%.

### Oral Work

How would you find the interest at 4%? $5\frac{1}{2}$%? $6\frac{1}{2}$%? 8%? 9%?

### Written Work

Rule and complete the following blank:

SIMPLE INTEREST

| RATE | 36 DA. | 45 DA. | 63 DA. | 19 DA. | 47 DA. | 184 DA. | 2 MO. 7 DA. |
|---|---|---|---|---|---|---|---|
| | $436.50 | $75.80 | $174.30 | $66.00 | $1,748.60 | $73.90 | $400.00 |
| 6% | | | | | | | |
| 8% | | | | | | | |
| 5% | | | | | | | |
| $4\frac{1}{2}$% | | | | | | | |
| $3\frac{1}{2}$% | | | | | | | |

**189. To find the interest when the time is stated in years, months, and days.**

In the preceding problems the time has been stated in days and months. When the time has been stated in months, we have reduced it to days, but when the time is stated in years, months, and days, it is too laborious to reduce it to days, and the following method should be used.

*Find the interest for the number of years at the given per cent. To this result add the interest for the given number of months and days.*

**Problem.** Find the interest on $375 for 2 yr. 3 mo. 21 da. at 5%.

SOLUTION. The interest for 1 yr. is 5% of the principal.
The interest for 2 yr. is 10% of the principal.
10% of $375 = $37.50.
The interest on $375 for 3 mo. 21 da. is $5.78.
Therefore, the interest for 2 yr. 3 mo. 21 da. is $43.28.

### Written Work

Rule a blank like the following model. Compute and enter the interest.

Complete the blank:

SIMPLE INTEREST

| TIME | | | $124.00 6% | $37.62 5% | $1,245.75 3½% | $463.80 4% | $2,000.00 5½% |
|---|---|---|---|---|---|---|---|
| Years | Months | Days | | | | | |
| 1 | 5 | 24 | | | | | |
| 1 | 8 | 12 | | | | | |
| 2 | 4 | 18 | | | | | |
| 3 | 8 | 6 | | | | | |
| 4 | 10 | 12 | | | | | |
| 10 | 2 | 7 | | | | | |
| 8 | 6 | 2 | | | | | |
| 5 | 7 | 9 | | | | | |
| 3 | 4 | 15 | | | | | |
| 2 | 7 | 28 | | | | | |
| 3 | 3 | 20 | | | | | |
| 4 | 7 | 27 | | | | | |
| 9 | 1 | 19 | | | | | |

**190. To compute the time between two dates.** In all of the problems given thus far, the time has been stated. You will often be called upon to compute the time. In such cases you will be given the date when the note was made and the date when it is due (date of maturity).

There are two methods of computing the time between dates:

*First Method.* Counting the actual number of days. (Not used if the interval is more than one year.)

*Second Method.* Compound subtraction. (May be used for any interval.)

*a.* **To find the time by counting the actual number of days.**

**Problem.** Find the number of days between February 15, 1921, and May 24, 1921.

SOLUTION. February has 28 days, of which 15 have expired, leaving

| | |
|---|---|
| in February | 13 days |
| March has | 31 days |
| April has | 30 days |
| The note matures on May | 24 |
| Total | 98 days |

From this problem we derive the following rule:

*Add the number of days remaining in the month after the day the note was made, the exact number of days in each complete month, and the number of days through which the note is to run in the last month.*

### Written Work

Find the exact number of days between the dates in the table:

| Made | Due | | | | | |
|---|---|---|---|---|---|---|
| | Oct. 26 | Nov. 13 | Dec. 27 | Jan. 3 | Jan. 17 | Feb. 26 |
| Oct. 13 | | | | | | |
| Sept. 27 | | | | | | |
| Aug. 18 | | | | | | |
| July 16 | | | | | | |
| July 3 | | | | | | |
| May 17 | | | | | | |
| May 1 | | | | | | |

*b.* **To find the interest period by compound subtraction of dates.**

**Problem.** Find the time between May 18, 1918, and February 16, 1921.

Solution.

| | | | |
|---|---|---|---|
| | 1921 | 2 | 16 |
| | 1918 | 5 | 18 |

Since 18 cannot be subtracted from 16, and since 5 cannot be subtracted from 2, reduce 1 month to days, and 1 year to months.

| | | | |
|---|---|---|---|
| | 1920 | 13 | 46 |
| | 1918 | 5 | 18 |
| | 2 yr. | 8 mo. | 28 da. |

### Written Work

Find the time by compound subtraction. The figures in the left-hand column are the dates on which the notes were given. The dates at the top are the dates of maturity.

| From | To | | | |
|---|---|---|---|---|
| | Maturity 10-22-1920 | Maturity 6-15-1921 | Maturity 11-14-1921 | Maturity 5-20-1922 |
| 6-17-1919 | | | | |
| 8- 5-1919 | | | | |
| 9-25-1918 | | | | |
| 11- 6-1918 | | | | |
| 4-19-1916 | | | | |
| 2-26-1915 | | | | |
| 4-30-1918 | | | | |
| 7-14-1919 | | | | |
| 10-11-1912 | | | | |
| 12-31-1918 | | | | |

The following table may be used in finding the exact number of days between any two dates:

For illustration, the time from Apr. 7 to Nov. 7 is found to be 214 da. From Sept. 3 to Jan. 3 is found to be 122 da. To find the time from Sept. 3 to Jan. 18, add 15 da. to 122 da.

| From Any Day of | To the Same Day of the Next | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
| January . . | 365 | 31 | 59 | 90 | 120 | 151 | 181 | 212 | 243 | 273 | 304 | 334 |
| February . | 334 | 365 | 28 | 59 | 89 | 120 | 150 | 181 | 212 | 242 | 273 | 303 |
| March . . | 306 | 337 | 365 | 31 | 61 | 92 | 122 | 153 | 184 | 214 | 245 | 275 |
| April . . . | 275 | 306 | 334 | 365 | 30 | 61 | 91 | 122 | 153 | 183 | 214 | 244 |
| May . . . | 245 | 276 | 304 | 335 | 365 | 31 | 61 | 92 | 123 | 153 | 184 | 214 |
| June . . . | 214 | 245 | 273 | 304 | 334 | 365 | 30 | 61 | 92 | 122 | 153 | 183 |
| July . . . | 184 | 215 | 243 | 274 | 304 | 335 | 365 | 31 | 62 | 92 | 123 | 153 |
| August . . | 153 | 184 | 212 | 243 | 273 | 304 | 334 | 365 | 31 | 61 | 92 | 122 |
| September . | 122 | 153 | 181 | 212 | 242 | 273 | 303 | 334 | 365 | 30 | 61 | 91 |
| October . . | 92 | 123 | 151 | 182 | 212 | 243 | 273 | 304 | 335 | 365 | 31 | 61 |
| November . | 61 | 92 | 120 | 151 | 181 | 212 | 242 | 273 | 304 | 334 | 365 | 30 |
| December . | 31 | 62 | 90 | 121 | 151 | 182 | 212 | 243 | 274 | 304 | 335 | 365 |

### Review

1. In the following exercise, enter the time (in red ink [1]) on the upper of the two lines, and the interest (in black ink) on the lower line.

[1] At teacher's discretion.

If the time is less than a year, compute the actual number of days.

If the time is more than a year, find the interval by compound subtraction, and state it in years, months, and days.

| From | Rate 6% | | | |
|---|---|---|---|---|
| | $216.80 | $625.00 | $923.40 | $87.86 |
| | To | | | |
| | 9–29–1921 | 11–26–1921 | 12–24–1921 | 1–26–1922 |
| 9–22–1921 | | | | |
| 6–24–1921 | | | | |
| 4– 2–1921 | | | | |
| 2–11–1921 | | | | |
| 11–13–1920 | | | | |
| 8– 6–1920 | | | | |
| 5–22–1920 | | | | |
| 4–25–1919 | | | | |
| 7–13–1919 | | | | |

2. Rule and complete the following, entering the time on the upper of the two lines, and the interest on the lower.

| From | 5% | 4½% | 7% | 7½% |
|---|---|---|---|---|
| | To | | | |
| | May 26, 1921 | Aug. 17, 1921 | Oct. 9, 1921 | Feb. 10, 1922 |
| Principal, $219.80<br>April 22, 1920 | | | | |
| Principal, $326.00<br>May 17, 1920 | | | | |
| Principal, $80.00<br>September 19, 1919 | | | | |
| Principal, $260.00<br>May 14, 1919 | | | | |

### Written Review

Use the simplest method to find the interest on the following:

1. Interest at 6% on \$321.65 for 24 da.
2. Interest at 6% on \$523.19 for 20 da.
3. Interest at 6% on \$413.00 for 75 da.
4. Interest at 6% on \$300.00 for 57 da.
5. Interest at 5% on \$450.00 for 35 da.
6. Interest at $7\frac{1}{2}$% on \$216.80 for 42 da.
7. Interest at 4% on \$625.00 for 117 da.
8. Interest at 6% on \$80.00 for 3 yr. 8 mo. 15 da.
9. Interest at 6% on \$50.00 from March 24, 1922, to May 26, 1922.
10. Interest at 7% on \$125.00 from July 19, 1918, to January 14, 1922.

**191. Accurate Interest.** The methods of computing interest already explained are based on the assumption that a year contains 360 days. The slight error is ignored because of the convenience of the method. However, the United States government and some business houses prefer to use exact interest.

When we compute interest on a basis of 360 days to the year, we ignore 5 days. Five days are $\frac{5}{365}$ or $\frac{1}{73}$ of a year. When, for example, we call 180 days, $\frac{180}{360}$ or $\frac{1}{2}$ a year, we obtain a result which is $\frac{1}{73}$ too large. Since accurate interest is $\frac{1}{73}$ smaller than the interest computed by the simple methods, we can find accurate interest by the following rule:

**To find accurate interest.**

*Find the interest by one of the common methods; diminish this result by $\frac{1}{73}$ of itself.*

**Problem.** What is the accurate interest on \$3,500.00 for 15 days at 6%?

SOLUTION. \$35.00 is the interest for 60 days.
\$35.00 ÷ 4 = \$8.75, the interest for 15 days.
$\frac{1}{73}$ of \$8.75 = \$.12.
\$8.75 − \$.12 = \$8.63, the accurate interest.

### Written Work

Find the accurate interest on the following:

1. \$584.00 for 24 days at 6%.
2. \$275.00 for 84 days at 6%.

3. $23.75 for 113 days at 5%.
4. $65.80 for 2 months 12 days at 6%.
5. $138.50 for 9 months 16 days at 7%.

**192. To find the time.** Situations sometimes arise in which it is necessary to determine the time, when the principal, the rate, and the interest are known.

**Oral Work**

The interest on $250.00 for 6 days at 6%, is $.25. $1.00 is the interest for how many days at 6%?

The interest on $1,200 for 1 day at 6% is $.20. $4.40 is the interest for what length of time at 6%?

From these problems the following rule will be understood.

**To find the time when the principal, rate, and interest are known.**

*Find the interest on the given principal at the given rate for one day. Divide the total interest by the interest for one day to determine the number of days.*

**Problem.** The interest on $4,500.00 at 6% for a certain time is $10.50. What is the time?

SOLUTION. The interest on $4,500.00 for 1 day at 6% is $.75.
$10.50 ÷ $.75 = 14, the number of days.

**Written Work**

Find the time in each of the following:

| | PRINCIPAL | RATE | INTEREST | TIME |
|---|---|---|---|---|
| 1. | $ 360.00 | 6% | $ 1.62 | |
| 2. | $1,500.00 | 6% | $ 9.75 | |
| 3. | $1,473.00 | 6% | $22.59 | |
| 4. | $ 279.80 | 6% | $ 1.91 | |
| 5. | $ 176.90 | 5% | $ 2.82 | |

**193. To find the principal.**

The interest on $1.00 for 72 days at 6% is $.012. $.24 is the interest on how many dollars for 72 days at 6%?

**To find the principal when the rate, the time, and the interest are known.**

*Find the interest on* $1.00 *for the given time at the given rate. Divide the given interest by the interest on* $1.00 *to determine the principal.*

**Problem.** The interest on a certain principal at 6% for 54 days is $2.48. What is the principal?

SOLUTION. The interest on $1.00 for 54 days at 6% is $.009.
$2.48 ÷ $.009 = 275.56, the number of dollars in the principal.

**Written Work**

Find the principal in each of the following.

| | TIME | RATE | INTEREST | PRINCIPAL |
|---|---|---|---|---|
| 1. | 75 days | 6% | $ 3.36 | |
| 2. | 35 days | 6% | $ 1.67 | |
| 3. | 92 days | 5% | $ .52 | |
| 4. | 72 days | 6% | $21.60 | |
| 5. | 90 days | 5% | $45.00 | |
| 6. | 60 days | 5% | $ 1.00 | |
| 7. | 30 days | 7% | $ 1.40 | |
| 8. | 90 days | 6% | $13.50 | |

**194. To find the rate when the principal, the interest, and the time are given.**

*Find the interest on the given principal for the given time at 1%. Divide the total interest by the interest at 1% to find the rate.*

**Problem.** The interest on $1,500.00 for 30 days at a certain rate is $6.25. What is the rate?

SOLUTION. The interest on $1,500.00 for 30 days at 1% is $1.25.
$6.25 ÷ $1.25 = 5, the rate per cent

**Written Work**

Find the rate in each of the following:

| | PRINCIPAL | TIME | INTEREST | RATE |
|---|---|---|---|---|
| 1. | $1,375.00 | 42 days | $ 9.63 | |
| 2. | $ 876.25 | 95 days | $11.56 | |
| 3. | $ 694.75 | 27 days | $ 3.65 | |
| 4. | $ 940.00 | 2 yr. 6 mo. | $94.00 | |

**195. Periodic Interest.** Periodic interest is the simple interest on the principal, plus the simple interest on each installment of interest which was not paid when due. The interest on the principal may be due at any stated interval, usually quarterly, semi-annually, or annually.

Periodic interest is not legal in some states. Special contracts are sometimes made which provide for its payment.

**Problem.** What is the periodic interest on a loan of $5,000.00 at 6% for 1 yr. 6 mo., interest due quarterly?

SOLUTION. The simple interest on $5,000.00 for 1 yr. 6 mo. at 6% is $450.00.

| | |
|---|---|
| The first interest payment of $75 is due at the end of 3 mo., and remains unpaid for | 1 yr. 3 mo. |
| The second interest payment is due in 6 mo. and is unpaid for | 1 yr. |
| The third interest payment is due in 9 mo. and is unpaid for | 9 mo. |
| Fourth interest payment is unpaid for | 6 mo. |
| Fifth interest payment is unpaid for | 3 mo. |
| The sixth interest payment is due at the maturity of the note, and is paid when due. | |
| The total time during which interest payments remain unpaid is | 3 yr. 9 mo. |

The interest on $75.00 for 3 yr. 9 mo. at 6% is $16.88

Simple interest on the principal = $450.00

Interest on unpaid interest = 16.88

Periodic interest = $466.88

The amount due at the end of 1 yr. 6 mo. (principal and interest) is $5,466.88.

### Written Work

Complete a form ruled like the following model. The results in the illustration are entered in the first column to illustrate how to fill in the various blanks.

| | Principal $5,000 Rate 6% Time 1 Yr. 6 Mo. Int. Due Quarterly | Principal $2,000 Rate 6% Time 2 Yr. 3 Mo. Int. Due Quarterly | Principal $1,200 Rate 6% Time 4 Yr. Int. Due Semi-annually | Principal $1,700 Rate 6% Time 6 Yr. 6 Mo. Int. Due Semi-annually | Principal $6,500 Rate 7% Time 1 Yr. 9 Mo. Int. Due Quarterly | Principal $1,500 Rate 5% Time 2 Yr. Int. Due Annually | Principal $3,000 Rate 8% Time 4 Yr. Int. Due Quarterly |
|---|---|---|---|---|---|---|---|
| Simple interest . . | $450.00 | | | | | | |
| Amount of each interest payment . | 75.00 | | | | | | |
| Sum of periods interest is unpaid . | 3 yr. 9 mo. | | | | | | |
| Interest on unpaid interest . . . . | $16.88 | | | | | | |
| Total interest . . | 466.88 | | | | | | |
| Amount of note at maturity . . . | 5,466.88 | | | | | | |

# CHAPTER XXII

## COMPOUND INTEREST AND ITS APPLICATIONS

**196. Compound Interest** is based on the assumption that the simple interest earned on an investment is added to the investment so that it thereafter earns interest at the same rate as the original investment. Interest may be compounded (that is, added to the investment) annually, semi-annually, quarterly, or at the end of any other period.

**Illustration.** What is the compound interest on $1,200.00 at 6% for two years, compounded semi-annually?

SOLUTION. This is equivalent to a rate of 3% per period of 6 months, and there are four semi-annual periods.

| | |
|---|---|
| Original principal | $1,200.00 |
| Add interest on $1,200.00 @ 3% | 36.00 |
| Amount at end of first period | 1,236.00 |
| Add interest on $1,236.00 @ 3% | 37.08 |
| Amount at end of second period | 1,273.08 |
| Add interest on $1,273.08 @ 3% | 38.19 |
| Amount at end of third period | 1,311.27 |
| Add interest on $1,311.27 @ 3% | 39.34 |
| Amount at end of fourth period | 1,350.61 |
| Deduct original principal | 1,200.00 |
| Compound interest | 150.61 |

**197. Compound Interest Tables.** Compound interest is usually computed with the aid of tables similar to the following:

**Illustration.** Using the table on page 231, compute the compound interest on $1,200.00 at 6% for two years, compounded semi-annually.

SOLUTION.

| | |
|---|---|
| The table shows the amount of $1.00 for four periods at 3% to be | $1.125509 |
| Deduct | 1.000000 |
| Compound interest on $1.00 | .125509 |
| Multiply by | 1200.00 |
| Compound interest on $1,200.00 | $ 150.61 |

## Compound Interest Table

Amounts of $1.00 compounded at various rates

| Periods | 1% | 1½% | 2% | 2½% | 3% | 3½% | 4% | 4½% | 5% | 6% | Periods |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.010000 | 1.005000 | 1.020000 | 1.025000 | 1.030000 | 1.035000 | 1.040000 | 1.045000 | 1.050000 | 1.060000 | 1 |
| 2 | 1.020100 | 1.030225 | 1.040400 | 1.050625 | 1.060900 | 1.071225 | 1.081600 | 1.092025 | 1.102500 | 1.123600 | 2 |
| 3 | 1.030301 | 1.045678 | 1.061208 | 1.076891 | 1.092727 | 1.108718 | 1.124864 | 1.141166 | 1.157625 | 1.191016 | 3 |
| 4 | 1.040604 | 1.061364 | 1.082432 | 1.103813 | 1.125509 | 1.147523 | 1.169859 | 1.192519 | 1.215506 | 1.262477 | 4 |
| 5 | 1.051010 | 1.077284 | 1.104081 | 1.131408 | 1.159274 | 1.187686 | 1.216653 | 1.246182 | 1.276282 | 1.338226 | 5 |
| 6 | 1.061520 | 1.093443 | 1.126162 | 1.159693 | 1.194052 | 1.229255 | 1.265319 | 1.302260 | 1.340096 | 1.418519 | 6 |
| 7 | 1.072135 | 1.109845 | 1.148686 | 1.188686 | 1.229874 | 1.272279 | 1.316932 | 1.360862 | 1.407100 | 1.503630 | 7 |
| 8 | 1.082857 | 1.126493 | 1.171659 | 1.218403 | 1.266770 | 1.316809 | 1.368569 | 1.422101 | 1.477455 | 1.593848 | 8 |
| 9 | 1.093685 | 1.143390 | 1.195093 | 1.248863 | 1.304773 | 1.362897 | 1.423312 | 1.486095 | 1.551328 | 1.689479 | 9 |
| 10 | 1.104622 | 1.160541 | 1.218994 | 1.280085 | 1.343916 | 1.410599 | 1.480244 | 1.552969 | 1.628895 | 1.790848 | 10 |
| 11 | 1.115668 | 1.177949 | 1.243374 | 1.312087 | 1.384234 | 1.459970 | 1.539454 | 1.622853 | 1.710339 | 1.898299 | 11 |
| 12 | 1.126825 | 1.195618 | 1.268242 | 1.344889 | 1.425761 | 1.511069 | 1.601032 | 1.695881 | 1.795856 | 2.012197 | 12 |
| 13 | 1.138093 | 1.213552 | 1.293607 | 1.378511 | 1.468534 | 1.563956 | 1.665074 | 1.772196 | 1.885649 | 2.132928 | 13 |
| 14 | 1.149474 | 1.231756 | 1.319479 | 1.412974 | 1.512590 | 1.618695 | 1.731676 | 1.851945 | 1.979932 | 2.260904 | 14 |
| 15 | 1.160969 | 1.250232 | 1.345858 | 1.448298 | 1.557967 | 1.675349 | 1.800944 | 1.935282 | 2.078928 | 2.396558 | 15 |
| 16 | 1.172579 | 1.268986 | 1.372786 | 1.484506 | 1.604706 | 1.733986 | 1.872981 | 2.022370 | 2.182875 | 2.540352 | 16 |
| 17 | 1.184304 | 1.288020 | 1.400241 | 1.521618 | 1.652848 | 1.794676 | 1.947901 | 2.113377 | 2.292018 | 2.692773 | 17 |
| 18 | 1.196148 | 1.307341 | 1.428246 | 1.559659 | 1.702433 | 1.857489 | 2.025817 | 2.208479 | 2.406619 | 2.854339 | 18 |
| 19 | 1.208109 | 1.326951 | 1.456811 | 1.597650 | 1.753506 | 1.922501 | 2.106849 | 2.307860 | 2.526950 | 3.025600 | 19 |
| 20 | 1.220190 | 1.346855 | 1.485947 | 1.638616 | 1.806111 | 1.989789 | 2.191123 | 2.411714 | 2.653298 | 3.207136 | 20 |
| 21 | 1.232392 | 1.367058 | 1.515666 | 1.679582 | 1.860295 | 2.059431 | 2.278768 | 2.520241 | 2.785963 | 3.399564 | 21 |
| 22 | 1.214716 | 1.387564 | 1.545980 | 1.721571 | 1.916103 | 2.131512 | 2.369919 | 2.633652 | 2.925261 | 3.603537 | 22 |
| 23 | 1.257163 | 1.408377 | 1.576899 | 1.764611 | 1.973587 | 2.206114 | 2.464716 | 2.752166 | 3.071524 | 3.819750 | 23 |
| 24 | 1.269735 | 1.429503 | 1.608437 | 1.808726 | 1.032794 | 2.283328 | 2.563304 | 2.876014 | 3.225100 | 4.048935 | 24 |
| 25 | 1.282432 | 1.450945 | 1.640606 | 1.853944 | 1.093778 | 2.363245 | 2.665836 | 3.005434 | 3.386355 | 4.291871 | 25 |

**198. Present Value of $1.00.** The present value of $1.00 is a sum smaller than $1.00 which, invested at compound interest, will amount to $1.00 at the end of a certain number of periods.

The present value of $1.00 is computed by dividing $1.00 by the amount of $1.00 at the given rate for the given number of periods, as shown in the table.

**Illustration.** What is the present value of $1.00 due in two years, interest at 6% per annum compounded semi-annually?

SOLUTION. The amount of $1.00 for 4 periods at 3% (see table) is $1.125509.
Then $1.00 ÷ 1.125509 = $.888487 Present value of $1.00

**Problem.** What is the present value of $1,500.00 due in two years, interest at 6% per annum, compounded semi-annually?

SOLUTION.

| | |
|---|---|
| Present value of $1.00 (as above) | $ .888487 |
| Multiply by | 1500 |
| Present value of $1,500.00 | $1332.73 |
| | $1332.72 |

The accumulation of interest by which $1,322.72 increases to $1,500.00 may be tabulated as follows:

TABLE OF ACCUMULATION

| | |
|---|---|
| Original investment | $1,332.73 |
| Add interest on $1,332.73 @ 3% | 39.98 |
| Total, end of first period | 1,372.71 |
| Add interest on $1,372.71 @ 3% | 41.18 |
| Total, end of second period | 1,413.89 |
| Add interest on $1,413.89 @ 3% | 42.42 |
| Total, end of third period | 1,456.31 |
| Add interest on $1,456.31 @ 3% | 43.69 |
| Total, end of fourth period | 1,500.00 |

**199. Compound Discount.** The compound discount on $1.00 is the difference between $1.00 and the present value of $1.00.

**Illustration.** What is the compound discount on $1.00 due in two years, interest at 6% per annum, compounded semi-annually?

SOLUTION.

| | |
|---|---|
| | 1.000000 |
| Deduct present value of $1.00 (as above) | .888487 |
| Compound discount | .111513 |

**Written Work**

**1.** State the total amount accumulated at the end of five years, and the compound interest earned, on \$5,000.00 invested at

(*a*) 6% per annum, compounded annually.
(*b*) 6% per annum, compounded semi-annually.
(*c*) 6% per annum, compounded quarterly.

**2.** What is the present value of \$5,000.00 due at the end of five years,

(*a*) at 6% per annum, compounded annually?
(*b*) at 6% per annum, compounded semi-annually?
(*c*) at 6% per annum, compounded quarterly?

# CHAPTER XXIII

## SAVINGS BANKS

**200. Checking and Savings Accounts.** Most banks receive deposits under two classes of accounts, checking accounts and savings accounts.

Money deposited in checking accounts can be drawn out by checks payable to the depositor or to any other party. These checks are payable on presentation at the bank. The cash on deposit usually does not bear interest.

Money deposited in a savings account cannot be drawn out by a check payable to any person other than the depositor himself. The law usually provides that the bank may require notice of from 10 to 60 days before paying money from a savings account. Banks rarely take advantage of this privilege, however. The cash on deposit bears interest at some rate fixed by the bank. 3%, 3½%, and 4% are common rates.

**201. Computing Interest on Savings Accounts.** There is no uniform procedure for computing interest on deposits in savings accounts, as each bank adopts its own rules. The following are the rules of a Chicago bank:

"On the first days of January and July in each year, interest shall accrue at the rate of three per cent per annum on all savings deposits then in the Bank, and which have remained on deposit for one month or more; but no interest shall be payable for periods less than a month (except as herein otherwise provided), nor upon sums less than one dollar, nor for any part of the half year included between the first days of January and July, or July and January, as the case may be, on sums withdrawn between those periods.

"Interest shall not be paid on average balances; and all withdrawals between the interest days shall be deducted from the first deposits.

"On deposits made during the first ten days of any calendar month, interest will be allowed from the first day of that month. On all deposits made after the first ten days of any calendar month, interest will be allowed from the first day of the succeeding month."

**Problem.** Following is a transcript of the pass book showing the savings account of W. M. Scott.

W. M. Scott.....*Book No.* 35836

In Account with *The Snowden Trust and Savings Bank* of Chicago.

| Date | | Withdrawals | Deposits | Balance |
|---|---|---|---|---|
| Jan. 3, 1917 | | | $100.00 | $100.00 |
| July 1, 1917 | Interest | | 1.50 | 101.50 |
| July 10, 1917 | | | 50.00 | 151.50 |
| Sept. 8, 1917 | | | 40.00 | 191.50 |
| Dec. 15, 1917 | | | 60.00 | 251.50 |
| Jan. 1, 1918 | Interest | | 2.66 | 254.16 |
| Feb. 8, 1918 | | $75.00 | | 179.16 |
| July 1, 1918 | Interest | | 2.68 | 181.84 |
| Aug. 5, 1918 | | | 60.00 | 241.84 |
| Sept. 19, 1918 | | | 70.00 | 311.84 |
| Dec. 21, 1918 | | 85.00 | | 226.84 |
| Jan. 1, 1919 | Interest | | 2.71 | 229.55 |
| Feb. 3, 1919 | | | 50.00 | 279.55 |
| Mar. 6, 1919 | | | 100.00 | 379.55 |
| May 19, 1919 | | 125.00 | | 254.55 |
| June 24, 1919 | | | 167.91 | 422.46 |
| July 1, 1919 | Interest | | 3.18 | 425.64 |
| July 20, 1919 | | 75.64 | | 350.00 |
| July 24, 1919 | | 100.00 | | 250.00 |
| July 31, 1919 | | 100.00 | | 150.00 |
| Aug. 29, 1919 | | 100.00 | | 50.00 |
| Sept. 6, 1919 | | 49.00 | | 1.00 |
| Oct. 16, 1919 | | | 301.75 | 302.75 |
| Oct. 18, 1919 | | 50.00 | | 252.75 |
| Oct. 25, 1919 | | 156.00 | | 96.75 |
| Dec. 14, 1919 | | | 48.00 | 144.75 |
| Dec. 26, 1919 | | | 105.25 | 250.00 |
| Jan. 2, 1920 | | | 100.00 | 350.00 |
| Jan. 7, 1920 | | | 75.00 | 425.00 |
| Jan. 1, 1920 | Interest | | .48 | 425.48 |

Explanation of Interest Computations

*July 1, 1917.* Since the $100.00 deposit was made during the first ten days of January, it bears interest from the first of January.

Interest on $100.00 for 6 months . . . . . $1.50

*January 1, 1918.* The $50.00 deposit of July 10th bears interest from July 1st, making the balance as of that date $151.50.

| | |
|---|---|
| Interest on $151.00 for 6 months | 2.26 |

The September 8th deposit bears interest from September 1st.

| | |
|---|---|
| Interest on $40.00 for 4 months | .40 |

The December 15th deposit does not begin to bear interest until January 1st.

| | |
|---|---|
| Total | $2.66 |

*July 1, 1918.* As all withdrawals are deducted from the first deposits, the $75.00 withdrawal at February 8th is deducted from the January 1st balance, reducing the balance as of that date to $179.16.

| | |
|---|---|
| Interest on $179.00 for 6 months | 2.68 |

*January 1, 1919.* The $85.00 withdrawal on December 21st reduces the balance as of July 1st to $96.84.

| | |
|---|---|
| Interest on $96.00 for 6 months | 1.44 |
| Interest on 60.00 for for 5 months | .75 |
| Interest on 70.00 for 3 months | .52 |
| Total | $2.71 |

*July 1, 1919.* The $125.00 withdrawal on May 19th reduces the balance as of January 1st to $104.55.

| | |
|---|---|
| Interest on $104.00 for 6 months | 1.56 |
| Interest on $50.00 for 5 months | .62 |
| Interest on $100.00 for 4 months | 1.00 |
| Total | $3.18 |

*January 1, 1920.* The withdrawals during the six months' period total $630.64. These exhaust the $425.64 balance at July 1st, and also cause a deduction of $205.00 from the deposit of October 16th. $301.75 minus $205.00 equals $96.75. Since this deposit was made after the first ten days of October, it does not begin to draw interest until November 1st.

| | |
|---|---|
| Interest on $96.00 for 2 months | $0.48 |

The December 14th and December 26th deposits do not begin to bear interest until January 1.

It will be noted that the January 1st interest is entered in the pass book after January 7th. This may seem incorrect, but it is caused by the operation of the following rule of the bank:

"Interest will be credited on the books of the Bank on the first days of January and July, but will not be entered upon the depositor's pass book nor be subject to withdrawal until on or after the tenth day following such credit."

### Written Work

1. Following is a statement of the deposits and withdrawals of J. O. Dorman in the First Trust and Savings Bank, which pays 4% interest according to the rules quoted above:

| Date | Withdrawals | Deposits |
|---|---|---|
| January 6, 1920 | | $200.00 |
| July 10, 1920 | | 100.00 |
| August 25, 1920 | | 50.00 |
| December 17, 1920 | | 75.00 |
| January 6, 1921 | | 40.00 |
| February 3, 1921 | $125.00 | |
| March 7, 1921 | | 105.50 |
| April 12, 1921 | | 60.00 |
| July 5, 1921 | 200.00 | |
| September 7, 1921 | | 50.00 |
| December 10, 1921 | 75.00 | |
| January 16, 1922 | | 500.00 |
| March 30, 1922 | 375.00 | |
| May 3, 1922 | | 105.45 |
| May 13, 1922 | | 155.20 |

Write up his pass book showing interest credits.

**2.** Following is a continuation of the pass book of W. M. Scott (see example). Compute interest credits and show balances.

| Date | Withdrawals | Deposits |
|---|---|---|
| February 1, 1920 | | $118.75 |
| February 7, 1920 | | 40.00 |
| February 28, 1920 | $300.00 | |
| March 13, 1920 | 84.23 | |
| April 2, 1920 | | 158.75 |
| April 8, 1920 | | 100.59 |
| April 12, 1920 | 25.00 | |
| April 24, 1920 | 433.00 | |
| May 14, 1920 | | 101.25 |
| May 17, 1920 | | 118.75 |
| May 19, 1920 | 50.00 | |
| May 29, 1920 | 25.00 | |
| May 31, 1920 | 25.00 | |
| June 10, 1920 | 75.00 | |
| July 11, 1920 | | 95.50 |
| July 17, 1920 | | 100.00 |
| August 2, 1920 | | 118.75 |

| | | |
|---|---|---|
| September 2, 1920 | | 499.85 |
| September 6, 1920 | 25.00 | |
| September 11, 1920 | 25.00 | |
| September 13, 1920 | 25.00 | |
| October 4, 1920 | | 114.45 |
| October 14, 1920 | 800.00 | |
| March 16, 1921 | 75.00 | |
| June 2, 1921 | | 150.00 |
| July 10, 1921 | | 498.87 |
| August 30, 1921 | 200.00 | |
| September 10, 1921 | 225.00 | |
| September 16, 1921 | 75.00 | |

No further deposits or withdrawals were made until July 10, 1922, when the entire balance was withdrawn.

**202. Postal Savings Banks.** The United States Government has provided a savings bank system operated in conjunction with the postal service, whereby savings may be deposited at interest with the security of the United States Government.

**Deposits.** Any person ten years of age or over may become a depositor. Sums less than $1.00 cannot be deposited. No person can deposit more than $100.00 in any one calendar month, or have a balance at any time of more than $500.00, exclusive of interest. Depositors receive a postal savings certificate for the amount of each deposit. Interest is allowed on these certificates at the rate of two per cent for each *full year* that the money remains on deposit, *beginning with the first day of the month following the one in which it was deposited.*

**Withdrawals.** Money may be withdrawn by surrendering to the officer where the deposit was made the savings certificates covering the desired amount.

**Bonds.** Under certain conditions a depositor may surrender certificates in amounts of $20.00 or any multiple of $20.00, up to and including $500.00 and receive in return government bonds bearing interest at 2½% per year. These postal savings bonds may be held in addition to the $500.00 deposit allowed to one depositor.

**Written Work**

1. How much interest would a man receive if he deposited $15.00 in a postal savings bank on March 1, 1921, and withdrew it three months later?

2. How much interest would he receive if he withdrew the money on March 11, 1922?

3. A man made the following deposits in a postal savings bank:

| | |
|---|---|
| May 20, 1922 | $15.00 |
| June 17, 1922 | 6.00 |
| August 11, 1922 | 26.00 |
| October 30, 1922 | 17.00 |

When would he be entitled to the full year's interest on each of his deposits, and how much interest would he receive?

# CHAPTER XXIV

## DISCOUNTING NOTES AND OTHER COMMERCIAL PAPER

**203. Borrowing from a Bank.** Banks prefer to have the borrower pay interest in advance. The interest may be paid in cash, or the bank may subtract it from the face of the note, giving the borrower the difference, which is called the **proceeds.**

If you gave a bank the following note:

$100.00 CHICAGO, ILL., March, 2, 1922.

Ninety days after date, I promise to pay to the order of the Bowen National Bank One Hundred Dollars.

.....

Your Signature

the bank would deduct ninety days' interest, $1.50, and you would receive the proceeds, $98.50.

NOTE. There is no promise to pay interest, because the interest is prepaid. Banks may charge any rate they choose, within the limits set by the state law. In this text, when no rate is mentioned use 6 %.

The above illustration shows the method of **discounting a note.**

**204. Terms Used in Bank Discount.** The following terms are used frequently, and you should become familiar with them:

**Maturity** means the date on which the note is due.

The **value** of the note is the amount due at maturity.

If the note draws interest, the *value* is the sum of the principal and the interest. Otherwise, the *value* is the face of the note.

The **discount period** is the exact number of days between the date of discounting the paper and its maturity.

The **bank discount** is the simple interest on the value for the discount period.

The difference between the value and the bank discount is called the *proceeds.*

If the proceeds of a discounted note exceed the face, the excess is interest; if the proceeds are less than the face, the difference is discount.

### Oral Work

In the illustration on the previous page what are each of the following:

The value? The maturity? The discount period?
The bank discount? The proceeds?

**205. To find the bank discount and the proceeds.**

*To compute the bank discount, compute the simple interest on the value of the note for the discount period.*

*To find the proceeds, subtract the bank discount from the value of the note.*

### Written Work

How much would you receive from the bank if you discounted the following notes?

**1.** A note for $500, without interest, due in 30 days, discounted at 6%.

**2.** A note for $725, without interest, due in two months, discounted at 6%.

**3.** A note for $85.50, without interest, due in 90 days, discounted at 7%.

**4.** The following blank shows the value of several notes and the periods for which they were discounted. Compute the bank discount at 6%, and find the proceeds.

| Note Due in | | Value | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $87.00 | $235.00 | $962.50 | $1,000.00 | $275.00 | $765.00 | $35.00 |
| 30 day | Bank discount | .44 | | | | | | |
| | Proceeds | 86.56 | | | | | | |
| 60 days | Bank discount | | | | | | | |
| | Proceeds | | | | | | | |
| 80 days | Bank discount | | | | | | | |
| | Proceeds | | | | | | | |
| 90 days | Bank discount | | | | | | | |
| | Proceeds | | | | | | | |
| 120 days | Bank discount | | | | | | | |
| | Proceeds | | | | | | | |

**206. Discounting the Paper of Other Persons.** If you have a note signed by some person of good financial standing, you can borrow money on it by discounting it in the same way that you would discount a note made by yourself.

Suppose John Doe had given you the following note:

$500.00 CHICAGO, ILL., January 18, 1922.

Ninety days after date I promise to pay to the order of (Your Name)
Five Hundred Dollars.
JOHN DOE.

By the following indorsement,

Pay to the order of
The First National Bank
(Your Name)

you transfer the note to the bank. Moreover you promise to pay the bank $500 on April 18, in case Doe fails to do so. The bank will loan you $500 minus the discount.

When you discounted your own note (in the exercise just given), you did so on the same day that the note was made. The discount period and the time of the note were therefore the same. But you may keep the notes of other people some time before discounting them. The bank will charge you interest from the day you discount a note until it is due.

Thus, if you had not discounted the above note until March 6, the bank would have charged you interest for the number of days from March 6 to April 18, the date of maturity. How many days in this discount period?

What would be the discount period if you discounted the note January 24? February 16? March 21?

**207. To find the discount period.**

*Find the maturity of the note.*

*Find the exact number of days between the date of discount and the date of maturity.*

**208. To find the maturity.** Notes due a certain number of *months* after date fall due on the same day of the month as the day on which they were made, with the following exceptions:

Notes made on the 31st, maturing after a specified number of months, falling due in a month having only 30 days, mature on the 30th.

Notes made on the 28th, 29th, 30th, or 31st, maturing after a specified number of months, falling due in February, mature on the last day of February.

**Illustrations.**

| Note Made | Due in | Maturity |
|---|---|---|
| Jan. 17, 1922 | 2 months | March 17, 1922 |
| August 31, 1922 | 3 months | November 30, 1922 |
| Dec. 31, 1921 | 2 months | February 28, 1922 |

Notes due a certain number of *days* after date, mature after the exact number of days has elapsed.

**Illustrations.**

| | Note Made | Due in | Maturity |
|---|---|---|---|
| | 1921 | | |
| *a.* | Feb. 16 | 30 days | March 18 |

Since February has only 28 days, the 30 days of the note will be divided as follows:

12 February days (after the 16th)
18 March days

*b.* March 18 — 30 days — April 17

March has 31 days; 13 left after the 18th; the remaining 17 of the 30 days must be in April.

*c.* April 17 — 30 days — May 17

April has 30 days; 13 left after the 17th; the remaining 17 of the 30 days will be in May.

From these illustrations, the following should be clear:

**To find the maturity:**

*Change the days to months;* (30 *days* = 1 *month*).

*Assume that the note will mature on the same day of some following month.*

*Correct this result by subtracting* 1 *day for each month of* 31 *days through which the note runs; and by adding* 2 *days if the note runs through February.* (*In case of leap year, add* 1 *day.*)

The following table provides a convenient method for indicating the months for which corrections are to be made.

| Month | Number of Month | | Days | Correction |
|---|---|---|---|---|
| January | 1 | 13 | 31 | − 1 |
| February | 2 | 14 | 28 | + 2 (or + 1) |
| March | 3 | 15 | 31 | − 1 |
| April | 4 | 16 | 30 | |
| May | 5 | 17 | 31 | − 1 |
| June | 6 | 18 | 30 | |
| July | 7 | 19 | 31 | − 1 |
| August | 8 | 20 | 31 | − 1 |
| September | 9 | 21 | 30 | |
| October | 10 | 22 | 31 | − 1 |
| November | 11 | 23 | 30 | |
| December | 12 | 24 | 31 | − 1 |

**Problems.** 1. Find the maturity of a note dated May 16, 1922, due in 4 months.

Solution. May is the fifth month. Add 4 months. The note will be due in the ninth month, which by the table is September.

Maturity, September 16.

2. Find the maturity of a note of the same date, due in 120 days.

Solution. Call 120 days 4 months.

May is the fifth month. The note is due in the ninth month, shown by the table to be September.

| Call the maturity September 16. | Correct as follows: |
|---|---|
| Note runs through May | Subtract 1 day |
| Note runs through July | Subtract 1 day |
| Note runs through August | Subtract 1 day |
| Total | 3 days |

Therefore the maturity is September 13.

3. Find the maturity of a note dated December 19, 1921, due in 150 days.

Solution. Call 150 days 5 months.

December is the twelfth month. The note is due in the seventeenth month, which, by the table, is May.

Call the maturity May 19, 1922.

Correct as follows:

| | |
|---|---|
| December | − 1 |
| January | − 1 |
| February | + 2 |
| March | − 1 |
| April (has 30 da.) | |
| Note does not run through May. | |
| Total | − 1 |

Maturity, May 18, 1922.

4. Find the maturity of a note dated June 16, 1922, due in 70 days.

SOLUTION. Call 70 days 2 months, 10 days.
June 16 plus two months is August 16; plus 10 days is August 26.
To correct:

| | |
|---|---|
| For June | |
| For July | − 1 |
| Total | − 1 |

Maturity, August 25, 1922.

### Written Work

Enter the maturity of each note in the proper column:

| NOTE DATED | DUE IN 1 MONTH | DUE IN 30 DAYS | DUE IN 2 MONTHS | DUE IN 60 DAYS | DUE IN 150 DAYS | DUE IN 45 DAYS |
|---|---|---|---|---|---|---|
| Sept. 23, 1922 | | | | | | |
| Oct. 10, 1922 | | | | | | |
| May 2, 1922 | | | | | | |
| July 20, 1922 | | | | | | |
| Dec. 15, 1922 | | | | | | |
| Feb. 9, 1922 | | | | | | |

**209. To find the discount period.** *Compute the actual number of days between the date of discount and the date of maturity.*

**Illustration.** A note, dated July 17, due in 4 months, was discounted on August 5. What was the discount period?

SOLUTION. Four months from July 17 is November 17, the date of maturity. The discount period, therefore, extends from August 5, the date of discount, to November 17, the date of maturity.

To find the number of days between August 5 and November 17:

| | |
|---|---|
| 31 | total number of days in August |
| 5 | number of August days expired before discounting |
| 26 | number of August days in the discount period |
| 30 | number of September days in the discount period |
| 31 | number of October days in the discount period |
| 17 | number of November days in the discount period |
| 104 | total number of days in the discount period |

### Oral Work

After studying the illustration, state a method for finding the number of days in the discount period.

### Written Work

1. On a form ruled like the following, enter the date of maturity on the upper line, and the discount period on the lower line.

The notes dated February 14 were discounted April 8.
The notes dated February 17 were discounted March 2.
The notes dated March 23 were discounted April 29.
The notes dated March 18 were discounted April 15.
The notes dated March 30 were discounted May 20.

| Dates Notes were Made | Time | | | | |
|---|---|---|---|---|---|
| | 60 Days | 75 Days | 100 Days | 120 Days | 6 Months |
| Feb. 14, 1922 | | | | | |
| Feb. 17, 1922 | | | | | |
| Mar. 23, 1922 | | | | | |
| Mar. 18, 1922 | | | | | |
| Mar. 30, 1922 | | | | | |

2. Complete a table ruled like the following model. Enter the date of maturity on the first line, the discount period on the second line, the

discount on the third line, the proceeds on the fourth line, as illustrated by the first problem in the table.

| Date of Paper and Face | Time, 90 Days | Time, 3 Months | Time, 120 Days | Time, 6 Months |
|---|---|---|---|---|
| | Discounted April 13 | Discounted April 11 | Discounted April 24 | Discounted May 20 |
| Jan. 17, 1922 $300 | April 16 | | | |
| | 3 days | | | |
| | $.15 | | | |
| | $299.85 | | | |
| Jan. 31, 1922 $285 | | | | |
| | | | | |
| | | | | |
| | | | | |
| Feb. 20, 1922 $126 | | | | |
| | | | | |
| | | | | |
| | | | | |

**210. Discounting Interest-bearing Notes.** If you had in your possession the following note and wished to discount it, you would first add 90 days' interest ($1.50) to the face of the note to determine its *value* at maturity. This note is really Roe's promise to pay you $101.50 at maturity, and the bank will give you as much for it as for a note for $101.50 without interest.

$100.00 Chicago, January 6th 1922

Ninety days after date I promise to pay to the order of (Your name)

One hundred and 00/100 Dollars

at my office, with interest @ 6%

Value received

No. Due Richard Roe

When discounting interest-bearing paper, compute the interest for the *full time* the note is to run, and add this interest to the face, to de-

termine the *value* at maturity. Find the discount on this value. Subtract the discount from the *value* to determine the proceeds.

### Written Work

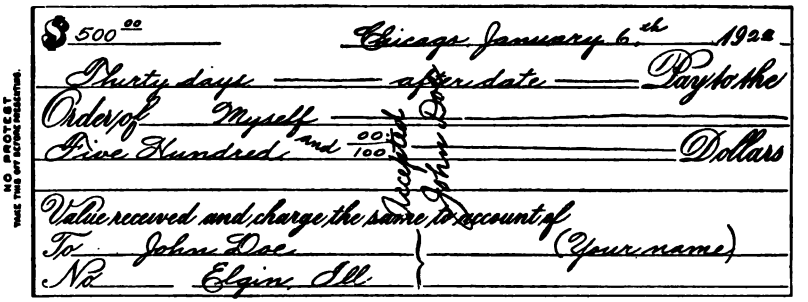
Complete a table ruled like the following model, entering the facts as illustrated in the first problem.

ll notes discounted at 6 %.

| | | Time, 90 Days | | Time, 4 Months | | Time, 6 Months | |
|---|---|---|---|---|---|---|---|
| | | Without Int. | With Int. 6% | Without Int. | With Int. 6% | Without Int. | With Int. 5% |
| Principal, $500.00 | Maturity | Sept. 1 | Sept. 1 | | | | |
| | Total Interest | | $ 7.50 | | | | |
| Date of Paper, June 3, 1921 | Value | $500.00 | $507.50 | | | | |
| | Dis. Period | 11 days | 11 days | | | | |
| Date of Discount Aug. 21, 1921 | Bank Dis. | $ .92 | $ .93 | | | | |
| | Proceeds | $499.08 | $506.57 | | | | |
| | Net Dis.* or Int. | .92* | 6.57 | | | | |
| Principal, $1,250.00 | Maturity | | | | | | |
| | Total Interest | | | | | | |
| Date of Paper, Aug. 6, 1921 | Value | | | | | | |
| | Dis. Period | | | | | | |
| Date of Discount Sept. 20, 1921 | Bank Dis. | | | | | | |
| | Proceeds | | | | | | |
| | Net Dis.* or Int. | | | | | | |
| Principal, $47.96 | Maturity | | | | | | |
| | Total Interest | | | | | | |
| Date of Paper, July 16, 1921 | Value | | | | | | |
| | Dis. Period | | | | | | |
| Date of Discount Aug. 27, 1921 | Bank Dis. | | | | | | |
| | Proceeds | | | | | | |
| | Net Dis.* or Int. | | | | | | |
| Principal, $55.00 | Maturity | | | | | | |
| | Total Interest | | | | | | |
| Date of Paper, Aug. 5, 1921 | Value | | | | | | |
| | Dis. Period | | | | | | |
| Date of Discount Oct. 5, 1921 | Bank Dis. | | | | | | |
| | Proceeds | | | | | | |
| | Net Dis.* or Int. | | | | | | |

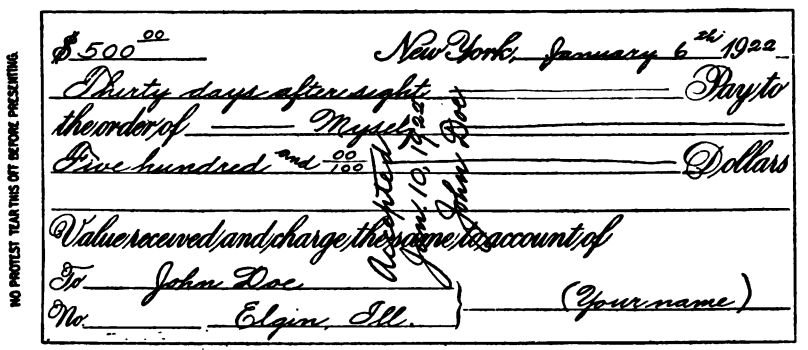
**211. Discounting Acceptances.** Time drafts were discussed in Chapter XVII. An accepted draft is a written promise of the acceptor to pay money at a definite date, and accepted drafts may be discounted, just as notes may be discounted. In case the person who has accepted the draft does not pay it when due, the person who discounted it will be required to pay it.

The following draft drawn by you on John Doe and accepted by him is just as truly his promise to pay $500.00 at a given date, as a promissory note signed by him would be.




This draft could be discounted in exactly the same manner as a note dated January 6, payable 30 days after date.

If the draft were drawn payable 30 days after sight, instead of after date, the bank discount would be different because the date of maturity would be different.


$500 00
New York, January 6th 1922
Thirty days after sight Pay to
the order of Myself
Five hundred and 00/100 Dollars
Value received and charge the same to account of
To John Doe
No Elgin, Ill
(Your name)
Accepted Jan 10 1922
John Doe
NO PROTEST TEAR THIS OFF BEFORE PRESENTING

**Written Work**

1. What is the date of maturity of the draft on this page payable 30 days after date?

2. What is the date of maturity of the draft on page 249 payable 30 days after sight?

3. What is the discount period of each draft if discounted on Jan. 14?

4. Find the bank discount and the proceeds of each draft.

### Review

Write the notes and drafts called for in the following. Show all acceptances and indorsements.

On October 3, you received from B. A. Anderson of Batavia, New York, a note dated October 1, due in two months, for $135.00, without interest.

On August 28, you sold to N. M. Davis, of Toledo, Ohio, an invoice of goods amounting to $84.00; terms 1/10; N/30. This invoice was due September 27, but Mr. Davis was unable to pay it. He agreed, however, to accept a thirty-day draft for the amount. You drew the draft on September 30, payable thirty days after date, and sent it to him for acceptance. He accepted it and returned it to you.

On October 5, you sold a bill of goods amounting to $68.00 to J. D. Robinson, of Burlington, Iowa. Terms, 30 day draft less 1%. On October 5, you drew a draft on him, payable 30 days after sight; he accepted the draft on October 7.

On July 10, you received from J. F. Cook of Lime Springs, Iowa, a note made by him on July 8, in your favor, for $500.00, due six months after date, with interest at 6%.

On June 4, you received a note for $600.00 from A. B. Hicks of Chicago. The note was drawn on April 16, by D. F. Fairchild of Omaha, Nebraska, in favor of Hicks, payable nine months after date, without interest, and was transferred to you by full indorsement.

On October 15, you took these notes and drafts to the Dairy State Bank to discount. The bank's discount rate was 6%. Finding that the proceeds would not be sufficient to meet your needs, you gave the bank your own note, dated October 15, due in 20 days, for $250.00, without interest.

1. Find the proceeds of each of the papers discounted.

Your balance at the bank before discounting this paper was $167.25. You deposited the proceeds of the discounted paper. What was your balance after discounting the paper?

**2.** On October 25, the bank wished to increase its deposit with its correspondent bank at Chicago. It therefore rediscounted the notes and drafts received from you. How much did the bank receive for the paper, when discounted on October 25 at 6%?

**3.** Marshall owed Daniels $485.90; on September 24 he gave Daniels the following note, properly indorsed.

$450.00 BRAINERD, IOWA, September 4, 1922

Sixty days after date, I promise to pay to the order of J. F. Marshall, Four Hundred and Fifty Dollars.
Value received
Without interest. F. S. CROSBY.

How much did Marshall owe Daniels after transferring the note?

### Written Review

**1.** Compute the interest on the following:

| | | | |
|---|---|---|---|
| $ 375.60 | for | 6 days | at 6% |
| 2,145.30 | for | 90 days | at 6% |
| 1,216.80 | for | 27 days | at 6% |
| 723.65 | for | 56 days | at 6% |
| 600.00 | for | 111 days | at 6% |
| 800.00 | for | 71 days | at 6% |
| 425.30 | for | 83 days | at 5% |
| 215.65 | for | 29 days | at 4½% |
| 90.00 | for | 7 months and 8 days | at 6% |
| 242.80 | for | 2 years, 4 months, and 9 days, | at 5% |

**2.** Compute the interest on $560.00 from May 1, 1919, to February 3, 1922, at 7%, finding the time by compound subtraction.

**3.** Compute accurate interest (on the basis of 365 days to the year) on $2,360.45 from January 3, 1921 to May 19, 1922, at 5%.

**4.** Find the balance due on April 7, 1922, on the following loan:

| | |
|---|---|
| $2,500.00 | borrowed January 6, 1921, at 6% interest |
| 265.00 | paid June 2, 1921 |
| 15.00 | paid December 4, 1921 |
| 745.00 | paid February 3, 1922 |

(*a*) Compute by United States Rule.
(*b*) Compute by Merchants' Rule.

5. A buys a piano for $850.00, paying $200.00 down and the balance in monthly installments of $25.00 plus long end interest at 6% per annum. How much is each of the first five payments? What is the balance of the debt after the fifth payment, ignoring accrued interest? How much interest is accrued and unpaid?

6. If the monthly payments are $25.00 each, long end interest being deducted and the balance applied on the principal, what is the balance of the debt after the fifth monthly payment?

7. If A pays $25.00 monthly plus short end interest, how much is each of the first five payments, and what is the balance of the debt after the fifth payment, ignoring accrued interest? How much interest is accrued and unpaid?

8. What is the discount at 6% on a $500.00 non-interest-bearing note dated May 3, due in 60 days, and discounted May 24? What are the proceeds?

9. What would be the proceeds if the note bore 5% interest?

# BUSINESS EXPENSES

## CHAPTER XXV

### WAGES AND PAY ROLLS

THERE are several wage systems in use, the most common of which will be discussed in this chapter.

**212. The Day or Hour Rate.** This system is the one generally used because it is the simplest. When the day or hour rate is used, the employee's wage depends upon the time he has worked. Work done overtime or on holidays is usually paid for at one and one-half times the regular rate. In many factories time clocks are installed. As each employee begins work, he registers the time on his card; when he stops work, he again registers the time. At the end of the week, each employee's card shows his actual hours of labor. A pay roll is prepared showing the total number of hours each employee has worked, his hourly rate, and his total wage.

#### Written Work

**1.** Supply the missing facts in the following pay roll.

| No. | Name | Hours worked per day | | | | | | Time | | | Overtime | | | Total Wages Earned | Money Advanced | Wages Due |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | M | T | W | T | F | S | Hrs. | Rate | Amount | Hrs. | Amount | | | | |
| 1 | Andrew White | 9 | 8 | 9 | 9 | 9 | 8 | 52 | .30 | 15 60 | | | | 15 60 | | 15 60 |
| 2 | August Piper | 9 | 9 | 11 | 9 | 12 | 9 | 54 | .28 | 15 12 | 5 | 2 10 | | 17 22 | | 17 22 |
| 3 | Edward Hazlett | 10 | 9 | 11 | 9 | 10 | 9 | 54 | .32 | ? | 4 | 1 92 | | ? | 4 — | ? |
| 4 | James Davis | 8 | 7 | 9 | 8 | 9 | 9 | ? | .29 | ? | | | | ? | | ? |
| 5 | Oscar Bennett | 9 | 9 | 10 | 10 | 11 | 9 | ? | .34 | ? | ? | ? | | ? | 1 20 | ? |
| 6 | James Belknap | 8 | 8 | 9 | 9 | 9 | 8 | ? | .32 | ? | | | | ? | | ? |
| 7 | Fred Calkins | 8 | 9 | 9 | 10 | 11 | 9 | ? | .30 | ? | ? | ? | | | 5 — | ? |
| 8 | Charles Davis | 9 | 6 | 10 | 11 | 9 | 10 | ? | .28 | ? | ? | ? | | ? | | ? |
| 9 | Harry McGee | 10 | 9 | 10½ | 9 | 10 | 11½ | ? | .25 | ? | ? | ? | | ? | 1 75 | ? |

In this factory a full day is 9 hours. Any over hours are paid for at $1\frac{1}{2}$ times the regular rate. Thus, Calkins gets $1\frac{1}{2}$ times regular pay

for the overtime on Thursday and Friday, even though he did not work full time on Monday.

**2.** Rule a pay roll similar to the model, and enter the following data, finding the amount of wages due each employee.

The superintendent supplies the cashier with the following list of the employees, together with their numbers, and the hourly wage rate paid each. A full day in this factory is 8 hours, and overtime is paid for at 1½ times the regular rate.

| No. | Employee's Name | Hourly Rate |
|---|---|---|
| 1. | Fred Smith | $.35 |
| 2. | Frank Bailey | .32 |
| 3. | Charles Hanchett | .30 |
| 4. | Harry Davis | .36 |
| 5. | Carl Bell | .38 |
| 6. | Lewis Clark | .40 |
| 7. | Arthur Helms | .29 |
| 8. | Tony Martin | .31 |
| 9. | Henry Dickinson | .26 |
| 10. | Nathan Wright | .37 |

During the week ending March 15, time tickets were turned in by the foreman, showing the hours worked by each employee.

Monday: 1, 9 (Workman No. 1, 9 hours); 2, 11; 3, 8; 4, 11; 5, 8; 6, 10; 7, 9; 8, 12; 9, 9; 10, 8.

Tuesday: 1, 9; 2, 11; 3, 9; 4, 9; 5, 10; 6, 9; 7, 9; 8, 9; 10, 9.

Wednesday: 1, 10; 2, 12; 3, 11; 4, 12; 5, 9; 6, 11; 7, 8; 8, 11; 9, 10; 10, 9.

Thursday: 1, 9; 2, 9; 3, 11; 4, 10; 5, 8; 6, 10; 7, 11; 8, 11; 9, 12; 10, 10.

Friday: 1, 5; 2, 9; 3, 11; 4, 10; 5, 9; 6, 9; 7, 9; 8, 9; 9, 10; 10, 8.

Saturday: 1, 9; 2, 11; 3, 11; 4, 10; 5, 11; 6, 9; 7, 12; 8, 9; 9, 4; 10, 9.

The cashier has already advanced Charles Hanchett $3.00; Carl Bell, $2.25; Tony Martin, $3.75; and Nathan Wright, $5.00.

**213. Piecework Wage System.** The piecework wage system is based on the theory that if the employee is paid for the actual amount of work he accomplishes, he will produce more in a day than if he is paid a straight hour or day wage.

The pay roll prepared under a piecework system is similar to that prepared under the hourly rate system, with the exception that overtime is usually paid for at the same rate that regular time receives, and the basis of the wage is the number of pieces produced and not the number of hours employed.

### Written Work

Find the missing facts in the following pay roll.

PIECEWORK PAY ROLL FOR WEEK ENDING JUNE 8, 192–

| No. | Name | Operation No. | Number Produced | | | | | | Total | Rate | Earned | | Advanced | | Due | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | M. | T. | W. | T. | F. | S. | | | | | | | | |
| 1 | John Sanford | 264 | 16 | 14 | 17 | 13 | 15 | 18 | 93 | .14 | 13 | 02 | 2 | | 11 | 02 |
| 2 | Irving Danner | 178 | 23 | 26 | 22 | 27 | 24 | 25 | ? | .12 | ? | | | | ? | |
| 3 | Byron Shepley | 361 | 37 | 33 | 34 | 39 | 36 | 30 | ? | .08 | ? | | | | ? | |
| 4 | Edward Magee | 422 | 28 | 27 | 29 | 28 | 26 | 27 | ? | .05 | ? | | 3 | 25 | ? | |
| 5 | Percival Conrad | 267 | 42 | 45 | 41 | 39 | 43 | 20 | ? | .07 | ? | | | | ? | |
| 6 | Ernest Anderson | 207 | 24 | 26 | 20 | 23 | 30 | 27 | ? | .09 | ? | | 2 | 60 | ? | |

**214. The Differential Rate.** The differential rate is a modification of the piecework system. A standard day's work is determined, and each employee is expected to produce this standard amount of work. If he produces the standard, he receives a certain rate per piece. If he produces less than the standard, he receives a lower rate per piece. If he produces more than the standard, he receives a higher rate per piece.

This system is based on the theory that the expenses for heat, light, rent, power, etc., remain about the same, whether the employees produce a small amount of product, or a large amount. By increasing the amount of output, these expenses are distributed over a larger quantity of manufactured goods, and the cost of making each article is thereby decreased. The saving effected by increasing the output is divided between the owner of the factory and the workmen who, by their skill or industry, increase the output. On the other hand, the employees who, by producing a small quantity, tend to increase the cost per article are paid at a lower rate.

**Problem.** A manufacturer learned by experience that the average workman in his factory could produce ten articles per day, and that

25 cents per article could be paid for the work. He then adopted a differential rate, as follows:

| Number of Pieces Produced | Rate per Piece |
|---|---|
| 7 | $.20 |
| 8 | .21 |
| 9 | .23 |
| 10, Standard | .25 |
| 11 | .26 |
| 12 | .27 |

Jones produced the standard quantity on Wednesday, and received $10 \times \$.25 = \$2.50$.

Smith produced 8 pieces, and received $8 \times \$.21 = \$1.68$.

Brown produced 12 pieces, and received $12 \times \$.27 = \$3.24$.

### Written Work

The following differential rate applies in a certain factory:

| Number of Pieces | Rate |
|---|---|
| 10 | $.12 |
| 11 | .13 |
| 12 | .14 |
| 13 | .15 |
| 14 | .17 |
| 15 | .19 |
| 16 | .21 |
| 17 | .22 |
| 18, Standard | .24 |
| 19 | .25 |
| 20 | .27 |
| 21 | .29 |
| 22 | .31 |
| 23 | .33 |
| 24 | .33½ |
| 25 | .34 |

The production sheet for the week ending September 17, 19— was as follows:

| No. | Name | Daily Production | | | | | |
|---|---|---|---|---|---|---|---|
| | | M. | T. | W. | T. | F. | S. |
| 1 | Edward Dye | 17 | 19 | 18 | 16 | 19 | 18 |
| 2 | Frank Bartlett | 14 | 15 | 14 | 16 | 15 | 14 |
| 3 | Arthur Bassett | 19 | 20 | 19 | 21 | 22 | 19 |
| 4 | James Hamilton | 22 | 21 | 21 | 23 | 20 | 21 |
| 5 | Den Slocum | 12 | 14 | 13 | 15 | 12 | 14 |
| 6 | John Gaylord | 13 | 16 | 15 | 16 | 15 | 17 |
| 7 | Sam Bayliss | 19 | 20 | 19 | 21 | 18 | 17 |
| 8 | George Bates | 19 | 22 | 23 | 20 | 19 | 18 |
| 9 | Wm. Osgood | 16 | 14 | 12 | 14 | 11 | 16 |

**1.** Devise a pay roll blank which will show all necessary facts, and enter each laborer's production and wages. See how good a pay roll form you can devise without asking for suggestions. It should be simple and convenient, and, while not being too elaborate, it should give all necessary information.

**2.** Find the average daily production of each workman. Prepare a graph, showing the standard production by a red line across the sheet, and the workmen's averages by a black curve.

## Commission

**215. Definitions.** An **agent** is a person who transacts business for another.

The **principal** is the person whose business is transacted by the agent.

Principals often pay their agents a commission for their services. The **commission** is usually a per cent of the money value involved in the transaction.

**Problem.** A wholesaler engaged a salesman, agreeing to pay him 8% of his gross sales as commission. How much did the salesman earn if he sold $3,425 worth of goods during a month?

Solution.

$3425.00 sales
.08 rate of commission
$ 274.00 commission

### Written Work

1. A real estate agent sold a farm belonging to Mr. Robinson for $20,450. The agent received 3% of the selling price as payment for his services. How much did the agent receive? How much did Mr. Robinson receive after paying his agent?

2. A manufacturer engaged Mr. Seeley as a traveling salesman. Mr. Seeley's contract stipulated that he should receive 7% of his gross sales as commission. During four weeks his sales were as follows:

$576.25; $498.49; $376.45; $723.50.

What was his commission each week?

3. Mr. Brooks, a merchant, engaged an attorney to collect a number of accounts. The attorney charged for his services 15% of the amounts collected. What did the attorney receive for collecting $354.75? What was the net amount received by Mr. Brooks?

4. James Hunter bought eggs for a Chicago produce merchant. For his labor in purchasing and hauling them to the station, he received $1\frac{1}{2}$% of the cost of the eggs. On Tuesday he bought 27 cases, of 30 dozen each, paying 29 cents per dozen for them. How much did Hunter pay for the eggs? How much was his commission? How much did the eggs cost the Chicago merchant, not including the transportation?

5. The salesmen of the Wickman Specialty Company receive the following commissions:

On goods sold in Department A, 9%.
On goods sold in Department B, 13%.
On goods sold in Department C, $12\frac{1}{2}$%.
On goods sold in Department D, 15%.

The following table shows the sales made by five salesmen.

| SALESMAN | DEPT. A | DEPT. B | DEPT. C | DEPT. D |
|---|---|---|---|---|
| C.G. PINES | 356.27 | 473.28 | 293.75 | 459.23 |
| F.L. SNOW | 523.74 | 625.90 | 483.37 | 824.84 |
| E.D. VICKS | 283.57 | 285.37 | 295.64 | 472.84 |
| L.S. BASSETT | 923.74 | 825.36 | 736.46 | 462.47 |
| FRANK HOLTZMAN | 735.84 | 375.47 | 375.47 | 462.85 |

Devise a form which will show the data and enter the following:

Each salesman's commission on goods sold in each department.
The total commissions paid to each salesman.
The total commissions paid for each department.
The total commissions paid to the five salesmen.
The total sales in each department.
The total sales of each salesman.
The total sales in all departments.

**216. Pay Roll Slips.** Laborers are sometimes paid by check and sometimes by the envelope system. When the latter method is used, each employee receives an envelope containing the exact amount of his wages. Each employee receives an envelope containing the exact amount of his wages.

**THE FIRST NATIONAL BANK**

CURRENCY FOR PAY ROLL

Depositor Hunter & Colby

| | DOLLARS | CENTS | |
|---|---|---|---|
| | | | $ 50 Bills, |
| | | | 20 |
| 12 | 120 | 00 | 10 |
| 4 | 20 | 00 | 5 |
| 5 | 10 | 00 | 2 |
| 4 | 4 | 00 | 1 |
| 3 | 1 | 50 | Halves, |
| 3 | | 75 | Quarters, |
| 7 | | 70 | Dimes, |
| 3 | | 15 | Nickels, |
| 10 | | 10 | Pennies, |
| | | | |
| | 157 | 20 | TOTAL |

PAY ROLL SLIP

The bills and coins required for the payroll are ascertained by preparing a coin sheet, showing the money needed for each envelope. The following coin sheet was drawn up to determine the money required for the payroll.

| No. | Name | Wages | | Bills | | | | $.50 | $.25 | $.10 | $.05 | $.01 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $10.00 | $5.00 | $2.00 | $1.00 | | | | | |
| 1 | James Downs | 22 | 67 | 2 | | 1 | | 1 | | 1 | 1 | 2 |
| 2 | Francis Enfield | 31 | 69 | 3 | | | 1 | 1 | | 1 | 1 | 4 |
| 3 | Arthur Hubbard | 26 | 38 | 2 | 1 | | 1 | | 1 | 1 | | 3 |
| 4 | Frank Munger | 29 | 35 | 2 | 1 | 2 | | | 1 | 1 | | |
| 5 | George Seiley | 28 | 96 | 2 | 1 | 1 | 1 | 1 | 1 | 2 | | 1 |
| 6 | Ezra Tolfax | 18 | 15 | 1 | 1 | 1 | 1 | | | 1 | 1 | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | 157 | 20 | 12 | 4 | 5 | 4 | 3 | 3 | 7 | 3 | 10 |

COIN SHEET

When the currency is delivered a check is made out to the bank for the amount of the pay roll.

### Written Work

Rule a coin sheet and enter the number of coins of each denomination necessary to pay each of the employees in the pay roll prepared in the differential wage rate problem on page 257.

Make out a pay roll slip on the First National Bank of your city, to obtain the money to pay these employees.

## CHAPTER XXVI

## POSTAGE, FREIGHT, AND EXPRESS RATES

### Postage

**217. The Rates for Domestic Postage.** Domestic mail matter includes mail addressed for local delivery, or for transmission from one place to another within the United States, or to or from or between the possessions of the United States, and to or from the United States postal agency at Shanghai, China. Domestic rates and conditions, with certain exceptions, apply to mail addressed to Canada, Cuba, Mexico, and the Republic of Panama.

### Classes of Domestic Mail

**First Class:** Written matter, letters and all mail which is sealed.

Rate: Letters and other first-class matter, 2 cents for each ounce or fraction thereof. Post cards and postal cards, 1 cent each.

Aeroplane mail, consisting of first-class matter, including sealed packages not exceeding 30 inches in length and girth combined, is chargeable with postage at the rate of 2 cents for each ounce or fraction thereof.

**Second Class:** Newspapers and periodicals bearing notice of entry as second-class matter.

Rate: When sent unsealed by others than the publisher or a news agent, 1 cent for each 4 ounces or fraction thereof.

**Third Class:** Circulars, newspapers and periodicals not admitted to the second class, nor embraced in the term "book." Books are included in fourth-class mail, as are also parcels of miscellaneous printed matter weighing more than 4 pounds.

Rate: 1 cent for each 2 ounces or fraction thereof. The limit of weight is 4 pounds. Parcels of printed matter

weighing more than 4 pounds which do not exceed the limit of weight and size for fourth-class matter come within that class and are mailable at the parcel-post rates.

**Fourth Class:** Merchandise, farm and factory products, books, miscellaneous printed matter weighing more than 4 pounds, and all other mailable matter not embraced in the first, second, and third classes.

Rates: Parcels weighing 4 ounces or less, except books, seeds, plants, etc., 1 cent for each ounce or fraction thereof, any distance.

Parcels weighing 8 ounces or less containing books, seeds, cuttings and plants, 1 cent for each 2 ounces or fraction thereof, regardless of distance.

Parcels weighing more than 8 ounces containing books, seeds, plants, etc., parcels of miscellaneous printed matter weighing more than 4 pounds, and all other parcels of fourth-class matter weighing more than 4 ounces are chargeable, according to distance or zone, at the pound rates shown in the following table, a fraction of a pound being computed as a full pound.

FOURTH-CLASS (DOMESTIC PARCEL) ZONE RATES

| WEIGHT IN POUNDS | LOCAL | ZONES | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1st, up to 50 mi. | 2d, 50 to 150 mi. | 3d, 150 to 300 mi. | 4th, 300 to 600 mi. | 5th, 600 to 1,000 mi. | 6th, 1,000 to 1,400 mi. | 7th, 1,400 to 1,800 mi. | 8th, over 1,800 mi. |
| 1 | $0.05 | $0.05 | $0.05 | $0.06 | $0.07 | $0.08 | $0.09 | $0.11 | $0.12 |
| 2 | .06 | .06 | .06 | .08 | .11 | .14 | .17 | .21 | .24 |
| 3 | .06 | .07 | .07 | .10 | .15 | .20 | .25 | .31 | .36 |
| 4 | .07 | .08 | .08 | .12 | .19 | .26 | .33 | .41 | .48 |
| 5 | .07 | .09 | .09 | .14 | .23 | .32 | .41 | .51 | .60 |
| 6 | .08 | .10 | .10 | .16 | .27 | .38 | .49 | .61 | .72 |
| 7 | .08 | .11 | .11 | .18 | .31 | .44 | .57 | .71 | .84 |
| 8 | .09 | .12 | .12 | .20 | .35 | .50 | .65 | .81 | .96 |
| 9 | .09 | .13 | .13 | .22 | .39 | .56 | .73 | .91 | 1.08 |
| 10 | .10 | .14 | .14 | .24 | .43 | .62 | .81 | 1.01 | 1.20 |
| 11 | .10 | .15 | .15 | .26 | .47 | .68 | .89 | 1.11 | 1.32 |
| 12 | .11 | .16 | .16 | .28 | .51 | .74 | .97 | 1.21 | 1.44 |
| 13 | .11 | .17 | .17 | .30 | .55 | .80 | 1.05 | 1.31 | 1.56 |

FOURTH-CLASS (DOMESTIC PARCEL) ZONE RATES

| WEIGHT IN POUNDS | LOCAL | ZONES | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1st, up to 50 mi. | 2d, 50 to 150 mi. | 3d, 150 to 300 mi. | 4th, 300 to 600 mi. | 5th, 600 to 1,000 mi. | 6th, 1,000 to 1,400 mi. | 7th, 1,400 to 1,800 mi. | 8th, over 1,800 mi. |
| 14 | .12 | .18 | .18 | .32 | .59 | .86 | 1.13 | 1.41 | 1.68 |
| 15 | .12 | .19 | .19 | .34 | .63 | .92 | 1.21 | 1.51 | 1.80 |
| 16 | .13 | .20 | .20 | .36 | .67 | .98 | 1.29 | 1.61 | 1.92 |
| 17 | .13 | .21 | .21 | .38 | .71 | 1.04 | 1.37 | 1.71 | 2.04 |
| 18 | .14 | .22 | .22 | .40 | .75 | 1.10 | 1.45 | 1.81 | 2.16 |
| 19 | .14 | .23 | .23 | .42 | .79 | 1.16 | 1.53 | 1.91 | 2.28 |
| 20 | .15 | .24 | .24 | .44 | .83 | 1.22 | 1.61 | 2.01 | 2.40 |
| 21 | .15 | .25 | .25 | .46 | .87 | 1.28 | 1.69 | 2.11 | 2.52 |
| 22 | .16 | .26 | .26 | .48 | .91 | 1.34 | 1.77 | 2.21 | 2.64 |
| 23 | .16 | .27 | .27 | .50 | .95 | 1.40 | 1.85 | 2.31 | 2.76 |
| 24 | .17 | .28 | .28 | .52 | .99 | 1.46 | 1.93 | 2.41 | 2.88 |
| 25 | .17 | .29 | .29 | .54 | 1.03 | 1.52 | 2.01 | 2.51 | 3.00 |
| 26 | .18 | .30 | .30 | .56 | 1.07 | 1.58 | 2.09 | 2.61 | 3.12 |
| 27 | .18 | .31 | .31 | .58 | 1.11 | 1.64 | 2.17 | 2.71 | 3.24 |
| 28 | .19 | .32 | .32 | .60 | 1.15 | 1.70 | 2.25 | 2.81 | 3.36 |
| 29 | .19 | .33 | .33 | .62 | 1.19 | 1.76 | 2.33 | 2.91 | 3.48 |
| 30 | .20 | .34 | .34 | .64 | 1.23 | 1.82 | 2.41 | 3.01 | 3.60 |
| 31 | .20 | .35 | .35 | .66 | 1.27 | 1.88 | 2.49 | 3.11 | 3.72 |
| 32 | .21 | .36 | .36 | .68 | 1.31 | 1.94 | 2.57 | 3.21 | 3.84 |
| 33 | .21 | .37 | .37 | .70 | 1.35 | 2.00 | 2.65 | 3.31 | 3.96 |
| 34 | .22 | .38 | .38 | .72 | 1.39 | 2.06 | 2.73 | 3.41 | 4.08 |
| 35 | .22 | .39 | .39 | .74 | 1.43 | 2.12 | 2.81 | 3.51 | 4.20 |
| 36 | .23 | .40 | .40 | .76 | 1.47 | 2.18 | 2.89 | 3.61 | 4.32 |
| 37 | .23 | .41 | .41 | .78 | 1.51 | 2.24 | 2.97 | 3.71 | 4.44 |
| 38 | .24 | .42 | .42 | .80 | 1.55 | 2.30 | 3.05 | 3.81 | 4.56 |
| 39 | .24 | .43 | .43 | .82 | 1.59 | 2.36 | 3.13 | 3.91 | 4.68 |
| 40 | .25 | .44 | .44 | .84 | 1.63 | 2.42 | 3.21 | 4.01 | 4.80 |
| 41 | .25 | .45 | .45 | .86 | 1.67 | 2.48 | 3.29 | 4.11 | 4.92 |
| 42 | .26 | .46 | .46 | .88 | 1.71 | 2.54 | 3.37 | 4.21 | 5.04 |
| 43 | .26 | .47 | .47 | .90 | 1.75 | 2.60 | 3.45 | 4.31 | 5.16 |
| 44 | .27 | .48 | .48 | .92 | 1.79 | 2.66 | 3.53 | 4.41 | 5.28 |
| 45 | .27 | .49 | .49 | .94 | 1.83 | 2.72 | 3.61 | 4.51 | 5.40 |
| 46 | .28 | .50 | .50 | .96 | 1.87 | 2.78 | 3.69 | 4.61 | 5.52 |
| 47 | .28 | .51 | .51 | .98 | 1.91 | 2.84 | 3.77 | 4.71 | 5.64 |
| 48 | .29 | .52 | .52 | 1.00 | 1.95 | 2.90 | 3.85 | 4.81 | 5.76 |
| 49 | .29 | .53 | .53 | 1.02 | 1.99 | 2.96 | 3.93 | 4.91 | 5.88 |
| 50 | .30 | .54 | .54 | 1.04 | 2.03 | 3.02 | 4.01 | 5.01 | 6.00 |
| 51 | .30 | .55 | .55 | 1.06 | ..... | ..... | ..... | ..... | ..... |
| 52 | .31 | .56 | .56 | 1.08 | ..... | ..... | ..... | ..... | ..... |

The limit of weight of fourth-class matter is 70 pounds for parcels mailed for delivery within the first, second, and third zones, and 50 pounds for all other zones. Parcels may not exceed 84 inches in length and girth combined.

**Special Delivery.** By affixing a "special delivery" stamp (cost 10 cents), or ten cents' worth of ordinary stamps in addition to the regular postage and writing the words "Special Delivery," delivery of letters and parcels by special messenger is assured.

**Registry.** The registry system provides greater security for valuable mail. The registry fee is ten cents. If registered mail is lost, the sender is indemnified for the actual value up to $50 for first-class mail, and up to $25 for third-class domestic mail.

**Insurance of Parcels.** Mail sent by parcel post may be insured by the payment of fees as follows:

| | | |
|---|---|---|
| 3 cents | for value not exceeding | $ 5.00 |
| 5 cents | for value not exceeding | 25.00 |
| 10 cents | for value not exceeding | 50.00 |
| 25 cents | for value not exceeding | 100.00 |

**C. O. D. Service.** Parcel post matter may be sent C. O. D. by affixing the following fees in stamps in addition to the regular postage:

10 cents when amount to be remitted to sender does not exceed $ 50.00
25 cents when amount to be remitted to sender does not exceed 100.00

The remittance is made by postal money order, the fee being collected from the addressee. The C. O. D. fees automatically insure parcels for their actual value up to $50.00 for a 10-cent fee, and up to $100.00 for a 25-cent fee.

**218. Rates for Foreign Postage.** The rates of postage applicable to articles for foreign countries are as follows:

| | Cents |
|---|---|
| Letters for Canada, Cuba, Mexico, Republic of Panama, city of Shanghai (China), England, Ireland, Newfoundland, Scotland, Wales, Bahamas, Barbados, British Guiana, British Honduras, Dominican Republic, Dutch West Indies, Leeward Islands, and New Zealand, Trinidad (including Tobago) and the Windward Islands (including Grenada, St. Vincent, The Grenadines and St. Lucia), per ounce | 2 |
| Letters for Germany by direct steamers, per ounce | 2 |
| Letters for all other foreign countries: | |
| For the first ounce or fraction of an ounce | 5 |
| For each additional ounce or fraction of an ounce | 3 |
| Single post cards (including souvenir cards), each | 2 |
| Reply postal cards, each | 4 |

| | |
|---|---|
| Printed matter of all kinds, for each 2 ounces or fraction of 2 ounces | 1 |
| Commercial papers, for the first 10 ounces or less | 5 |
| For each additional 2 ounces or fraction of 2 ounces | 1 |
| Samples of merchandise, for the first 4 ounces or less | 2 |
| For each additional 2 ounces or fraction of 2 ounces | 1 |
| Registration fee, in addition to postage | 10 |

(Newfoundland is not included in the Dominion of Canada.)

**Written Work**

Compute the cost of postage on the following:

Domestic mail.

1. Letter weighing $1\frac{1}{2}$ oz.
2. Book weighing 5 oz.
3. Book weighing 19 oz., to a point in the fourth zone.
4. Books weighing 5 pounds to a point in the second zone.
5. Three separately wrapped newspapers mailed by a person not a publisher; weight, 3 oz., 5 oz., and 8 oz.
6. Ten separately addressed printed circulars, each weighing $1\frac{1}{2}$ oz.
7. Package of merchandise weighing 3 oz.
8. Package of seeds weighing 12 oz. to a point in the seventh zone.
9. Merchandise weighing 16 lb. 7 oz., to a point in the sixth zone.
10. Merchandise weighing 35 pounds, to a point 130 miles distant.
11. Registered letter weighing 7 oz.
12. Books weighing 11 lb. 9 oz., to a point in the fifth zone; package insured. (Ins. fee 5¢.)

Foreign mail.

13. Letter to London, weighing $1\frac{1}{2}$ oz.
14. Letter to Paris, weighing $1\frac{1}{2}$ oz.
15. Package of merchandise samples weighing 11 oz., to London.

## Freight Rates

A freight rate is the charge made by a railroad company for conveying a specified quantity of property between two stations.

The rate is usually stated in cents per hundred pounds; but it may be stated in dollars and cents per ton or other unit of weight.

**219. Factors Determining Rates.** Different rates are charged on different kinds of freight, the rate depending in general upon two things:

*a.* **The expense of handling.**

Suppose two boxes, each weighing 100 pounds, are sent by freight. One is a large box occupying 40 cubic feet of car space; the other occu-

pies only 16 cubic feet. The rate per 100 pounds on the first box will probably be more than on the second because fewer such boxes can be placed in one car.

*b.* **The risk of loss or damage.**

Railroads are required by law to pay for goods lost or damaged. The rate will therefore depend upon:

1. The value. For example, the rate on a prize-winning cow, worth $900, will be higher than on a cow worth $60.
2. The risk of damage. The rate on glassware, for example, is higher than on wooden ware. Careful packing usually lowers the rate.
3. The risk of loss. The rate on small packages which may easily be misplaced or lost is higher than on those of larger size having the same value.
4. The risk of an article damaging other articles in the same car.

**220. Classifications.** For convenience in determining rates, commodities are grouped into ten different classifications. These classifications are made by the railroads after considering the elements of cost of transportation and risk. When goods are to be shipped from one state to another, the classification must be approved by the Interstate Commerce Commission.

A book called a " Classification " is published, containing an alphabetical list of commodities which can be shipped by freight, together with the classification in which each commodity is placed.

The following list is selected from a page of a Classification:

Notice that two classifications are given for some articles. The first applies when goods are shipped in less than car loads (L. C. L.); the second applies to car load shipments.

| | C. L. |
|---|---|
| Cutlery, blades or handles plated or sterling silver, boxed . . . . . | 1½ |
| Cutters, Sage Brush (knife attachments for grading machines): | 5 Min. wt. 30,000 lbs. |
| In bundles . . . . | 2 |
| In boxes or barrels . | 3 |
| Designs, Floral, Artificial or Natural, dried, in boxes . . . . . | D 1 |
| Display Cases, window shade, boxed or crated . | 1 |
| Dog Benches (for exhibition purposes only), K. D. flat, min. C. L. wt. 30,000 lbs.. . . . . | 4 |
| Door Hangers and Rail: | B |
| Barn Door Hangers (iron), boxed or crated . . . | 3 |
| Barn Door Rails (iron), in bundles . . . . . | 3 |
| Barn Door Hanger Rails (wooden), in bundles . | 3 |

| C. L. |
|---|
| |
| |
| |
| |
| |
| |
| |
| 5 Min. wt. 36,000 lbs. |

| | | |
|---|---|---|
| Dextrine, in bags, bbls. or boxes, min. C. L. wt. 30,000 lbs. . . . . | 2 | 4 |
| Dish-Washing Machines, N. O. S., boxed or crated. | 1 | |
| Formaldehyde, in barrels or kegs or in glass bottles, boxed, min. C. L. wt. 30,000 lbs. . . . . | 2 | 4 |
| Parlor Door Hangers, boxed . . . . . | 2 | |
| Parlor Door Hanger Rail, wooden, in bundles . | 3 | |
| Tubing, for Barn Door Hanger Rail, nested in bundles . . . . . | 4 | |

TERMS EXPLAINED: N. O. S. means Not Otherwise Specified.

$1\frac{1}{2}$ means one and one half times first-class.
D 1 means double the first-class rate.
C. L. means car loads.
K. D. means knocked down, taken apart.

**221. Freight Tariff.** The tariff is a list of the rates applying to different classifications of freight when shipped between the points named.

The following illustration is selected from a freight tariff.

| BETWEEN | AND | CLASS RATES IN CENTS PER 100 POUNDS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1st Class | 2d Class | 3d Class | 4th Class | 5th Class | Class A | Class B | Class C | Class D | Class E |
| Stoddard, Wis. Calvert, Wis. La Crosse, Wis. Grand Crossing, Wis. Onalaska, Wis. Lytle, Wis. Trempealeau, Wis. East Winona, Wis. Winona, Minn. | Chicago, Ill. Group No. 1 | 50 | 42 | 33 | 23 | 18 | 23 | 18 | 15 | 12 | 11 |
| | Springfield, Ill. Group No. 2 | 53 | 43 | 33 | 25 | 20 | 23 | 18 | 15 | $12^{0}$ | 11 |
| | St. Louis, Mo. Group No. 3 | 50 | 42 | 33 | 23 | 18 | 23 | 18 | 15 | 12 | 11 |
| | Danville, Ill. Group No. 4 | 63 | $52^{5}$ | 42 | 26 | 21 | 26 | 21 | 18 | 15 | $13^{5}$ |
| | Cairo, Ill. Group No. 4 | 80 | 65 | 52 | 33 | 26 | 33 | 26 | 22 | 19 | 17 |

NOTE. $52^{5}$ means 52½ cents.

**222. To find the freight charge.** The rates differ for each of the ten classifications. Therefore, when a shipment of goods is made, the following steps are necessary to determine the freight charge:

*a. Weigh the commodity.*

*b. Find its classification.*

*c. Determine from the tariff the rate applying to that classification between the points of shipment.*

*d. Multiply the number of hundreds of pounds by the rate.*

**Problem.** A shipment of barn door hangers (crated) is sent from Chicago to La Crosse, Wis. The weight crated is 185 pounds. Find the freight charges.

SOLUTION. The classification for less than car load lots is 3.

The rate on third-class commodities between Chicago and La Crosse, as shown by the tariff, is 33 cents per hundred.

$0.33 freight rate per hundred pounds.
1.85 number of hundreds of pounds.
$0.61 freight charge.

**223. Minimum Charges.** A minimum charge is fixed for shipment of any commodity between two points. When the charge is computed and is less than this minimum charge, the minimum charge is made. The minimum charge is sometimes the third-class rate per hundred, or it may be an arbitrarily fixed number of cents.

**224. Car Loads.** When goods are shipped in car load lots, they frequently go under a different classification and at a lower rate. Minimum weights for car loads are specified. For example, 36,000 pounds is the minimum weight for a car of door hangers. This does not mean that the car must contain 36,000 pounds, but the shipper will be charged freight on at least that weight, and more if the car contains a greater weight.

**Problem.** Shipped from Chicago to La Crosse a car load 39,000# of door hangers. Find the freight charges.

SOLUTION. Classification C. L., 5 (see classification, page 266).
Rate, $0.18.
390.00 × $0.18 = $70.20, the freight charge.

If the door hangers weighed only 32,380#, a charge would be made for the minimum weight of 36,000#.

SOLUTION. Classification, 5.
Rate, $0.18 per 100 pounds.
360 × $0.18 = $64.80, freight charge.

**225. Commodity Rates.** There are some articles which are not listed in the classification, but which have a rate of their own, called a "commodity rate." The most important of these commodities are lumber, grain, live stock, coal, coke, and hay.

### Written Work

Find the freight on each of the following:

1. 360# of formaldehyde in glass bottles, boxed, shipped from St. Louis, Mo., to Winona, Minn.
2. 500# dextrine in bags, shipped from Winona, Minn., to Danville, Ill.
3. 260# display cases (wood and wire, crated), shipped from Springfield, Ill., to Onalaska, Wis.
4. Car load of dextrine 28,000#, shipped from Winona, Minn., to Cairo, Ill.
5. Box of silver-plated cutlery, 60#, shipped from Chicago, to La Crosse, Wis.
6. Box of dish washing machines, shipped from Chicago to Stoddard, Wis. Weight 175#.
7. Box of parlor door hangers, 80#, shipped from Springfield, Ill., to Grand Crossing, Wis.
8. Flower design, 35#, from Chicago to East Winona, Wis.
9. Car load of iron door hangers, 41,350#, from St. Louis, Mo., to La Crosse, Wis.

## Express Rates

**226. The Classification.** The express classification is simpler than the freight classification, most articles being unspecified and taking the first-class rate.

**Rate Scale Numbers.** The agent at each express station is supplied with a directory showing the scale number to be used in computing the cost of shipment to every other express station in the United States. The scale numbers run from 1 to 294, depending on the distance between point of shipment and point of delivery.

The following table shows the rate scale numbers from the Chicago directory for shipment to certain Iowa points.

| Station | Scale No. | Station | Scale No. |
|---|---|---|---|
| Burdette | 27 | Campbell | 26 |
| Burlington | 15 | Carlisle | 26 |
| Burt | 31 | Carnes | 39 |
| Buxton | 21 | Carney | 26 |
| | | Carpenter | 27 |
| C | | Carroll | 29 |
| Calmar | 17 | Casey | 30 |
| Calumet | 33 | Cedar Falls | 23 |
| Cambridge | 26 | Cedar Rapids | 15 |

**227. Schedule of Rates.** The table on the following page gives the rates per pound on shipments between points where scale numbers from 26 to 33 apply. The complete table states the rates on shipments weighing from 1 to 100 pounds.

**228. To find the express charge for shipments of 100 pounds or under.**

*a. Find the weight of the commodity.*

*b. Determine the classification.*

*c. Find the rate scale number applicable between the points of shipment and delivery.*

*d. The table of rates will show the charge for first and second-class commodities weighing* 100 *pounds or less.*

**Problem.** What is the express charge on a case of medicine shipped from Chicago to Burdette, Iowa? Weight, 40 pounds.

Solution. Classification, 1.
Rate scale number (see table) 27.

The schedule shows the charge under rate scale number 27, first class, 40 pounds, to be $1.23.

**229. To find the express charge for shipments over 100 pounds.**

(*a*) First-class Rates:

*Find the charge for a shipment of* 100 *pounds, first class. Multiply this charge by the number of pounds in the shipment, and divide by* 100.

**Problem.** Find the charge on 216 pounds of first-class matter when the scale rate number is 31.

## Schedule of First and Second Class Express Rates in Cents

Scale Numbers

| Lbs. | 26 | | 27 | | 28 | | 29 | | 30 | | 31 | | 32 | | 33 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1st Class | 2d Class | 1st Class | 2d Class | 1st Class | 2d Class | 1st Class | 2d Class | 1st Class | 2d Class | 1st Class | 2d Class | 1st Class | 2d Class | 1st Class | 2d Class |
| 25 | 88 | 66 | 89 | 67 | 92 | 69 | 93 | 70 | 96 | 72 | 97 | 73 | 98 | 74 | 100 | 75 |
| 26 | 89 | 67 | 93 | 70 | 95 | 71 | 96 | 72 | 97 | 73 | 100 | 75 | 101 | 76 | 102 | 77 |
| 27 | 93 | 70 | 95 | 71 | 97 | 73 | 98 | 74 | 100 | 75 | 102 | 77 | 103 | 77 | 106 | 80 |
| 28 | 95 | 71 | 97 | 73 | 98 | 74 | 101 | 76 | 102 | 77 | 103 | 77 | 107 | 80 | 108 | 81 |
| 29 | 97 | 73 | 98 | 74 | 101 | 76 | 102 | 77 | 106 | 80 | 107 | 80 | 110 | 83 | 111 | 83 |
| 30 | 98 | 74 | 101 | 76 | 102 | 77 | 106 | 80 | 107 | 80 | 110 | 83 | 111 | 83 | 113 | 85 |
| 31 | 101 | 76 | 103 | 77 | 106 | 80 | 108 | 81 | 110 | 83 | 112 | 84 | 113 | 85 | 116 | 87 |
| 32 | 103 | 77 | 106 | 80 | 108 | 81 | 110 | 83 | 112 | 84 | 115 | 86 | 116 | 87 | 120 | 90 |
| 33 | 106 | 80 | 108 | 81 | 110 | 83 | 112 | 84 | 115 | 86 | 116 | 87 | 120 | 90 | 122 | 92 |
| 34 | 108 | 81 | 110 | 83 | 112 | 84 | 115 | 86 | 116 | 87 | 120 | 90 | 122 | 92 | 125 | 94 |
| 35 | 110 | 83 | 112 | 84 | 115 | 86 | 116 | 87 | 120 | 90 | 122 | 92 | 125 | 94 | 126 | 95 |
| 36 | 112 | 84 | 115 | 86 | 116 | 87 | 120 | 90 | 122 | 92 | 125 | 94 | 127 | 95 | 129 | 97 |
| 37 | 113 | 85 | 116 | 87 | 120 | 90 | 122 | 92 | 125 | 94 | 127 | 95 | 129 | 97 | 131 | 98 |
| 38 | 116 | 87 | 120 | 90 | 122 | 92 | 125 | 94 | 126 | 95 | 129 | 97 | 131 | 98 | 135 | 101 |
| 39 | 117 | 88 | 121 | 91 | 123 | 92 | 126 | 95 | 129 | 97 | 131 | 98 | 135 | 101 | 137 | 103 |
| 40 | 121 | 91 | 123 | 92 | 126 | 95 | 129 | 97 | 131 | 98 | 135 | 101 | 137 | 103 | 140 | 105 |

SOLUTION. Rate for 100 pounds, first-class matter, scale number 31, is found by the complete table to be $2.84.

$$\frac{216 \times \$2.84}{100} = \$6.1344$$

Under the express company's rule that all fractions of a cent must be equalized as one cent, the charge would be $6.14.

(*b*) Second-class Rates:

*Multiply the first-class rate on* 100 *pounds by the number of pounds in the shipment, and divide by* 100.

*Deduct* 25 % *of this result to find the second-class charge.*

**Problem.** What is the charge on 216 pounds of second-class matter, when the rate scale number is 31?

SOLUTION. Charge for 216 pounds, scale number 31, first class, is $6.14.
75 % of $6.14 = $4.605, or $4.61.

**Written Work**

Find the charge for the following shipments by express from Chicago:

1. 26-pound box of cameras to Burdette, Iowa. Classification 1 (first class).
2. 25 pounds of candy to Calumet, Iowa. Classification 1.
3. 30 pounds of seeds to Carpenter, Iowa. Classification 2.
4. 28 pounds of telephone instruments, unboxed, to Casey, Iowa. Classification $1\frac{1}{2}$ times first class.
5. 163 pounds of statuary to Carroll, Iowa. Classification 1. First-class rate for 100 pounds, scale number 29, $2.70.
6. 191 pounds of shrubs to Burt, Iowa. Classification 2. First-class rate for 100 pounds, scale number 31, $2.84.

## CHAPTER XXVII

### PROPERTY INSURANCE

PROPERTY is in danger of destruction from many causes; insurance companies have been organized to assume the risks of destruction, thus relieving the owners from anxiety and loss. Any person having a financial interest in a property may insure himself against its loss or destruction by paying an insurance company a certain per cent of the value of the property insured.

The person who purchases insurance is called the **Insured.**

The company which assumes the risk is called the **Underwriter.**

The contract between the insured and the underwriter is called a **Policy.**

**230. The Value of Insurance.**

*a.* In case of loss the insured receives a certain sum, called an indemnity, which wholly or partially repays his loss.

*b.* Even though the property is not destroyed, the insured has the security which the insurance gives.

*c.* Some wholesale houses are unwilling to sell goods on credit to persons whose property is inadequately insured.

**231. Risks for which Insurance is Written.** Insurance companies are organized to furnish protection against numerous kinds of risks. Some of the most common classes of insurance are:

Fire.
Lightning and Tornado.
Marine, on vessels and cargoes at sea.
Live stock.
Transportation.
Burglary.
Plate Glass.
Boiler Explosion.
Automobile.
Crop.

The principles underlying all types of property insurance are the same, and for the purposes of this text fire insurance only will be explained. It will be understood that other forms of insurance may be written, settled, or canceled in a similar manner.

**232. Process of Insuring.** When a person wishes to secure a policy insuring his property, he should first ascertain the value of the property. To do so, it may be necessary to take an inventory, applying to the goods a price at which they would sell in the market at the time of taking the inventory.

After the valuation has been determined, the next thing to decide is the amount of insurance it is desirable to carry. If the property is insured at its total valuation, the owner will bear no loss in case of fire. But since, in most fires, a portion of the property is saved, it is customary to insure only a fraction of the total valuation.

It is a matter of choice how much risk the insured wishes to take. If he wishes to be free from all risk, he should insure his property at its full valuation. Insurance companies, however, are rarely willing to insure property at its full value. Business men usually insure from $\frac{1}{2}$ to $\frac{4}{5}$ of the valuation of their property.

In some communities where the fire protection is poor, insurance companies sometimes refuse to insure more than $\frac{3}{4}$ of the valuation of property. On the other hand, in cities where the fire protection is good and the probability of total loss is comparatively slight, people are not inclined to insure a large fraction of the valuation. To counteract this tendency, the insurance companies frequently offer a lower rate, or price per hundred dollars of insurance carried, to those who insure 80 % or more of the valuation than they offer to persons insuring less than 80 %.

**233. The Cost of Insurance.** Insurance companies charge a certain number of cents or dollars for insuring each $100.00 worth of property. Thus, if the insurance rate were 60 cents per hundred dollars of insurance carried, and you insured a building worth $5,000.00 at its full value, the insurance cost would be computed as follows:

$.60 the rate per $100 insurance.
50 the number of hundred dollars of insurance purchased.
$30.00 the premium, or cost of the insurance per year.

**234. To find the premium:**

*Multiply the valuation of the property by the fraction of the value insured.*

*Point off two places in this product, to determine the number of hundred dollars of insurance purchased.*

*Multiply by the rate per* $100.

**Problem.** What is the premium for insuring, at 80% of its value, a building worth $10,000.00, if the rate is 90 cents per year?

SOLUTION.

| | |
|---|---|
| $10,000.00 | value of the property |
| .80 | fraction of property to be insured |
| $ 8,000.00 | insured value |
| $ .90 | rate per hundred |
| 80 | number of hundred dollars of insurance carried |
| $72.00 | premium |

**235. Insurance Rates.** The insurance rate per hundred dollars per year varies from about $.30 on brick dwellings in cities, where the fire protection is good, to $7 or more on very dangerous risks, such as garages storing gasoline, and buildings located in districts where the fire protection is poor.

Thus it is seen that the rate depends upon the hazard, or the danger of the property burning. This hazard is determined by four things, which are always taken into account in fixing the rate on different pieces of property. They are:

*a.* The **Construction** of the building. If it is built of wood, a higher rate is charged than if it is built of brick, or other slow-burning material. If it has many stories, so that a fire starting near the roof will be difficult to extinguish, a higher rate will be charged.

*b.* Its **Occupancy.** If the building is occupied by some hazardous business, such as a woodworking mill in which wood shavings accumulate, or a clothes-cleaning establishment containing dangerous chemicals, a higher rate will be charged than if the business caused no particular hazard.

*c.* Its **Exposure.** By "exposure" is meant: first, the distance from other buildings from which it might take fire; and second, any hazard which might arise from its being in the neighborhood of a dangerous building.

*d.* Its **Protection.** The rate will be lower if the building is close to a fire hydrant, in a city with an efficient fire department, or if the building is equipped with automatic sprinklers which are opened by

the heat from a fire. If a night watchman is engaged, this also may reduce the rate.

The rate depends also upon the length of time for which the insurance is purchased.

**236. Typical Rates.** The following table will illustrate the rates which apply on different classes of property. On business houses and factories the rate is fixed after an inspection of the property made by a representative of the insurance company. The insurance companies of a state frequently join in organizing an "inspection bureau" which fixes the rates.

The rates on city property will be lower than those which apply generally throughout the country, because of the more adequate fire protection of a large city.

A City Insurance Tariff

DWELLINGS

| | One Year, per $100 |
|---|---|
| Brick, stone, tile, cement-block, or concrete dwellings | $0.30 |
| Single frame dwellings and contents, detached not less than 50 feet in all directions | .50 |
| Single frame dwellings and contents, detached not less than 25 feet in all directions | .60 |
| Single frame dwellings and contents, detached 50 feet on one side | .60 |
| All other frame dwellings not less than | .75 |
| Brick-veneered or tile-veneered dwellings | .40 |
| Dwellings plastered outside, with tile or other noncombustible roof | .40 |
| Dwellings plastered outside, with shingle or other combustible roof, rate same as frame dwellings. | |
| Dwellings in part brick and frame rate as frame, subject to survey and rating by the board. | |
| Streets, without reference to their width, to be considered as cutting off charge for exposure. | |

BRICK BUILDINGS OCCUPIED FOR APARTMENT HOUSES OR FLATS

| | One Year, per $100 |
|---|---|
| Three stories in height | $0.40 |
| Four stories in height and 50 feet or less wide | .50 |
| Five stories in height and 50 feet or less wide | .60 |
| Six stories or more in height and 50 feet or less wide | .75 |

BRICK BUILDINGS (OTHER THAN APARTMENT HOUSES, CHURCHES, SCHOOLS, FLATS AND WOODWORKING RISKS) IN PROCESS OF CONSTRUCTION

| | One Year, per $100 |
|---|---|
| Two stories or less in height | $0.40 |
| Three stories in height | .50 |
| Four stories in height | .60 |
| Five stories in height | .75 |

Over five stories in height, add 25 cents for each story.

SCHOOLHOUSES

| | One Year, per 100 |
|---|---|
| Brick or stone, with metal, slate, or composition roof | $0.60 |
| Brick or stone, with shingle roof | .75 |
| Frame | 1.00 |

CHURCHES

| | One Year, per $100 |
|---|---|
| Brick or stone | $0.75 |
| Frame | 1.00 |

Contents of buildings take a rate 10 % higher than the buildings themselves.

TERM RATES

When policies are written for several years, the following will apply:

2 years, $1\frac{3}{4}$ annual rates.
3 years, $2\frac{1}{2}$ annual rates.
4 years, $3\frac{1}{4}$ annual rates.
5 years, 4 annual rates.

FARM PROPERTY

| | One Year, per $100 | Three Years, per $100 | Five Years, per $100 |
|---|---|---|---|
| Dwellings, barns, outbuildings, when written under same policy | $0.50 | $1.00 | $1.50 |
| When farm barns are written without the dwellings | .75 | 1.50 | 2.25 |

## Written Work

Compute the premiums on the following fire insurance policies. The proper rates will be found by a study of the preceding tariffs.

### One-year Policies — City Property

1. Single frame building, value $4,000. No buildings nearer than 60 feet. Insured at 80 % of its valuation.

2. Single frame dwelling, value $5,400. House on north, 70 feet away; store on south, 25 feet away. Insured at full value.

3. Brick dwelling, value $6,800. Insured at 75% of its value. Nearest building, 50 feet.

4. Dwelling, plastered outside; shingle roof. Value, $4,200. Insured for $3,600. No building nearer than 35 feet.

5. Five-story apartment house, 48 feet wide. Value, $36,000. Insured at $\frac{3}{4}$ of value.

6. Four-story brick store in process of construction. Value, $3,900. Insured at 80% of its valuation.

TERM POLICIES — CITY PROPERTY

7. Three-year policy on brick schoolhouse, with tile roof. Value $40,000. Insured at full value.

8. Five-year policy on frame church. Value, $7500. Insured at full value.

POLICIES ON HOUSEHOLD GOODS — CITY

9. Three-year policy of $1,000 on household goods in frame dwelling. Building on north, 10 feet; building on south, 60 feet.

10. One-year policy of $500 on household goods in brick-veneered building.

FARM POLICIES

11. One-year policy of $8,500, covering dwelling and barns.

12. Five-year policy of $3,000 on barn and silo.

**237. Insurance Agent's Commission.** Local agents of the fire insurance companies are located in almost every city and town. They act as the representatives of the companies, soliciting the business, and collecting the premiums. For this service they receive a certain per cent of the premiums, usually varying from 15% on mercantile risks to 25% on dwellings.

**To find the agent's commission.**

**Problem.** Suppose that a store building valued at $6,000 is insured for $\frac{3}{4}$ of its value, the rate being 70 cents per hundred. What is the agent's commission if he receives 15% of the premium?

SOLUTION. $\frac{3}{4}$ of $6,000 = $4,500, the insured value.
$45 \times \$.70 = \$31.50$, the premium.
15% of $31.50 = $4.73, the agent's commission.

**Written Work**

In each of the following problems find

The premium;
The agent's commission;
The amount of the premium received by the insurance company.

1. A one-year policy of $3,000 on a store building. Rate, 90 cents. Agent's commission, 20 %.

2. A three-year policy on a dwelling valued at $6,500, insured at 80 % of its valuation. Rate, 65 cents. Three-year policy for 2½ annual rates. Agent's commission, 18 %.

3. A one-year policy of $800 on household goods. Rate, 50 cents. Agent's commission, 15 %.

**238. Settlement of Losses.** When a fire occurs, causing a small loss, settlement may be made by the local agent. When the loss is large, a special **adjuster** is sent by the company to make the settlement.

The adjuster first determines with the owner the actual cash value of the property immediately preceding the fire. This may be done by finding the cost of the property when new, and subtracting a certain amount for **depreciation;** that is, the decrease in value due to use, age, change of style, or any other cause. This cash value is called the **sound value.**

The adjuster next determines the value of goods which were saved from the fire. In determining this value he deducts any loss due to damage by the fire, or by water used in extinguishing the fire. The property saved is called **salvage.**

It is evident that:

Sound Value − Salvage = Loss.

The companies pay this actual loss up to the value of the insurance carried.

**Problems.**

1. Valuation of property by insured when taking the policy . . . . . $10,000.00
Insured for 80 % of its value; Insured value . . . . 8,000.00
Sound value agreed upon by adjuster and insured after allowing $500 for depreciation . . . . . 9,500.00
Salvage . . . . . 3,000.00
Actual loss. . . . . $ 6,500.00

Since the insured carried $8,000 insurance, the loss would be paid in full.

| | |
|---|---|
| 2. Valuation of property by insured when taking policy . | $ 8,000.00 |
| Insured for ¾ of its value; Insured value. . . . . | 6,000.00 |
| Sound value, after deducting 10 % for depreciation . . | 7,200.00 |
| Salvage . . . . . | 500.00 |
| Actual loss. . . . . | $6,700.00 |

Since the insured carried only $6,000 insurance, the company is liable for only $6,000 of the loss; the insured bearing the remaining $700 loss.

In adjusting losses, the insurance companies reserve the right to:

*a.* Settle the loss as shown above; or

*b.* Pay the sound value of the property, taking possession of all the salvage; or

*c.* Replace the property by other property of like kind and quality.

**239. The Co-Insurance Clause.** Policies often contain what is called a co-insurance clause, worded somewhat as follows: "In consideration of the premium for which this policy is issued, it is expressly stipulated that in event of loss this company shall be liable for no greater proportion thereof than the sum hereby insured bears to eighty per cent of the cash value of the property described herein at the time when such loss shall happen."

This is an "80 % co-insurance clause." The clause may contain any per cent, but 80 % is customary. The theory of this clause is as follows: the insured agrees that if he carries a policy less than 80 % of the sound value of the property he becomes a co-insurer with the company. For example, if the face of the policy is only 70 % of the sound value, the insured is a co-insurer for 10 %. Any losses will be divided ⅞ to the company and ⅛ to the insured, except that the company will not be liable for more than the face of the policy. If the policy is 80 % of the sound value of the property, the company will be liable for all losses up to the face of the policy.

*Example A.*

| | |
|---|---|
| Sound value of property | $10,000.00 |
| Policy | 8,000.00 (or 80 %) |

The company is liable for all losses up to $8,000.00.

*Example B.*

| | |
|---|---|
| Sound value of property | $10,000.00 |
| Policy | 8,500.00 (Or more than 80 %) |

The company is liable for all losses up to $8,500.00

*Example C.*

| | |
|---|---|
| Sound value of property | $10,000.00 |
| Policy | 7,000.00 (Less than 80 %) |
| Loss | 4,000.00 |

Company is liable for $\frac{7}{8}$ of $4,000.00, or $3,500.00.

*Example D.*

| | |
|---|---|
| Sound value of property | $10,000.00 |
| Policy | 6,500.00 (Less than 80 %) |
| Loss | 4,000.00 |

Company is liable for $\frac{65}{80}$ of $4,000.00, or $3,250.00.

*Example E.*

| | |
|---|---|
| Sound value | $10,000.00 |
| Policy | 6,500.00 |
| Total loss | |

Company is liable for $6,500.00. $\frac{65}{80}$ of $10,000.00 is $8,125.00, but as this is more than the face of the policy, the company is liable for only $6,500.00.

From these examples, the following rule should be clear:

To determine the company's liability under a policy with the 80 % co-insurance clause, when the policy is less than 80 % of the sound value, multiply the total loss by a fraction, the numerator of which is the face of the policy, and the denominator of which is 80 % of the sound value of the property. The product is the liability of the company, except that the liability cannot be greater than the face of the policy.

*Example F.*

| | |
|---|---|
| Sound value | $12,500.00 |
| Policy | 7,000.00 |
| Loss | 4,000.00 |

Numerator of fraction = 7,000

Denominator of fraction = 80 % of 12,500 = 10,000

Liability = $\frac{7,000}{10,000}$ of $4,000 = $2,800.00

### Written Work

(No co-insurance clause in Problems 1 to 8)

**1.** An apartment building worth $45,000 was insured at 80% of its valuation. A fire occurred resulting in a total loss. If the sound value was considered to be the same as the original valuation, how much insurance did the owner of the property receive, and what was his loss?

**2.** A dwelling valued at $3,500.00 was insured at its full valuation. A fire resulted in a partial loss, the salvage being estimated at $1,250.00. If the sound value of the property was $3,325.00, how much insurance did the owner of the property receive?

**3.** If the property in Problem 2 had been insured for $1,500.00, how much insurance would have been paid?

**4.** A policy of $1,500.00 was written on a man's household goods. When adjustment of the loss was made, the following facts were determined: Value of household goods immediately preceding the fire, $1,855.00; salvage, $575.00. How much money did the insured receive as a settlement?

**5.** If the salvage in Problem 4 had been $215.00, how much would the insured have received?

**6.** A building worth $5,000.00 was insured under a five-year policy at its full value. Four years after taking the policy, the house was completely destroyed by fire. In making adjustment of the loss, it was agreed that the sound value was 5% less than the insured value. How much did the owner of the house receive in settlement?

**7.** If the salvage in Problem 6 had been $1,350.00, how much insurance would the insured have received?

**8.** Cost of building, $6,000; insured for 80% of its value under five-year policy secured by the payment of four annual rates of 75 cents. Burned at end of third year. Sound value determined by deducting 4% annual depreciation. Salvage, $1,200. Find premium and loss, if any, to owner.

**9.** Property is insured for $7,500.00 under a policy with an 80% co-insurance clause. The property was totally destroyed. Its sound value was $12,000.00. What was the liability of the insurance company?

**10.** What would the liability of the company have been if only a partial loss of $7,000.00 had occurred?

**11.** Sound value of property $25,000.00. Insurance carried

$20,000.00. Co-insurance clause, 80%. Total loss. What is the liability of the company?

12. Same facts except that the loss was only $15,000.00. What is the liability of the company?

13. Sound value of property $25,000.00. Insurance carried, $15,000.00. Co-insurance clause, 80%. Total loss. What is the liability of the company?

14. Same facts except that the loss was only $10,000.00. What is the liability of the company?

**240. Canceling Policies.** Both the insurance company and the insured have the right to cancel an insurance policy at any time.

When the policy is canceled by the *insurance company*, the portion of the premium to be repaid to the insured is determined *pro rata*.

The exact fraction of the time of the policy which has not expired is determined and this fraction of the premium is returned to the insured.

**Problem.** On July 7, 1922, the owner of a building insured his property for one year. The premium was $24.00. On September 17, the policy was canceled by the insurance company. What rebate did the insured receive for the unexpired term?

SOLUTION. Time from September 17, 1922, to July 7, 1923, 293 days, unexpired term of policy.

$\frac{293}{365}$ of $24.00 = $19.27, amount of premium returned.

The table on page 224 will be of assistance in finding the unexpired term of a policy.

**Problem.** An insurance policy for one year was purchased on July 9, and was canceled by the company on the 17th of the following December. How many days in the unexpired term of this policy? What fraction of the premium will be refunded?

SOLUTION. The number of days between December 17 and July 17 is shown by the table to be 212. Subtracting 8, we have 204, the number of days from December 17 to July 9.

$\frac{204}{365}$ of the premium will be refunded.

When the policy is canceled by the *insured*, the amount of premium returned is determined by the "short rate." The short rate is an arbitrary per cent fixed by the insurance companies, and is shown by a table, a portion of which is shown on page 284.

PER CENTS OF THE ANNUAL PREMIUMS TO BE CHARGED OR RETAINED FOR PERIODS LESS THAN ONE YEAR. ARRANGED BY DAYS FROM ONE TO THREE HUNDRED SIXTY DAYS.

| DAYS | PER CENT | DAYS | PER CENT | DAYS | PER CENT | DAYS | PER CENT |
|---|---|---|---|---|---|---|---|
| 66 | 34.32 | 71 | 36.30 | 76 | 37.50 | 81 | 38.85 |
| 67 | 34.97 | 72 | 36.58 | 77 | 37.90 | 82 | 39.13 |
| 68 | 35.46 | 73 | 36.79 | 78 | 38.20 | 83 | 39.34 |
| 69 | 35.79 | 74 | 36.93 | 79 | 38.40 | 84 | 39.48 |
| 70 | 35.95 | 75 | 37.00 | 80 | 38.50 | 85 | 39.55 |

**Problem.** I. On October 28, 1922, a one-year policy for $2500.00 was written on a dwelling. Premium, $12.50. On January 15, 1923, the policy was canceled at the request of the insured. What rebate did the insured receive?

SOLUTION. Time from October 28, 1922, to January 15, 1923, 79 days.

The table shows that 38.40 % of the premium is to be *retained* when the policy has been in force 79 days.

38.40 % of $12.50 = $4.80.

$12.50 − $4.80 = $7.70, amount of premium returned

### Written Work

Find the premium rebates on the following canceled policies:

1. One-year policy of $2,400.00. Premium, $16.00. Policy written May 19, 1922; canceled by the company on July 29, 1922.

2. Same facts as Problem 1, except that the policy is canceled at the request of the policyholder.

# CHAPTER XXVIII

## TAXATION

**241. Purpose of Taxation.** The national government requires money to support the army and navy, to pay the salaries of government employees, to pay pensions, and to finance other activities carried on by the nation.

The state governments require money for the expense of their officers, and to support their various institutions, schools, universities, asylums, and penitentiaries.

The counties require money for the building of bridges, the trial of criminal cases, the salaries of officers, the relief of the poor, etc.

Cities must pay for police and fire protection, care of streets, etc.

School districts contribute to the support of the public schools.

The money required for these expenses is raised by taxes, licenses, fees, assessments, and fines.

**242.** A **tax** is a sum of money levied by the proper officers of a government to defray government expenses.

The funds of the national government are raised largely in three ways:

Customs Duties on Imports;
Internal Revenue;
Income Tax.

The funds of the state and local governments are raised largely by direct taxes on real estate and personal property.

### State and Local Taxes

**243. Taxable Property.** The amount of property tax paid by any individual to state and local governments depends on the assessed value of the property which he owns and on the tax rate.

**Real estate** consists of land and buildings.

**Personal property** consists of movable property, such as merchandise, furniture, machinery, live stock, cash, notes, stocks, bonds, and mortgages.

**244. Determining the Assessed Value of Property.** The value of each person's taxable property is determined by the assessor. The **assessed value** is usually a fractional part of the real value.

For example, in a certain state, property is assessed at $\frac{1}{3}$ of its real value. Jones owns a farm which the assessor considers to have a real value of $30,000. He therefore assesses it at $10,000.

Although the law gives the assessor the power to determine the taxable value of property, property owners usually have the privilege of appearing before a Board of Equalization to prove a claim that their property has been assessed at too high a valuation. The Board of Equalization judges between the values fixed on property by the assessor and by the owner.

**245. The Tax Rate.** The tax rate may be expressed in several ways, the most common of which are:

A per cent;

A certain number of mills on each dollar of assessed valuation;

A certain number of dollars or cents on each hundred dollars of assessed valuation.

The following tax rates are equivalent:

1.6%;
16 mills (on the dollar);
$1.60 (on each hundred dollars).

**246. To find the amount of tax.** To find the amount of tax to be paid by any property owner:

*Multiply the assessed value of the property by the tax rate.*

(*a*) When the rate is stated as a per cent.

**Problem.** Bennet's property is assessed at $3,000; the tax rate is 1.6%.

SOLUTION.

$3,000 assessed valuation
.016 tax rate
$48.00 tax

(*b*) When the rate is stated as a certain number of mills on the dollar.

**Problem.** Taylor's property is assessed at $3,800. The rate is 24 mills.

SOLUTION.
$3,800 assessed valuation
.024 tax rate
$91.20 tax

(*c*) When the rate is stated as a certain number of dollars on each hundred dollars of assessed value.

**Problem.** Finch's property is assessed at $5,470. The tax rate is $1.95.

SOLUTION.
$1.95 the rate per hundred dollars
54.70 the number of hundreds of dollars assessed value
$106.67 the tax

### Oral Work

**1.** State each of the following tax rates in two other ways:

3.6% 27 mills $2.83

**2.** A certain state assesses property at $\frac{2}{5}$ of its real value. What is the assessed value of a house worth $5,000? Of a factory worth $15,000? Of a building worth $4,000?

**3.** Dickinson has a house assessed at $2,000. The rate is 2.5%. What is his tax?

**4.** Barnes has a store building assessed at $3,000 and a house assessed at $2,000. The tax rate is 30 mills. What is his tax?

**5.** If property is assessed at $\frac{1}{3}$ of its real value, what is the assessed value of a city lot worth $1,200? How much tax will the owner pay if the tax rate is 22 mills?

**6.** Property valued at $6,000 is assessed at $\frac{1}{3}$ of its value and taxed at a rate of $2.50. What tax does the owner pay?

**7.** Three men living in different cities were comparing their tax rates. A's rate was $3.56; B's rate was 28 mills; C's rate was 3.14%. Which had the largest rate? How much tax should each pay on an assessed value of $2,000?

**8.** Bailey owns property worth $15,000 and is taxed $4.50 per hundred dollars on $\frac{1}{2}$ of the real value. Osborne owns property worth $15,000 and is taxed 32 mills on $\frac{1}{3}$ the real value. Which pays the larger tax? How much does each pay?

**9.** Powell pays 3% on $\frac{1}{3}$ of the real value of his property. What per cent of the value of his property does he pay annually as taxes?

**10.** A loaned money at 6% and took a mortgage. Since a mortgage is personal property, it is taxable. The mortgage was taxed at a rate

of \$4 on $\frac{1}{3}$ the real value. What per cent of the value of the mortgage did A pay annually as taxes? What was his per cent of net income on the loan?

**Written Work**

1. Complete the following table:

| Real Value of Property | Fraction of Value Assessed | Assessed Value | Rate | Tax |
|---|---|---|---|---|
| \$16,250 | $\frac{1}{5}$ | | \$5.42 | |
| 7,920 | $\frac{1}{3}$ | | 3.18% | |
| 22,000 | $\frac{2}{5}$ | | \$2.89 | |
| 960 | $\frac{1}{5}$ | | 64 mills | |
| 4,000 | $\frac{1}{3}$ | | 2.92% | |

2. Hill rents his house for \$30.00 per month. The house cost him \$3850.00. What is the per cent of net profit on his investment, after paying the following annual expense?

Insurance, \$15.40.

Repairs, \$60.00.

Taxes: Property assessed at $\frac{1}{3}$ its value and taxed at \$6.23 per hundred dollars.

**247. To determine the tax rate.** Each of the taxing governments (state, county, township, city, school district, etc.) prepares a budget or estimate of the money needed for the following year.

The assessed value of the property available for taxes is found from the assessors' lists.

The amount of funds needed, divided by the assessed value of the property, gives the rate which must be levied in order to obtain the required amount.

The rates for the state, county, city, school district, etc., are added to form a single rate.

The following illustration shows the method of determining a tax rate.

State Tax

State budget . . . . . \$3,600,000

Assessed value of the property in the state . \$1,000,000,000

$$\frac{\$36,000,000}{3,600,000} \div \$1,000,000,000 = .36\% \text{ state rate}$$

COUNTY TAX

County budget . . . . . $30,000
Assessed value of the property in the county . $4,000,000
$30,000 ÷ $4,000,000 = .75 % county rate

CITY TAX

City budget . . . . . $12,000
Assessed value of property in the city . . . . $1,000,000
$12,000 ÷ $1,000,000 = 1.20 % city rate

SCHOOL TAX

School district budget . . . . . $24,000
Assessed value of the property in school district $1,000,000
$24,000 ÷ $1,000,000 = 2.40 % school tax rate
4.71 % total tax rate

**Problem.** Mr. Owen has a building worth $15,000.00, which is assessed at $\frac{1}{3}$ of its real value. What is the tax on the building at the above rate?

SOLUTION. Assessed value, $5,000.00.
Rate, 4.71 %.
.0471 × $5,000.00 = $235.50, tax.

### Oral Work

Explain why the tax rate in your city may be different from the rate in a neighboring city.

### Written Work

1. Find the tax rate (approximate).
State budget, $2,850,000.
Assessed value of property in the state, $1,450,000,000.
State rate?
County budget, $33,000.
Assessed value of property in county, $5,600,000.
County rate?
City budget, $14,800.
Assessed value of property in city, $1,345,000.
City rate?
School district budget, $32,500.
Assessed value of property in school district, $1,345,000.
School tax rate?
Total rate?

2. The following is the tax rate in the town of M.

| | |
|---|---|
| State tax | $ .60 |
| County tax | 1.25 |
| Township tax | .26 |
| City tax | 2.18 |
| School district tax | 2.86 |
| | $7.15 |

In this town, property is assessed at $\frac{1}{3}$ of its real value. Snyder owns the following property:

| | Real Values |
|---|---|
| Residence | $ 6,500.00 |
| Store | 4,950.00 |
| Office building | 13,000.00 |

What is his total tax?

3. How much of this tax goes to the state, and how much to each of the other taxing units?

4. What per cent of his tax goes to each of the taxing units?

5. The assessed value of Miller's real estate is $13,395.00. What is his total tax?

6. How much school tax does Miller pay?

# TABULATIONS TO PROMOTE EFFICIENT MANAGEMENT

## CHAPTER XXIX

### BUYING AND SELLING EXPENSES; NET PROFIT

**248. Expenses and Profits.** In Chapter XX the principles of gross earnings were discussed. You have now studied many of the expenses incurred in conducting a business, and are ready to consider the problems which arise in computing net profit.

**Net Profit** is the difference between the gross earnings of an enterprise and the total expense of conducting it.

**249. Buying Expenses** include all the costs of buying goods, such as freight and drayage, customs duties on goods imported, commissions of purchasing agents, insurance, and the cost of keeping goods in warehouses and placing them on the shelves of the salesroom. Buying expenses are added to the original purchase price of goods to determine the total cost. Thus:

Prime Cost of Merchandise is the original or purchase price;

Total Cost of Merchandise is the prime cost plus the buying expenses.

**Problem.** A retail furniture store purchases goods during the year amounting to $36,450.00. This is the prime cost. The freight, drayage, rent of warehouse, labor of warehouse employees, and other costs of placing the goods in the salesroom are $2,187.00.

What is the total cost of the merchandise purchased?

SOLUTION.

| | |
|---|---|
| $36,450.00 | Prime cost |
| 2,187.00 | Buying expenses |
| $38,637.00 | Total cost |

**250. Marking Cost of Goods.** When the cost price is marked on goods, some merchants add a certain per cent to the prime cost to

cover the buying expenses. The records of previous years are used as a basis to determine the per cent to be added.

In the illustration on page 291, the prime cost is $36,450.00, and the buying expenses $2,187.00, or 6% of the prime cost. The furniture dealer might say, "Next year's buying expenses will probably be about the same as they were this year, and I will therefore add 6% to the prime cost to cover them."

A safer method, however, is to base the computation on the average expenses of several years.

| Year | Prime Cost | Buying Expenses |
|---|---|---|
| 1919 | $ 35,275.00 | $1,840.13 |
| 1920 | 41,391.00 | 2,069.55 |
| 1921 | 62,387.00 | 3,618.45 |
| | $139,053.00 | $7,528.13 |

$7,528.13 ÷ $139,053.00 = 5.41+%.

For convenience, 5.4% probably would be adopted as the per cent of buying expenses.

**Illustration.** An article cost $8. At what price is it marked to cover buying expenses?

Solution.

| | |
|---|---|
| $8 | Prime cost |
| .055 | Per cent of buying expenses |
| $ .44 | Buying expenses |
| $8.00 | Prime cost |
| .44 | Buying expenses |
| $8.44 | Total marked cost |

### Written Work

The prime cost of a merchant's purchases and the buying expenses for several years are given below. Find the per cent of buying expenses for each year and the average for the entire interval.

| | Year | Purchases | Buying Expenses |
|---|---|---|---|
| 1. | 1917 | $ 9,426.90 | $ 643.92 |
| | 1918 | 12,318.26 | 812.29 |
| | 1919 | 14,126.29 | 830.12 |
| | 1920 | 11,341.72 | 690.19 |
| | 1921 | 14,985.29 | 752.29 |

| | Purchases | Buying Expenses |
|---|---|---|
| 2. 1920 | 39,286.26 | 2,326.80 |
| 1921 | 47,562.83 | 3,146.29 |
| 1922 | 59,286.28 | 4,210.08 |

**3.** The prime cost of merchandise in a certain store is increased 8 % to include buying expenses; the selling price is determined by adding 9 % to the total cost. Mark the total cost (using " blacksmith " as the key and " g " as a repeater) and the selling price (in figures) of articles, the prime cost of which was as follows:

| Stock No. | Prime Cost |
|---|---|
| A 326 | $2.19 |
| A 394 | 6.25 |
| F 246 | .90 |
| A 385 | 2.78 |

**4.** The manager of one of the departments in a large store adds 6 % to the prime cost to cover the buying expenses. He receives quotations of prices from manufacturers who offer different discounts. He prepares a table showing the total cost after deducting the discount offered and adding the 6 % buying expenses. The following is a similar table which you will complete:

| $ | .00 | | | .25 | | | .50 | | | .75 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Less 2 % Plus 6 % | Less 5 % Plus 6 % | −7 % +6 % | −2 % +6 % | −5 % +6 % | −7 % +6 % | −2 % +6 % | −5 % +6 % | −7 % +6 % | −2 % +6 % | −5 % +6 % | −7 % +6 % |
| 0 | | | | .2597 | .25175 | .24645 | | | | | | |
| 1 | 1.0388 | 1.007 | .9858 | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | |

**5.** By referring to the table, find the total cost, including buying expenses, of an article quoted at $2.25 less 5 %; $3 less 2 %.

Refer to the table, and find

6. Which is the better price, and how much better: \$1.75 each, less 2%, or \$2 each, less 7%? How much will be saved by purchasing one gross at the cheaper quotation?

**251. Selling Expenses.** After the goods have been placed on sale, all costs such as rent of store, salaries of clerks, advertising, heat and light, delivery, store supplies, insurance and taxes, depreciation and bad debts are considered as selling expenses.

**252. Finding the per cent of selling expenses.** The per cent of selling expenses is a matter of valuable information to a merchant. It is determined by the formula:

Selling Expenses ÷ Net Sales = % of Selling Expenses.

**Problem.** A merchant's net sales during one year are \$124,265; the selling expenses for the same year are \$29.078.01. The selling expenses are what per cent of the net sales?

SOLUTION. \$29,078.01 ÷ \$124,265.00 = 23.4%.

### Written Work

The following statistics are reprinted from *System*, and show the result of an investigation made by that magazine to determine *average* or *standard* per cents of selling expenses in various businesses.

Find what per cent each item of expense is of the total sales.

Find what per cent the total selling expenses are of the sales.

Prepare graphs as directed by your teacher.

| | DRY GOODS STORE | GENERAL STORE | SHOE STORE |
|---|---|---|---|
| | Sales, \$50,000.00 | Sales, \$50,000.00 | Sales \$25,000.00 |
| Rent | \$1,550.00 | \$1,154.85 | \$ 778.13 |
| Salaries | 4,800.00 | 4,067.09 | 2,786.21 |
| Advertising | 750.00 | 351.48 | 376.51 |
| Heat and light | 200.00 | 251.04 | 225.91 |
| Delivery | 450.00 | 954.01 | 75.30 |
| Supplies | 200.00 | 150.63 | 100.40 |
| Insurance and taxes | 550.00 | 200.84 | 301.21 |
| General expense | 2,200.00 | 150.63 | 1,029.14 |
| Depreciation and shrinkage | 700.00 | 301.27 | 150.61 |
| Bad debts | 150.00 | 150.63 | 25.10 |

| | Vehicle Store | Hardware Store | Furniture Store |
|---|---|---|---|
| | Sales, $57,000.00 | Sales, $45,000.00 | Sales, $100,000.00 |
| Rent | $1,094.63 | $1,305.61 | $3,507.36 |
| Salaries | 5,818.81 | 5,042.35 | 8,317.60 |
| Advertising | 633.73 | 405.20 | 2,906.15 |
| Heat and light | 345.67 | 225.10 | 801.70 |
| Delivery | 518.51 | 270.13 | 1,102.33 |
| Supplies | 230.45 | 180.08 | 501.06 |
| Insurance and taxes | 460.90 | 495.23 | 1,402.97 |
| General expense | 230.45 | 360.17 | 1,803.82 |
| Depreciation and shrinkage | 403.28 | 225.10 | 1,904.03 |
| Bad debts | 115.22 | 180.08 | 1,202.54 |

| | Clothing Store | Drug Store | Jewelry Store |
|---|---|---|---|
| | Sales, $60,000.00 | Sales, $20,000.00 | Sales, $30,000.00 |
| Rent | $1,322.24 | $ 924.65 | $1,080.40 |
| Salaries | 5,469.29 | 2,191.01 | 3,361.23 |
| Advertising | 2,043.47 | 522.64 | 1,050.38 |
| Heat and light | 180.31 | 160.81 | 180.07 |
| Delivery | 360.61 | 80.40 | 30.01 |
| Supplies | 120.20 | 60.30 | 270.10 |
| Insurance and taxes | 661.12 | 281.40 | 540.20 |
| General expense | 1,681.84 | 482.42 | 630.23 |
| Depreciation and shrinkage | 721.22 | 100.50 | 360.13 |
| Bad debts | 240.41 | 40.20 | 90.03 |

Save your results for use in the problems which follow.

**253. Efficient Management to Determine Economies and "Leaks."** When a merchant has a reliable standard with which to compare the various per cents of selling expenses in his business, he is able to determine the items of expense which are too large.

### Written Work

A dry goods store has sales during one year amounting to $40,000.00. The expenses are as follows:

| | | | |
|---|---|---|---|
| Rent | $1,200.00 | Supplies | $165.00 |
| Salaries | 3,240.00 | Insurance and taxes | 180.00 |
| Advertising | 250.00 | General expense | 155.00 |
| Heat and light | 225.00 | Depreciation and shrinkage | 120.00 |
| Delivery | 1,135.00 | Bad debts | 42.00 |

1. Each item of expense is what per cent of the sales?
2. The total selling expenses are what per cent of the sales?
3. Accepting the per cents found in the dry goods store problem on page 294 as a standard, determine which expenses are larger than the standard, and which are smaller.
4. Complete the table on the following page.
5. Plot a curve in black ink to show the per cent of gross earnings in each department. Plot a curve on the same paper in red ink to show the per cent of net profit or net loss in each department.

SUGGESTION. Draw a line horizontally across the paper representing 0 %; scale the paper to show per cents of gain above this line, and per cents of loss below the line.

**254. Common Omissions in Computing Selling Expenses.** Merchants sometimes underestimate their selling expenses by omitting the following:

(*a*) Depreciation.

Store buildings and fixtures depreciate in value, and while such depreciation may not be noticeable, it should be computed annually. Goods also become shopworn or out of style and the depreciation in value should be charged off in the year in which it occurs.

(*b*) Bad debts.

Careful merchants make every effort to collect their accounts, but unless a strictly cash business is conducted, some accounts are almost certain to prove uncollectable. In estimating the probable loss from bad debts, some merchants compute a certain per cent of the sales, while others compute a certain per cent of past due accounts.

### Written Work

A merchant invests $5,000.00 in a general store. His sales during a certain year are $17,350.00. He owns his store building valued at $3,000.00. He lists as his selling expenses the items at the top of page 298.

PROFITS IN 12 DEPARTMENTS

| DEPT. | STOCK, JAN. 1, 1922 | PURCHASES, DURING 1922 | TOTAL COST FOR 1922 | STOCK, DEC. 31, 1922 | COST OF GOODS SOLD | SALES, 1922 | GROSS PROFITS | % | EXPENSES | NET PROFITS | % | NET LOSS | % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | $15,223 40 | $118,427 56 | | $17,327 72 | | $162,341 72 | | | $26,514 95 | | | | |
| B | 21,714 32 | 168,427 92 | | 25,314 71 | | 184,231 76 | | | 25,601 51 | | | | |
| C | 26,361 71 | 176,204 91 | | 25,747 39 | | 197,432 65 | | | 21,430 13 | | | | |
| D | 13,247 91 | 112,347 29 | | 15,261 03 | | 120,261 32 | | | 29,675 55 | | | | |
| E | 20,403 71 | 150,265 97 | | 20,374 76 | | 174,231 61 | | | 14,512 53 | | | | |
| F | 27,349 61 | 160,231 39 | | 28,417 19 | | 174,628 32 | | | 16,669 17 | | | | |
| G | 25,732 92 | 237,511 86 | | 27,961 31 | | 284,162 39 | | | 13,635 20 | | | | |
| H | 21,113 65 | 107,221 13 | | 22,213 47 | | 135,261 34 | | | 15,509 01 | | | | |
| I | 23,416 47 | 121,376 41 | | 24,316 19 | | 142,221 32 | | | 13,459 85 | | | | |
| J | 30,265 45 | 164,213 96 | | 29,262 47 | | 174,227 42 | | | 11,581 19 | | | | |
| K | 18,462 31 | 174,216 91 | | 19,417 13 | | 202,346 21 | | | 9,420 57 | | | | |
| L | 15,271 34 | 121,216 09 | | 16,237 41 | | 161,213 47 | | | 14,748 47 | | | | |
| Total | | | | | | | | | | | | | |

Per cent of Gross Profit is found by dividing the Gross Profit by the Sales.
Per cent of Net Profit is found by dividing the Net Profit by the Sales.
Per cent of Net Loss is found by dividing the Net Loss by the Sales.

| | |
|---|---|
| Salaries (one clerk) | $600.00 |
| Advertising | 85.00 |
| Heat and light | 125.00 |
| Delivery | 140.00 |
| Insurance and taxes | 95.00 |
| General expenses | 110.00 |

1. What are the total expenses as shown by the list?

2. These expenses are what per cent of the sales?

The following items have been omitted:

7% would be a fair rate for depreciation on the building.

An examination of this merchant's books shows that his annual loss from bad debts is about 1% of his sales.

3. Make a list of the selling expenses of this business, including those which the merchant omitted.

4. What are the total expenses?

5. The total expenses are what per cent of the sales?

6. The cost of the goods sold is $14,520.00.

What is the gross profit?

7. What does the merchant consider to be his net profit after deducting the expenses as stated in his list?

8. After deducting his real expenses as shown by your corrected list, did he have a net gain or a net loss? How much?

9. Find what per cent each item of selling expenses is of the sales.

10. By comparing these per cents with the average or standard per cents, we may find the "leaks" of this business.

Which of these per cents are smaller than the standard?

Which of the per cents are larger than the standard?

Although the general expenses are large, they include supplies for which no separate charges are made.

11. What is the worst "leak" in this business?

**255. Reducing the Per Cent of Selling Expenses by Increasing the Gross Sales.** It is evident that as the per cent of selling expenses increases, other factors remaining the same, the per cent of profit on the sales decreases. It is therefore desirable to decrease the per cent of selling expenses. The per cent of selling expenses may be reduced by

Reducing the selling expenses, or

Increasing the sales.

**Written Work**

1. If the sales were \$50,000.00 and the selling expenses \$10,000.00, what was the per cent of selling expenses?

2. If the selling expenses could be reduced to \$8,000.00, without reducing the sales, what would be the per cent of selling expenses?

3. If the selling expenses remained \$10,000.00, and the sales were increased to \$60,000.00, what would be the per cent of selling expenses?

4. If an increase of the sales to \$80,000.00 resulted in an increase of selling expenses to \$12,000.00, what would be the per cent of selling expenses?

**Increasing the Sales.** The sales may be increased in three ways:
By increasing the selling price; the turnover remaining the same.
By increasing the turnover; the selling price remaining the same.
By increasing the selling price and the turnover.

Increasing the selling price of a commodity may result, however, in decreasing the sales by causing a decreased demand. For this reason the more popular method of increasing sales is to sell at a small profit and to make frequent "turnovers."

The "turnover" means the number of times by which the sales exceed the average inventory.

Thus, if the average value of the stock kept is \$5,000.00 (selling price) and the gross sales for a year are \$5,000.00, there is one turnover. If the gross sales are \$15,000.00, there are three turnovers.

**256. Small Profits and Frequent Turnovers.** The following illustration shows how a larger profit may be made from several turnovers at a small markup,[1] than by a single turnover at a large markup.

Let us suppose that a merchant's stock inventories at cost prices \$10,000.00 (which includes buying expenses). If the selling expenses are \$5,000.00, the merchant must obtain \$15,000.00 before he begins to make any profit. This means a markup of 50%. In order to make a 5% profit on the cost, he must mark his goods at 55% above cost. An article costing \$1.00 must sell for \$1.55. Thus, a single turnover at 55% markup will net a profit of \$500.00.

Now let us suppose that a 30% markup (selling an article costing \$1.00 for \$1.30) will result in two turnovers without increasing the selling expenses.

[1] By "markup" is meant the per cent added to prime cost and buying expenses to cover selling expenses and profit.

# CHAPTER XXX

## LIFE INSURANCE

INSURANCE against loss due to sickness, accident, or death is called **Personal Insurance.**

There are several classes of Personal Insurance of which the following are the most common: Accident Insurance, Health Insurance, Liability Insurance, and Life Insurance. Only Life Insurance will be considered in this chapter.

In consideration of certain payments, called **Premiums**, a Life Insurance Company agrees to pay, at the death of the insured, or at some other time stipulated in the contract, called the **Policy**, a stated sum of money to the person designated by the insured. The person to whom payment is to be made is called the **Beneficiary.** There are many kinds of policies, and policies of the same name often differ in certain details.

While it will be impossible to present the subject of Life Insurance in great detail in this chapter, the fundamental principles of the various types of policies will be discussed.

**257. Principal Kinds of Life Insurance Policies.** The principal kinds of policies are:

*a.* Ordinary Life policy.
*b.* Limited Life policy.
*c.* Endowment policy.
*d.* Term policy.

These differ in three important particulars.

*a.* The number of premiums paid by the insured.
*b.* The amount of each premium.
*c.* The time when payment is made by the company.

COMPARISON OF SEVERAL KINDS OF POLICIES

| KIND OF POLICY | NUMBER OF YEARS PREMIUMS ARE PAID | TIME WHEN PAYMENT IS MADE BY THE COMPANY |
|---|---|---|
| Ordinary Life . . . . | During life of insured [In some companies, if dividends are allowed to accumulate with the company, this period may be shortened.] | At death of insured |
| * Limited Life — | | |
| 20-payment life . . . | 20 years | At death of insured |
| 10-payment life . . . | 10 years | At death of insured |
| * Endowment Policy — | | |
| 20-year endowment . | 20 years | At death of insured payment made to beneficiary; or at expiration of 20 years payment made to insured if still living |
| 10-year endowment . | 10 years | At death of insured or at expiration of 10 years |
| 20-payment, 30-year endowment . . . | 20 years | At death of insured or at expiration of 30 years |
| Term Policy — | | |
| 20 years . . . . . | 20 years | At death of insured if he dies within 20 years. If he lives beyond this term, no payment is made |

* The policies listed under this heading are illustrative only. Policies are issued for other periods than those mentioned.

**258. Insurance Rates.** Insurance rates are usually quoted at a certain price per $1000 of insurance. The rate depends upon the age of the insured when he "takes out the policy," and upon the kind of policy. The premiums are payable in advance.

The desirability of insuring at as early an age as practicable, is shown by a study of the comparative premiums at different ages.

The following table shows the premium rates per thousand on various kinds of policies for various ages.

ANNUAL PREMIUM ON DIFFERENT KINDS OF INSURANCE PER $1000.00

| AGE | ORDINARY LIFE | 20-PAY. LIFE | 10-PAY. LIFE | 10-YEAR ENDOWMENT | 20-YEAR ENDOWMENT |
|---|---|---|---|---|---|
| 15 | $ 17.40 | $27.34 | $ 44.62 | $100.60 | $47.79 |
| 20 | 19.21 | 29.39 | 47.85 | 101.57 | 48.48 |
| 25 | 21.49 | 31.83 | 51.67 | 102.73 | 49.33 |
| 30 | 24.38 | 34.76 | 56.18 | 104.14 | 50.43 |
| 35 | 28.11 | 38.34 | 61.53 | 105.87 | 51.91 |
| 40 | 33.01 | 42.79 | 67.90 | 108.07 | 54.06 |
| 45 | 39.55 | 48.52 | 75.57 | 111.03 | 57.34 |
| 50 | 48.48 | 56.17 | 84.99 | 115.28 | 62.55 |
| 55 | 60.72 | 66.69 | 96.66 | 121.48 | 70.81 |
| 60 | 77.69 | 81.60 | 111.47 | 130.76 | 83.82 |
| 65 | 101.48 | | 131.13 | 145.08 | |

**Written Work**

From the preceding table find the annual premium required for:

1. An ordinary life policy for $4000, taken by a man 20 years of age.

2. A ten-payment life policy for $3000, taken by a man 25 years of age.

3. A ten-year endowment policy for $5000, taken by a man 35 years of age.

4. A twenty-year endowment policy for $5000, taken by a man 25 years of age.

5. A man 25 years of age took out a twenty-year endowment policy for $5000, and died after making eight payments. How much less would the combined premiums have been on an ordinary life policy?

6. A man on his 30th birthday took out a twenty-payment life policy for $3000, and 5 years later he took out an ordinary life policy for $5000. He died after making fifteen payments on his first policy. By how much did the insurance received by his beneficiary exceed the premiums paid (not considering accumulations)?

**259. Dividends.** Policies written on the "participating plan" provide that a portion of the annual profits of the company shall be shared by the holders of the policies. These annual payments of a share of the profits are called "dividends." Since the profits of the

company vary from year to year, no specific amount of dividend is guaranteed to the policyholder. Policyholders usually receive no dividend the first year. Dividends may, at the option of the insured, be

*a.* Paid to him in cash;

*b.* Applied to the payment of his premium;

*c.* Left with the company and allowed to accumulate at compound interest;

*d.* Left with the company to increase the amount of insurance carried;

*e.* Left with the company in order to decrease the number of payments of premiums.

The following table shows the effect of the dividends in reducing the annual premiums.

ANNUAL CASH DIVIDENDS AND NET COST OF INSURANCE ON POLICIES OF $1000 — AGE 35

| | Ordinary Life Policy Annual Premium, $27.30 | | 20-Premium Life Policy Annual Premium, $36.00 | | 20-Year Endowment Policy Annual Premium, $50.80 | |
|---|---|---|---|---|---|---|
| | Dividend at End of Year | Cost for Each Year | Dividend at End of Year | Cost for Each Year | Dividend at End of Year | Cost for Each Year |
| 1 | $3.00 | $24.30 | $3.30 | $32.70 | $3.80 | $47.00 |
| 2 | 3.10 | 24.20 | 3.45 | 32.55 | 4.05 | 46.75 |
| 3 | 3.20 | 24.10 | 3.60 | 32.40 | 4.35 | 46.45 |
| 4 | 4.10 | 23.20 | 4.40 | 31.60 | 4.95 | 45.85 |
| 5 | 4.25 | 23.05 | 4.60 | 31.40 | 5.25 | 45.55 |
| 6 | 4.70 | 22.60 | 5.30 | 30.70 | 6.60 | 44.20 |
| 7 | 5.25 | 22.05 | 6.05 | 29.95 | 7.70 | 43.10 |
| 8 | 5.80 | 21.50 | 6.75 | 29.25 | 8.65 | 42.15 |
| 9 | 6.30 | 21.00 | 7.30 | 28.70 | 9.30 | 41.50 |
| 10 | 6.50 | 20.80 | 7.55 | 28.45 | 9.65 | 41.15 |

**260. Lapses.** If the annual premium is not paid when due, the policy lapses. However, most companies allow one month's grace, during which time the overdue premium may be paid with interest. If death occurs during this period of grace, the amount of the overdue premium and the accrued interest are deducted in making settlement with the beneficiary.

If the premium remains unpaid after the expiration of the period of grace, the policy may be reinstated, but the policyholder must furnish satisfactory evidence of health and pay all back premiums with accrued interest.

**261. Cash Surrender, Loan, and Paid-up Insurance.** After a policy has been in force for a certain number of years (usually two or three), many companies offer the insured the following privileges:

*a.* Borrowing money from the company;

*b.* Surrendering the policy for a cash payment;

*c.* Receiving a "paid-up" policy which gives a fixed amount of insurance during the remainder of life without further payment of premiums.

*d.* Being insured for the face of the policy for a fixed number of years and months.

For example, after an ordinary life policy for $1000.00, taken at the age of 35, in a certain company, has been in force 8 years, the insured can —

*a.* Borrow $108.00 from the company on the security of the policy.

*b.* Surrender the policy and receive $108.00 in cash.

*c.* Cease the payment of premiums and be insured for the remainder of his life for $223.00.

*d.* Cease the payment of premiums and be insured for $1000.00 for 10 years and 9 months.

**262. Modes of Settlement.** Upon proof of the death of the insured (or, in case of an endowment policy, at the expiration of the endowment period), the policy becomes payable by the company. Settlement may be made in a variety of ways, the most common of which are:

*a.* Payment of cash to the beneficiary;

*b.* Annual payment of interest during the life of the beneficiary, and payment of the face of the policy at the death of the beneficiary;

*c.* Payment of equal annual installments for the number of years specified to the beneficiary;

*d.* Payment of equal annual installments for a period of twenty years, and for as many years thereafter as the beneficiary shall live. The amount of each annual payment depends upon the age of the beneficiary at the death of the insured.

### Oral Work

1. Explain the reason for the difference in the premiums of the different kinds of policies.
2. Why is it often wise to take insurance at an early age?
3. What is participating insurance?

### Written Work

Refer to the table of annual premiums on different kinds of insurance and find the annual premium on the following:

1. $2000, ordinary life policy, taken at the age of 35.
2. $5000, 20-year endowment policy, taken at the age of 35.
3. $3000, 10-payment life policy, taken at the age of 35.
4. $6000, 20-payment life policy, taken at the age of 35.
5. $10,000, 10-year endowment policy, taken at the age of 35.
6. On his 25th birthday Mr. Anderson took out a 20-payment life policy for $3000; on his 30th birthday he took out a 20-year endowment policy for $5000; on his 35th birthday he took out an ordinary life policy for $7000. He died at the age of 38 years 7 months. How much more did his beneficiary receive (dividends excepted) than he paid the company? (Use the table on page 305.)

## CHAPTER XXXI

### FINAL REVIEW

1. A manufacturer sells an article to a wholesaler at a price 20% above cost; the wholesaler sells it to a retailer at a price 15% above the amount he paid for it. The retailer adds a profit of $11.32 and sells it for $130.00. What was the cost to the manufacturer?

2. A manufacturer sold an article to gain $33\frac{1}{3}$% on the selling price; the wholesaler sold to make 25% on sales; the retailer sold to make 25% on sales. The retail selling price was $160.00. What was the manufacturing cost?

3. You purchase an invoice of goods for $30.00 and find that 10% of the goods are unsalable. You price the remaining goods to sell so that you will make 10% on the invoice cost. What is the selling price? the rate of profit on cost, and the rate of profit on sales?

4. Mr. Hawkins purchases a house for $10,000.00 and rents it for a year at $85.00 per month. Taxes, insurance, and incidental expenses are $205.00 for the year and he estimates the depreciation at 3% per annum. What net rate of return does he receive on his investment?

5. A merchant purchases a commodity for $3.00 and sells it for $4.00. In order to meet competition he is forced to reduce the selling price to $3.75. What per cent must he increase his sales to make the same gross profit as before?

6. I mark up a certain kind of merchandise 30% on cost, but am unable to sell it at that price, therefore I reduce the selling price 10%. After reducing the selling price, what rate of profit will I make on cost and what rate on selling price?

7. An article cost $8.00 to manufacture and is sold for $12.00. Because of increases in the cost of materials, labor and manufacturing expenses, the cost to manufacture goes up to $10.00 and the selling price is increased to $14.00. What per cent of profit is made before the increase in cost, and what per cent is made after the increase?

8. At what price must the article sell to make the same rate of profit on sales as before?

**9.** I offered to sell goods at a profit of 25 % but was unable to make the sale. In order to dispose of the goods I sold them for $1,800.00, making a profit of 20 %. What was the cost of the goods, and at what price did I originally offer them?

**10.** Two concerns offer you the same goods; the first quotes a price of $340.00 less trade discounts of 15 % and 10 %; the second quotes the same price, with discounts of 10 %, 10 %, and 5 %. Which is the better offer and how much?

**11.** What difference is there in the following discount series?

15 %, 10 %, and 5 %
10 %, 15 %, and 5 %
5 %, 10 %, and 15 %

**12.** What is the difference between a discount series of 10 %, 10 %, and 5 % and a single discount of 25 %? What is the difference on an invoice with a list price of $1,500.00?

**13.** R. S. Dutton marks goods at an increase of 30 % on cost and allows a discount of 10 % from the list price. What rate of profit does he make on sales?

**14.** If a merchant buys goods at 20 %, 25 %, and 10 % off the list price and sells them at 15 % off the list price, what rate of profit does he make on sales?

**15.** If a merchant sells goods at 20 % and 10 % off from the list price, what per cent must he add to the cost of the goods in order to make 20 % on the selling price?

**16.** At the end of February the Second National Bank sends a statement to D. B. Warren showing a balance of $376.85. Warren examines his returned checks and finds that the following checks issued in February have not been returned:

| | |
|---|---|
| No. 2,019 | $6.50 |
| No. 2,023 | $7.55 |

A deposit of $93.68 made on February 28 does not appear on the bank's statement. What balance should be shown on Warren's check stub at the end of February in order to have the stub and the bank statement reconcile?

**17.** What is the cost of transferring $215.60 by postal money order?

**18.** What is the cost of transferring the same amount by express money order?

**19.** A manufacturer sells washing machines through an agent to whom he pays 15 % of the selling price, and makes a gross profit of 25 % on sales. He decides to increase the selling price 10 % and allow the agent 20 % on sales. What is his per cent of profit on sales?

**20.** I owe \$409.50 on a promissory note which has run 5 months and 10 days and is due today. The note bears 5½ % interest. What total payment will be required to pay the note?

**21.** Bailey and Wharton allow 1 % discount if bills are paid within 10 days from the date of invoice; bills are due 30 days after date and interest at 6 % is charged thereafter. If I buy goods on March 17 for \$621.80 what amount will settle the bill on March 23? on April 12? on June 23?

**22.** A loan of \$15,000.00 at 4½ % made on May 16 is called on July 23. How much interest is payable?

**23.** Dalton loans \$764.00 on March 18 at 6 %. The note is due on August 18. How much interest is due if computed by the customary commercial method? How much is the accurate interest?

**24.** The interest on a \$630.00 note for 85 days is \$10.41. What is the rate?

**25.** A bank discounts a \$600.00 sixty day note and credits the borrower's account with proceeds of \$593.50. At what rate was the note discounted?

**26.** The interest on a certain loan of \$750.00 at 6 % is \$5.63. How many days did the note run?

**27.** The interest for 75 days at 6 % on an unknown principal is \$16.88. What is the principal?

**28.** What is the bank discount on a non-interest bearing note discounted at 7 % fifty days before maturity?

**29.** You held a note signed by G. L. Kenton dated July 15. The note bears 5 % interest and is due in 90 days. On August 20 you discounted the note at the bank at 7 %. State the discount period and the proceeds.

**30.** A manufacturer decides to use the differential piece work method of paying wages. After careful study he fixes his rates as follows:

30 cents each for 20 articles if made in a day;

a reduction of 5 % in the rate for each article less than 20 made per day; that is, if 19 are made, the rate is 95 % of \$.30; if 18 are made, the rate is 90 % of \$.30 each.

an increase of 10 % in the rate for each article in excess of 20, the first 20 being paid for at the standard rate; that is, if 23 are made in a day, the first 20 are paid for at \$.30 each. The 21st article is paid for at the rate of 110 % of \$.30; the 22d article at 120 % of \$.30; the 23d article at 130 % of \$.30.

Draw up a wage scale showing the amount of daily wages to be paid for the work on 10 to 30 pieces inclusive.

**31.** Draw up a coin sheet for the payment of a week's payroll of the above factory:

| Operative's Number | Name | Number Produced | | | | | |
|---|---|---|---|---|---|---|---|
| | | M | T | W | T | F | S |
| 1 | D. H. Mather | 21 | 23 | 22 | 24 | 20 | 19 |
| 2 | Oscar Hart | 27 | 26 | 24 | 29 | 25 | 23 |
| 3 | Harold Barclay | 18 | 19 | 20 | 22 | 17 | 21 |
| 4 | George Benton | 22 | 22 | 29 | 28 | 24 | 25 |

**32.** The owner of a brick residence takes an insurance policy of \$8,000.00 on March 15 at a rate of 48 cents. He also takes a policy of \$4,000.00 on the contents of the residence, the rate being 10 % higher. Both are one year policies. What is the cost of the insurance?

**33.** If the policies are cancelled by the insurance company on June 10, what portion of the premium is returned?

**34.** What was the agent's commission on these policies at 20 %?

**35.** M has insured his residence for \$7,500.00 under a policy containing the 80 % co-insurance clause. The building is burned and at the time of the fire it is estimated that its sound value was \$10,500.00 How much insurance will M recover?

**36.** Property in a certain state is assessed at $\frac{2}{5}$ of its real value; the rate in one town in this state is \$2.60. The assessor considers a piece of real estate worth \$18,500.00. What is the amount of the property tax?

**37.** What is the compound interest at 5 % on \$940.00 for 2 years, 3 months, 15 days, interest compounded semi-annually?

**38.** The statistics of a certain business show that 4 % should be added to the cost of merchandise purchased to cover buying expenses. An invoice costing \$28.75 wholesale is marked to sell for \$35.00. What is the gross profit, and the rate of profit on sales?

**39.** A man deposits \$15.00 monthly in a savings bank which pays 4 % annually. Interest is computed quarterly on all amounts which

have been on deposit for a month or more. How much will his balance be at the end of two years if no withdrawals are made?

**40.** Keep a record of the way you spend your time for twenty-four hours, and prepare a graph showing the time spent in sleep, at recitations, in studying, in recreation, at outside work, at meals, etc. Prepare this graph on the basis of hours and minutes.

**41.** Use the same data to prepare a graph showing the per cent of time devoted to each.

**42.** For this problem divide the class into groups of about five or six members each. Each individual except one in each group represents a bank which will be known by his name, as Smith National Bank. Let each individual bank prepare a statement purporting to represent the total checks which it has paid during the preceding day for each other bank in the group; these payments shall not be more than $10,000.00 for each other bank in the group. In addition to the statement, each banker should prepare a slip the size of a check for each bank for which it has made payment; on this slip write the name of the bank and the amount of checks paid. These slips will represent the checks paid by the bank. One member of each group should be appointed as clearing house manager and the bankers should turn over their statements and checks to the manager. Each banker should also retain a copy of his statement. The clearing house manager should then prepare a manager's sheet to determine the debit balances due from some banks and the credit balances due to other banks. The manager should also turn over to each banker the checks paid for him by the other banks, and each banker should determine whether he has a debit or a credit balance. Compare the balances computed by the manager with the various balances computed by the bankers. The manager should then require each banker with a debit balance to give him a bank draft for the amount of the balance and he should endorse these drafts to be deposited in some city bank to the credit of the clearing house. The bankers may draw their drafts on a bank in any city where they are assumed to have deposits. The clearing house manager should now draw checks on the bank where the clearing house makes its deposit, these checks to be payable to the banks with credit balances.

# APPENDIX A

## DENOMINATE NUMBERS

### SURVEYORS' LONG MEASURE

| | |
|---|---|
| 7.92 inches | = 1 link (lk.) |
| 25 links | = 1 rod |
| 4 rods, or 100 links | = 1 chain (ch.) |
| 80 chains | = 1 mile |

### SURVEYORS' SQUARE MEASURE

| | |
|---|---|
| 625 square links | = 1 square rod |
| 10 square rods | = 1 square chain |
| 16 square chains | = 1 acre |
| 640 acres | = 1 square mile |
| 36 square miles | = 1 township |

The following units of measure are used by sailors:

| | |
|---|---|
| 6 feet | = 1 fathom (Used for measuring depths at sea.) |
| 120 fathoms | = 1 cable length (Used for measuring depths at sea.) |
| About 1.15 statute miles | = 1 knot, or 1 nautical mile, or 6080.27 ft. |

### CIRCULAR OR ANGULAR MEASURE

| | |
|---|---|
| 60 seconds (60″) | = 1 minute (′) |
| 60 minutes | = 1 degree (1°) |
| 90 degrees | = 1 right angle |
| 360 degrees | = 1 circumference |

### TROY WEIGHT

| | |
|---|---|
| 24 grains (gr.) | = 1 pennyweight (pwt. or dwt.) |
| 20 pennyweights | = 1 ounce (oz.) |
| 12 ounces | = 1 pound (lb.) |

## APOTHECARIES' WEIGHT

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb.)

Apothecaries' weight is used by physicians and druggists. Troy weight is used in the measurement of precious metals.

## COMPARISON OF TROY AND AVOIRDUPOIS WEIGHTS

1 pound Troy = 5760 grains
1 pound avoirdupois = 7000 grains
1 ounce Troy = 437½ grains
1 ounce avoirdupois = 480 grains

The term *carat* has two meanings:

In weighing precious stones, a carat usually means 3.2 Troy grains.

In expressing the purity of gold, 24 carats means pure gold; 18 carats means $\frac{18}{24}$ pure gold, and $\frac{6}{24}$ alloy.

## PAPER MEASURE

24 sheets = 1 quire (qr.)
20 quires = 1 ream (rm.)

Although a ream contains 480 sheets, 500 sheets are usually sold as a ream.

## STANDARD UNITS OF WEIGHT

| | |
|---|---|
| 1 barrel flour | weighs 196 pounds |
| 1 barrel salt | weighs 280 pounds |
| 1 barrel pork or beef | weighs 200 pounds |
| 1 keg of nails | weighs 100 pounds |

## STANDARD BUSHELS IN MANY STATES

| | |
|---|---|
| 1 bushel shelled corn | weighs 56 pounds |
| 1 bushel ear corn | weighs 70 pounds |
| 1 bushel wheat | weighs 60 pounds |
| 1 bushel barley | weighs 48 pounds |
| 1 bushel rye | weighs 56 pounds |
| 1 bushel oats | weighs 32 pounds |

## ABBREVIATIONS COMMONLY USED IN BUSINESS

(The singular form is commonly used for both the singular and plural.)

A. . . . . acre
acct. or a/c. account
agt. . . . . agent
amt. . . . . amount
ans. . . . . answer
Apr. . . . April
Aug. . . . August
av. . . . . average
bg. . . . . bag; bags
bal. . . . . balance
bbl. or brl. . barrel
bdl. . . . . bundle
bkt. . . . . basket
bl. . . . . bale
B/L . . . . bill of lading
bot. . . . . bought
bu. . . . . bushel
bx. . . . . box
C . . . . . one hundred
cd. . . . . cord; card
cg. . . . . centigram
ch. . . . . chain; chest
chg. . . . . charge
c.i.f. . . . . carriage and insurance free
ck. . . . . check
cm. . . . . centimeter
cml. . . . . commercial
c/o . . . . care of
Co. . . . . company; county
c.o.d. . . . collect on delivery
coll. . . . . collection
com. . . . . commission
consg't . . . consignment
cr. . . . . creditor; credit; crate
cs. . . . . case
csk. . . . . cask
c., ct. . . . cent; centime
cu. ft. . . . cubic foot
cu. in. . . . cubic inch
cu. yd. . . . cubic yard
cwt. . . . . hundredweight
*d.* . . . . . pence
da. . . . . day
Dec. . . . . December
dep't . . . department
dft. . . . . draft
disc. . . . . discount
do. . . . . ditto (the same)
doz.; dz. . . dozen; dozens
Dr. . . . . debit; debtor; doctor
E. . . . . East
ea. . . . . each
*e.g.* . . . . for example
e.o.e. . . . errors and omissions excepted
etc. . . . . and so forth
ex. . . . . example; express
exch. . . . exchange
exp. . . . . expense
f. . . . . franc
far. . . . . farthing
Feb. . . . February
fir. . . . . firkins
f.o.b. . . . free on board
frt. . . . . freight
ft. . . . . foot
gal. . . . . gallon
gi. . . . . gill
gr. . . . . grain
gro. . . . . gross
gr. gro. . . great gross
guar. . . . guaranty; guarantee
hf. . . . . half
hf. cht. . . half chest
hhd. . . . hogshead
hr. . . . . hour
*i.e.* . . . . that is
in. . . . . inch; inches
ins. . . . . insurance
inst. . . . . instant; the present month
int. . . . . interest
I; inv. . . invoice
inv't . . . inventory
Jan. . . . . January
kg. . . . . keg
km. . . . . kilometer
l. . . . . link
lb. . . . . pound
l.p. . . . . list price
ltd. . . . . limited

## ABBREVIATIONS COMMONLY USED IN BUSINESS — (*continued*)

M . . . . one thousand
m. . . . . mill; meter
Mar. . . . March
mdse. . . . merchandise
Messrs. . . *Messieurs;* Gentlemen; Sirs
mi. . . . . mile
min. . . . minute
mo. . . . . month
mortg. . . . mortgage
Mr. . . . . Mister
Mrs. . . . . Mistress
N. . . . . North
no. . . . . number
Nov. . . . November
Oct. . . . . October
O.K. . . . all correct
oz. . . . . ounce
p. . . . . page
pay't . . . payment
pc. . . . . piece
pd. . . . . paid
per . . . . by; by the
per cent . . per centum; by the hundred
pfd. . . . . preferred
pk. . . . . peck; pecks
pkg. . . . . package
pp. . . . . pages
pr. . . . . pair
prox. . . . the following month
pt. . . . . pint
pwt. . . . . pennyweight
qr. . . . . quire
qt. . . . . quart
rd. . . . . rod
rec'd . . . received
rec't . . . receipt
rm. . . . . ream
Rm. (or M.) Reichmark; Mark
*s.* . . . . . shilling; shillings
S. . . . . south; sales
sec. . . . . second
sec'y . . . secretary
Sept. . . . September
set. . . . . settlement
ship. . . . shipment
shipt. . . . shipped
sig. . . . . signed; signature
sq. ch. . . . square chain
sq. ft. . . . square foot
sq. mi. . . square mile
sq. rd. . . . square rd.
sq. yd. . . square yard
stk. . . . . stock
sund. . . . sundries
T. . . . . ton
tb. . . . . tub
Tp.; Twp. township; townships
tr.; trans. . transfer
treas. . . . treasurer; treasury
ult. . . . . last month
via . . . . by way of
viz. . . . . namely; to wit
vol. . . . . volume
wk. . . . . week
wt. . . . . weight; weigh
yd. . . . . yard
yr. . . . . year

## TABLE OF SYMBOLS

a/c . . account
a/s . . account sales
+ . . . addition
( ) . . aggregation
& . . . and
... . . . and so on
@ . . . at; each; to
' . . . foot; feet; minutes
c/o . . . . . care of
¢ . . . . . cent
√ . . . . . check mark; correct
° . . . . . degree
÷ . . . . . division
$ . . . . . dollar; dollars
= . . . . . equal; equals
o/d . . . . . on demand

**TABLE OF SYMBOLS** — (*continued*)

| | | | |
|---|---|---|---|
| $^1, ^2, ^3$ | fourths (written as exponents, thus, $3^1 = 3\frac{1}{4}$) | % | per cent |
| | | £ | pounds sterling |
| > | greater than | : | ratio |
| C | hundred | ∵ | since |
| ″ | inch; inches; seconds | − | subtraction |
| < | less than | ∴ | therefore |
| × | multiplication | M | thousand |
| ♯ | if written before figures, means number; if written after figures, means pounds | | |

# APPENDIX B

## FARM RECORDS

**The Cash Book.** One of the most important records for a farmer is the cash book, in which he can tabulate the various sources of his income and the amounts of his various expenditures.

The following are three of the forms which may be used for this purpose:

*A*

| | | | Receipts | Expenditures |
|---|---|---|---|---|
| 1922 | | | | |
| Oct. | 2 | Balance | 123 75 | |
| | 2 | 15 bu. apples @ .70 | 10 50 | |
| | 2 | 12 doz. eggs @ .30 | 3 60 | |
| | 3 | Groceries | | 5 95 |
| | 5 | Lumber | | 26 25 |

*B*

| Receipts | | | | | | Expenditures | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1922 | | | | | | 1922 | | | | | |
| Oct. | 2 | Balance | | | 123 75 | Oct. | 3 | Groceries | | 5 95 | |
| | 2 | 15 bu. apples @ .70 | | 10 50 | | | 5 | Lumber | | 26 25 | |
| | 2 | 12 doz. eggs @ .30 | | 3 60 | | | | | | | |

*C*

Receipts

| 1922 | | Item | Dairy | Poultry | Crops | General |
|---|---|---|---|---|---|---|
| May | 1 | 2 cows to Owen | 125 00 | | | |
| | 2 | 10 doz. eggs @ .28 | | 2 80 | | |
| | 3 | 4 T. hay | | | 60 00 | |

EXPENDITURES

| 1922 | | ITEM | DAIRY | POULTRY | CROPS | HOUSEHOLD | GENERAL |
|---|---|---|---|---|---|---|---|
| May | 1 | 500 # bran | 8 00 | | | | |
| | 2 | 10 bu. seed oats | | | 6 00 | | |
| | 3 | Groceries | | | | 3 60 | |
| | 4 | Oyster shell | | 1 50 | | | |
| | 5 | Cement | | | | | 8 00 |

**Written Work**

Rule a form for a cash book similar to the last illustration.

Include an extra column in both "Receipts" and "Expenditures" labeled "Stock" and enter the following facts:

March 1, Balance, $280.00.
1, Sold for cash 1 carload hogs, $960.00.
1, Bought 2 cows, $45.00 and $58.00
2, Bought 1 set double work harness, $21.00.
3, Bought 1 work horse, $120.00.
4, Sold 30 doz. eggs at 45 cents.
5, Sold 8 T. timothy hay at $14.00 per ton.
6, Sold 1 load oats to elevator, 95 bu., at 36 cents.
6, Bought groceries, $7.95.
8, Sold one second-hand riding plow, $15.00.
9, Bought 20 sacks cement at $.50.
10, Bought 90 bu. seed oats at 48 cents.
11, Bought 500# middling for hogs at $30.00 per ton.
12, Bought 2 riding plows at $55.00 each.
15, Received milk check for 30 da., $226.47.
16, Bought groceries, $6.83.
17, Bought wheat screening for chickens, $2.50.
20, Sold 2 steer calves at $9.00 and $12.00.
22, Sold 18 young roosters, 57# at 14¢ per pound.
24, Rented a three-horse team to a neighbor for 2½ da. at $3.75 per day.
26, Bought 10 T. rock phosphate at $3.00 per ton.
28, Sold 28 doz. eggs at 37 cents.
28, Bought 1 pair shoes, $3.85; suit of clothes, $18.00.

Find the total receipts and expenditures for each column, and the balance at the end of the month.

**Farm Profits.** The annual increase or decrease of the farmer's wealth may be determined by taking an inventory each year, and comparing it with the inventory of the preceding year. The inventory should be classified to show the capital invested in land, building, live stock, machinery, and other property. An inventory is reproduced on pages 324 and 325.

**What is Farm Profit?** The annual increase shown by the two inventories is not the farmer's net gain for the year. The real annual profit is found by the following method:

Increase shown by inventories, plus living supplied by farm to family, plus interest paid on indebtedness, minus interest on capital invested, minus wages of farmer and family, equals actual net profit.

**Problem.** Let us suppose the inventories show an increase for the year of $2,000. The farm has also supplied a residence and provisions for the farmer's family. If the house rent is estimated at $120 and the supplies are valued at $900, the total, $1,020, should be credited to the farm. Adding $80 interest paid on the farm mortgage, we have the following:

| | |
|---|---|
| $2,000 | Increase of inventory |
| 1,020 | Household expenses |
| 80 | Interest paid |
| $3,100 | Produced by farm |

The farmer has $10,000 invested in the farm. If this money were loaned at 5% interest, it would produce $500 interest annually. $500 interest must therefore be subtracted.

If the farmer had worked for some one else, he might have earned wages of, say, $700. This amount must also be subtracted in order to determine real profit.

Thus, we have

$3,100 − $500 (interest) − $700 (wages) = $1,900, profit.

Real profit may be determined by the preparation of a statement similar to the following:

| Item | March 1, 1921 | March 1, 1922 |
|---|---|---|
| *Resources* | | |
| Real estate | $15,000 | $15,000 |
| Live stock | 3,160 | 3,590 |
| Machinery and tools | 530 | 575 |
| Feed and supplies | 860 | 735 |
| Cash | 140 | 275 |
| Bills receivable | | 50 |
| Accounts receivable | 93 | 126 |
| Total resources | 19,783 | 20,351 |
| *Liabilities* | | |
| Accounts payable | 135 60 | 215 20 |
| Mortgage | 1,000 | 1,000 |
| Total liabilities | 1,135 60 | 1,215 20 |
| Net worth | 18,647 40 | 19,135 80 |
| Increase in net worth | | 488 40 |
| *Add* | | |
| Interest paid on mortgage 5 % on $1,000 | | 50 |
| Personal and household expenses paid in cash | | 750 |
| Supplies furnished by farm for household | | 363 |
| Rent of farmhouse | | 120 |
| Gross farm gain | | 1,771 40 |
| *Deduct* | | |
| Interest on net investment at 5 % | 932 37 | |
| Labor of owner and family (estimated) | 600 | |
| Total deduction | | 1,532 37 |
| True net gain | | 239 03 |

### Written Work

From the following facts prepare a statement similar to the preceding illustration.

Condition of farmer's affairs on March 1, 1922.

He owns a farm of 160 acres, worth $225 per acre, on which he has given a mortgage of $6,000, bearing 5 % interest. Cash on hand and in bank, $345.85. His live stock is worth $5,280; his poultry, $286; his machinery, $1,250; unsold crops, $900. He has just sold a car of hogs to a local stock buyer, who owes him $975 for them. He also has

on hand fertilizer, lumber, and other supplies worth $387.50. He owes sundry accounts, amounting to $236.25.

During the year between March 1, 1921 and March 1, 1922, he paid the interest on his mortgage, and also paid $1,000 on the principal of the mortgage. He paid by cash and by butter, eggs, and other produce, for household expenses, $596.50. He estimated the rent of his house at $15 per month; the produce of the farm consumed by the family, $645; and his wages, $900. He paid taxes and insurance, $275. Compute interest on net investment at the beginning of the year at 5%.

On March 1, 1922, he had, cash $467.20; live stock, $6,140; poultry, $270; machinery, same as previous year, less 10% depreciation, plus $165 worth of new machinery; crops on hand, $650; miscellaneous supplies, $295. He owes personal accounts amounting to $193.70 and the unpaid balance of the mortgage.

**Finding the Profits by Crops.** In order to determine the profit from any particular crop, the farmer must know:

*a.* The value of the crop produced;

*b.* The cost of producing the crop, including seed, fertilizer, labor, and a share of the taxes, interest, and general expenses.

**Determining the Expenses.** The cost of seed and fertilizer may be determined with comparative ease, because measurable quantities are put on each field.

*Taxes, interest, and general expenses* are prorated among the different crops in the ratio of the land occupied.

**Problem.** A farmer who has 40 acres of hay, 60 acres of corn, and 30 acres of oats under cultivation, finds that his total interest, taxes, and expenses are $1,200. How should this be prorated?

SOLUTION.

| | |
|---|---|
| $1,200 ÷ 130 = $9.23 | Charge per acre. |
| 40 × $9.23 = $369.20 | Share for hay crop. |
| 60 × $9.23 = $553.80 | Share for corn crop. |
| 30 × $9.23 = $276.90 | Share for oat crop. |

*Labor of men and horses* must be determined by records of the actual labor spent on each crop.

**Finding the Cost of Man Labor and Horse Labor per Hour.** At the end of a month or other period of time, the total number of

man hours and horse hours can be determined from the records. The cost per hour may be found by the method illustrated below:

Cost of man labor per year

| | |
|---|---|
| Wages, | $625.00 |
| Board, | 200.00 |
| Total, | $825.00 |

Hours worked during year, 2,500.

$825.00 ÷ 2,500 = $.33 labor cost per man hour.

Cost of a horse hour is determined in a similar way.

| | |
|---|---|
| Total value of 2 horses, $400.00 | |
| Int. on $600.00 at 6 %, | $36.00 |
| Feed and expense, | 450.00 |
| Total cost per year, | $486.00 |

Total number of horse labor hours, 6,000.

$486.00 ÷ 6,000 = 8.1 cents, labor cost per horse hour.

After the labor cost records have been made, the profit or loss from the crops may be determined.

The labor records show the number of horse hours and man hours spent on each crop. After determining the hourly rate, it is necessary to multiply as follows:

The number of man hours by the rate per man hour; the number of horse hours by the rate per horse hour; to determine the total labor cost of each.

The costs and returns of each crop may then be summarized, and the gain computed, as illustrated in the table on page 323.

**Written Work.**

1. Find the labor cost to be charged against a corn crop, in the production of which 136 man hours and 312 horse hours of labor were expended. The total number of hours of man labor for the year was 4,824, and the annual cost was $1206; total number of hours of horse labor, 4,312; total cost of horse labor for the year, $388.08.

2. Use the facts on page 323 to prepare a table similar to the model on page 323.

CORN CROP ON SOUTH 40

| ITEM | | 40 ACRES | PER ACRE |
|---|---|---|---|
| Seed | | $24.00 | .60 |
| Fertilizer | | 70.00 | 1.75 |
| Taxes | | 28.30 | .7075 |
| Interest | | 450.00 | 11.25 |
| Expenses | | 40.00 | 1.00 |
| Man labor, | | 109.01 | 2.725 |
| Horse labor, | | 228.60 | 5.715 |
| Total cost | | 949.91 | 23.7475 |
| Yield, 2552 bu. corn at $.48 | $1,224.96 | | |
| Stalks, 75¢ per acre | 30.00 | | |
| Total returns | | 1254.96 | 31.374 |
| | | 949.91 | 23.7475 |
| Gain | | 305.05 | 7.6265 |

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## CORN CROP ON 20 ACRES

Yield: 10 bushels of seed corn worth $2.50 per bushel.
Corn crop harvested, 67 bushels per acre, worth 54 cents per bushel.
Stalks were estimated at a value of 60 cents per acre.

Costs: 3½ bushels of seed at $2.60 per bushel.
Fertilizer, $38.00.
Taxes: The farm was worth $175.00 per acre, and was taxed on ⅓ the real value at a rate of $1.65. The farm contained 160 acres, and the taxes were prorated among the various fields in proportion to their acreage.
Interest: 6% on the value of the land.
Expenses: $31.25.
Labor: 560 man hours at 22 cents.
1,348 horse hours at 9 cents.

*Sample farm inventory: Farm of ——*

| Property | April 1, 1921 | | | | April 1, 1922 | | | |
|---|---|---|---|---|---|---|---|---|
| | No. | Rate | Valuation | | No. | Rate | Valuation | |
| REAL ESTATE | | | | | | | | |
| Farm of 180 acres (155 tillable), including buildings (dwelling $1,600, barns $1,800, other buildings $600), fences, and other improvements | | | | $13,500.00 | | | | $13,500.00 |
| LIVE STOCK | | | | | | | | |
| Dairy cattle: | | | | | | | | |
| Cows, dry and in milk | 24 | $50.00 | $1,200.00 | | 26 | $50.00 | $1,300.00 | |
| Bull | 1 | | 50.00 | | 1 | | 45.00 | |
| Calves | 6 | 14.00 | 84.00 | | 8 | 15.00 | 120.00 | |
| Two-year-olds | 4 | 28.00 | 112.00 | | 6 | 20.00 | 120.00 | |
| Total value of dairy cattle | | | | 1,446.00 | | | | 1,585.00 |
| Hogs: | | | | | | | | |
| Brood sows | 2 | 22.00 | 44.00 | | 2 | 21.00 | 42.00 | |
| Pigs | 8 | 4.00 | 32.00 | | 6 | 3.00 | 18.00 | |
| Total value of hogs | | | | 76.00 | | | | 60.00 |
| Horses: | | | | | | | | |
| Horse, Jim, 7 years old | 1 | | 120.00 | | 1 | | 100.00 | |
| Team, Nell and Bess, 5 and 6 years old | 1 | | 325.00 | | 1 | | 325.00 | |
| Team, Jack and Prince, 6 and 7 years old | 1 | | 300.00 | | 1 | | 300.00 | |
| Colt, 1 year old | 1 | | 75 00 | | 1 | | 135.00 | |
| Total value of horses | | | | 820.00 | | | | 860 00 |
| Poultry: | | | | | | | | |
| Hens | 160 | .60 | 96.00 | | 125 | .60 | 75.00 | |
| Roosters | 5 | 1.00 | 5.00 | | 4 | 1.00 | 4.00 | |
| Turkeys | 2 | 3.00 | 6.00 | | 3 | 3.00 | 9.00 | |
| Total value of poultry | | | | 107.00 | | | | 88.00 |
| Total value of live stock | | | | 2,449.00 | | | | 2,593.00 |
| MACHINERY AND TOOLS | | | | | | | | |
| Grain binder | 1 | | 90.00 | | 1 | | 82.00 | |
| Sulky plow | 2 | 45.00 | 90.00 | | 2 | 41.00 | 82.00 | |
| Disk harrow | 2 | 28.00 | 56.00 | | 2 | 25.00 | 50.00 | |
| Mower | 1 | | 35.00 | | 1 | | 30.00 | |
| Hay rake | 1 | | 20.00 | | 1 | | 19.00 | |
| (List all items of farm machines, wagons, harness, and small tools.) | | | | | | | | |
| Total investment in machinery and tools (not all listed here) | | | | 475.00 | | | | 461.00 |

*Sample farm inventory: Farm of* —— (Continued)

| Property | April 1, 1921 | | | | April 1, 1922 | | | |
|---|---|---|---|---|---|---|---|---|
| | No. | Rate | Valuation | | No. | Rate | Valuation | |
| **FEED AND SUPPLIES** | | | | | | | | |
| Farm products: | | | | | | | | |
| Corn.....bushels | 80 | .60 | 48.00 | | 125 | .60 | 75.00 | |
| Oats..... " | 200 | .42 | 84.00 | | 90 | .50 | 45.00 | |
| Potatoes..... " | 40 | .75 | 30.00 | | 80 | .60 | 48.00 | |
| Hay, timothy.... tons | 10 | 16.00 | 160.00 | | 20 | 15.00 | 300.00 | |
| Hay, mixed..... " | 5 | 12.00 | 60.00 | | 4 | 12.00 | 48.00 | |
| Silage..... " | 40 | 4.00 | 160.00 | | 40 | 4.00 | 160.00 | |
| Bran..... " | 0½ | ..... | 15.00 | | ..... | ..... | ..... | ..... |
| Mixed feed..... " | 1 | ..... | 31.00 | | 2½ | 30.00 | 75.00 | |
| Seed oats.....bushels | 30 | .80 | 24.00 | | 35 | .80 | 28.00 | |
| Seed potatoes.... " | 45 | .80 | 36.00 | | 50 | 1.00 | 50.00 | |
| Seed corn..... " | 3 | 2.00 | 6.00 | | 3 | 2.00 | 6.00 | |
| Cement..... sacks | 4 | .50 | 2.00 | | ..... | ..... | ..... | |
| Twine.....pounds | 20 | .10 | 2.00 | | 10 | .10 | 1.00 | |
| Total value of feed and supplies..... | ..... | ..... | ..... | 658.00 | ..... | ..... | ..... | 836.00 |
| **BILLS RECEIVABLE** | | | | | | | | |
| J. A. Brown, hay... tons | 2 | 13.00 | 26.00 | | | | | |
| R.S.Jones, potatoes, bushels | 40 | .50 | 20.00 | | | | | |
| Total..... | ..... | ..... | ..... | 46.00 | | | | |
| **CASH** | | | | | | | | |
| On hand..... | ..... | ..... | 90.00 | | ..... | ..... | 210.00 | |
| In bank..... | ..... | ..... | 580.00 | | ..... | ..... | 1,938.00 | |
| Total..... | ..... | ..... | ..... | 670.00 | ..... | ..... | ..... | 2,148.00 |
| **BILLS PAYABLE** | | | | | | | | |
| Farm mortgage ..... | ..... | ..... | ..... | 2,000.00 | ..... | ..... | ..... | 1,500 00 |
| **SUMMARY** | | | | | | | | |
| Real estate..... | ..... | ..... | 13,500.00 | | ..... | ..... | 13,500.00 | |
| Live stock..... | ..... | ..... | 2,449.00 | | ..... | ..... | 2,593.00 | |
| Machinery and tools..... | ..... | ..... | 475.00 | | ..... | ..... | 461.00 | |
| Feed and supplies..... | ..... | ..... | 658.00 | | ..... | ..... | 836.00 | |
| Bills receivable..... | ..... | ..... | 46.00 | | ..... | ..... | ..... | |
| Cash on hand and in bank. | ..... | ..... | 670.00 | | ..... | ..... | 2,148.00 | |
| Total investment..... | ..... | ..... | ..... | 17,798.00 | ..... | ..... | ..... | 19,538.00 |
| Bills Payable..... | ..... | ..... | ..... | 2,000.00 | ..... | ..... | ..... | 1,500.00 |
| Net Worth..... | ..... | ..... | ..... | 15,798.00 | ..... | ..... | ..... | 18,038.00 |
| Increase in inventory $2,240. | | | | | | | | |

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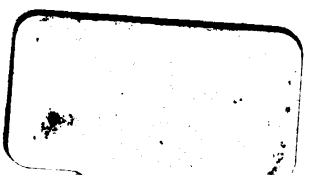
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